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New Exact Solutions for the MNV Equation

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Abstract: By the extended-$tanh$ method , we establish exact traveling wave solutions of the Mizhnik-Novikov-Veselov (MNV) equation. The obtained solutions include solitary wave solutions and periodic wave solutions and combined formal solutions.

Key words: extended-$tanh$ method; MNV equation; solitary wave solutions; nonlinear equations

1 Introduction

The nonlinear partial differential equations are widely used to describe many important phenomena and dynamic processes in physics, mechanics, chemistry, chemical kinetics, geochemistry and biology. In order to well understand various nonlinear phenomena, many methods for obtaining analytical solutions of nonlinear evolution equations have been proposed, among them are the inverse scattering method [1,2], the Hirota's method, the Backlund transformation [3], the Darboux transformation method [4], the F-expansion method [5], the Riccati equation [6], the sine-cosine method [7], the Adomian decomposition method [8], the variational iteration method [9] and the homotopy perturbation method [10], and so on. Relative reseaches canbe found in [11,12].

The Mizhnik-Novikov-Veselov (MNV) equation [13] is given by

$$u_t + u_{xxx} + u_{yyy} + 3(u\partial_y^{-1}u_x)_x + 3(u\partial_x^{-1}u_y)_y = 0 \tag{1}$$

M. Lakshmanan and P. Kaliappan in [13] presented the results of a systematic investigation of invariance properties in a large class of nonlinear evolution equations under the one-parameter continuous (Lie) group of transformations.

The rest of the paper is organized as follows. In Section 2, we briefly describe the extended-$tanh$ method [14]. In Section 3, we apply this method to Eq. (1). Conclusions will be finally presented.

2 The extended-*tanh* method

A PDE

$$P(u, u_t, u_{xx}, u_{xxx}, ...) = 0$$

can be converted to an ODE

$$Q(u, u', u'', u''', ...) = 0 \tag{2}$$

by using a wave variable $\xi = (x - ct)$. Eq.(2) is then integrated as long as terms contain derivatives where integration constants are considered zeros.

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In the $tanh$ method developed by Malfliet in [15], an independent variable

$$Y = \tanh(\mu\xi), \quad \xi = x - ct$$

is introduced which leads to the change of derivatives:

$$\frac{d}{d\xi} = \mu(1 - Y^2)\frac{d}{dY}$$
$$\frac{d^2}{d\xi^2} = -2\mu^2 Y(1 - Y^2)\frac{d}{dY} + \mu^2(1 - Y^2)^2\frac{d^2}{dY^2}$$

The extended-$tanh$ method admits the use of the finite expansion

$$u(\mu\xi) = S(Y) = \sum_{i=-N}^{N} a_i Y^i \quad , \tag{3}$$

where N is a positive integer, in most cases, that will be determined. Expansion (3) reduces to the standard $tanh$ method [16,17] for $a_i = 0$, $-N \le i \le -1$.The parameter N is usually obtained by balancing the linear terms of highest order in the resulting equation with the highest order nonlinear terms. Substituting (3) into (2) results in an algebraic system of equations in powers of Y that will lead to the determination of the parameters $a_i, (i = -N, \ldots, N)$ μ, and c.

The function $Y = Y(\xi)$ satisfies the Riccati equation

$$\frac{dY}{d\xi} = b + Y^2$$

where b is a constant.

3 Mizhnik-Novikov-Veselov equation

The Mizhnik-Novikov-Veselov equation is

$$u_t + u_{xxx} + u_{yyy} + 3(u\partial_y^{-1}u_x)_x + 3(u\partial_x^{-1}u_y)_y = 0$$

To obtain traveling wave solutions of Eq. (1) we will set

$$u(x, y, t) = u(\xi), \quad \xi = x + \alpha y + \lambda t \tag{4}$$

where α and λ are constants to be determined later.

Thus

$$u_t = \lambda u_\xi, \quad u_{xxx} = u_{\xi\xi\xi}, \quad u_{yyy} = \alpha^3 u_{\xi\xi\xi}$$
$$\partial_y^{-1}u_x = \partial_y^{-1}u_\xi = \frac{1}{\alpha}\partial_\xi^{-1}u_\xi = \frac{1}{\alpha}u,$$
$$\partial_x^{-1}u_y = \partial_\xi^{-1}(\alpha u_\xi) = \alpha u.$$

Substituting (4) into (1) ,we obtain

$$\alpha\lambda u_\xi + \alpha u_{\xi\xi\xi} + \alpha^4 u_{\xi\xi\xi} + 3(u^2)_\xi + 3\alpha^3(u^2)_\xi = 0. \tag{5}$$

Integrating Eq.(5) and setting the constant of integration to zero, we give

$$\alpha\lambda u + \alpha u_{\xi\xi} + \alpha^4 u_{\xi\xi} + 3u^2 + 3\alpha^3 u^2 = 0 \tag{6}$$

Balancing the order of $u_{\xi\xi}$ with the order of u^2 in Eq.(6) gives

$$2N = N + 2$$

so that

$$N = 2$$

Using the extended $tanh$ expansion (3) we set

$$u(\xi) = a_2 Y^2 + a_1 Y + a_0 + a_{_{-1}} Y^{-1} + a_{-2} Y^{-2}, \tag{7}$$

so that

$$u_\xi = \frac{du}{dY}\frac{dY}{d\xi} = -\frac{2ba_{-2}}{Y^3} - \frac{2a_{-2}}{Y} - a_{-1} - \frac{ba_{-1}}{Y^2} + ba_1 + Y^2 a_1 + 2bY a_2 + 2Y^3 a_2$$

$$u_{\xi\xi} = 2a_{_{-2}} + \frac{6b^2 a_{-2}}{Y^4} + \frac{8ba_{-2}}{Y^2} + \frac{2b^2 a_{-1}}{Y^3} + \frac{2ba_{-1}}{y} + 2bY a_1 + 2Y^3 a_1 + 2b^2 a_2 + 8bY^2 a_2 + 6Y^4 a_2$$

Substituting (7) into (6), we obtain a system of algebraic equations, for $a_2, a_1, a_0, a_{-1}, a_{-2}, \alpha$ and λ as the following form:

$$6b^2\alpha a_{-2} + 6b^2\alpha^4 a_{-2} + 3a_{-2}^2 + 3\alpha^3 a_{-2}^2 = 0$$

$$2b^2\alpha a_{-1} + 2b^2\alpha^4 a_{-1} + 6a_{-2}a_{-1} + 6\alpha^3 a_{-2}a_{-1} = 0$$

$$8b\alpha a_{-2} + 8b\alpha^4 a_{-2} + \alpha\lambda a_{-2} + 3a_{-1}^2 + 3\alpha^3 a_{-1}^2 + 6a_{-2}a_0 + 6\alpha^3 a_{-2}a_0 = 0$$

$$2b\alpha a_{-1} + 2b\alpha^4 a_{-1} + \alpha\lambda a_{-1} + 6a_{-1}a_0 + 6\alpha^3 a_{-1}a_0 + 6a_{-2}a_1 + 6\alpha^3 a_{-2}a_1 = 0$$

$$2\alpha a_{-2} + 2\alpha^4 a_{-2} + \alpha\lambda a_0 + 3a_0^2 + 3\alpha^3 a_0^2 + 6a_{-1}a_1 + 6\alpha^3 a_{-1}a_1 + 2b^2\alpha a_2 + 2b^2\alpha^4 a_2 + 6a_{-2}a_2 + 6\alpha^3 a_{-2}a_2 = 0$$

$$2b\alpha a_1 + 2b\alpha^4 a_1 + \alpha\lambda a_1 + 6a_0 a_1 + 6\alpha^3 a_0 a_1 + 6a_{-1}a_2 + 6\alpha^3 a_{-1}a_2 = 0$$

$$3a_1^2 + 3\alpha^3 a_1^2 + 8b\alpha a_2 + 8b\alpha^4 a_2 + \alpha\lambda a_2 + 6a_0 a_2 + 6\alpha^3 a_0 a_2 = 0$$

$$2\alpha a_1 + 2\alpha^4 a_1 + 6a_1 a_2 + 6\alpha^3 a_1 a_2 = 0$$

$$6\alpha a_2 + 6\alpha^4 a_2 + 3a_2^2 + 3\alpha^3 a_2^2 = 0$$

Solving the system of the algebraic equations with the aid of Mathematica we can distinguish six cases namely

(i) The first set of parameters is given by

$$\lambda = -4(b + b\alpha^3), \quad a_{-2} = -2b^2\alpha, \quad a_0 = -\frac{2b\alpha}{3}, \quad a_2 = 0, \quad a_{-1} = 0, \quad a_1 = 0, \tag{8}$$

(ii) The second set of parameters is given by

$$\lambda = 4(b + b\alpha^3), \quad a_{-2} = -2b^2\alpha, \quad a_0 = -2b\alpha, \quad a_2 = 0, \quad a_{-1} = 0, \quad a_1 = 0, \tag{9}$$

(iii) The third set of parameters is given by

$$\lambda = -16(b + b\alpha^3), \quad a_{-2} = -2b^2\alpha, \quad a_0 = \frac{4b\alpha}{3}, \quad a_2 = -2\alpha, \quad a_{-1} = 0, \quad a_1 = 0, \tag{10}$$

(iv) The fourth set of parameters is given by

$$\lambda = -4(b + b\alpha^3), \quad a_{-2} = 0, \quad a_0 = -\frac{2b\alpha}{3}, \quad a_2 = -2\alpha, \quad a_{-1} = 0, \quad a_1 = 0, \tag{11}$$

(v) The fifth set of parameters is given by

$$\lambda = 4(b + b\alpha^3), \quad a_{-2} = 0, \quad a_0 = -2b\alpha, \quad a_2 = -2\alpha, \quad a_{-1} = 0, \quad a_1 = 0, \tag{12}$$

(vi) The sixth set of parameters is given by

$$\lambda = 16(b + b\alpha^3), \quad a_{-2} = -2b^2\alpha, \quad a_0 = -4b\alpha, \quad a_2 = -2\alpha, \quad a_{-1} = 0, \quad a_1 = 0, \tag{13}$$

For $b < 0$, from (8)-(13) we obtain the following twelve solitary wave solutions

$$u_1(x, y, t) = -\frac{2b\alpha}{3} + 2b^2\alpha\sqrt{-b}\coth^2\sqrt{-b}[x + \alpha y - 4(b + b\alpha^3)t]$$

$$u_2(x, y, t) = -\frac{2b\alpha}{3} + 2b^2\alpha\sqrt{-b}\tanh^2\sqrt{-b}[x + \alpha y - 4(b + b\alpha^3)t]$$

$$u_3(x, y, t) = -2b\alpha + 2b^2\alpha\sqrt{-b}\coth^2\sqrt{-b}[x + \alpha y + 4(b + b\alpha^3)t]$$

$$u_4(x, y, t) = -2b\alpha + 2b^2\alpha\sqrt{-b}\tanh^2\sqrt{-b}[x + \alpha y + 4(b + b\alpha^3)t]$$

$$u_5(x, y, t) = 2b^2\alpha\sqrt{-b}\coth^2\sqrt{-b}[x+\alpha y-16(b+b\alpha^3)t]+\frac{4b\alpha}{3}+2\alpha\sqrt{-b}\tanh^2\sqrt{-b}[x+\alpha y-16(b+b\alpha^3)t]$$

$$u_6(x, y, t) = 2b^2\alpha\sqrt{-b}\tanh^2\sqrt{-b}[x+\alpha y-16(b+b\alpha^3)t]+\frac{4b\alpha}{3}+2\alpha\sqrt{-b}\coth^2\sqrt{-b}[x+\alpha y-16(b+b\alpha^3)t]$$

$$u_7(x, y, t) = -\frac{2b\alpha}{3} + 2\alpha\sqrt{-b}\tanh^2\sqrt{-b}[x + \alpha y - 4(b + b\alpha^3)t]$$

$$u_8(x, y, t) = -\frac{2b\alpha}{3} + 2\alpha\sqrt{-b}\coth^2\sqrt{-b}[x + \alpha y - 4(b + b\alpha^3)t]$$

$$u_9(x, y, t) = -2b\alpha + 2\alpha\sqrt{-b}\tanh^2\sqrt{-b}[x + \alpha y + 4(b + b\alpha^3)t]$$

$$u_{10}(x, y, t) = -2b\alpha + 2\alpha\sqrt{-b}\coth^2\sqrt{-b}[x + \alpha y + 4(b + b\alpha^3)t]$$

$$u_{11}(x, y, t) = 2b^2\alpha\sqrt{-b}\coth^2\sqrt{-b}[x+\alpha y+16(b+b\alpha^3)t]-4b\alpha+2\alpha\sqrt{-b}\tanh^2\sqrt{-b}[x+\alpha y+16(b+b\alpha^3)t]$$

$$u_{12}(x, y, t) = 2b^2\alpha\sqrt{-b}\tanh^2\sqrt{-b}[x+\alpha y+16(b+b\alpha^3)t]-4b\alpha+2\alpha\sqrt{-b}\coth^2\sqrt{-b}[x+\alpha y+16(b+b\alpha^3)t]$$

For $b > 0$, from (8)-(13) we obtain the following periodic solutions

$$u_{13}(x, y, t) = -\frac{2b\alpha}{3} - 2b^2\alpha\sqrt{b}\cot^2\sqrt{b}[x + \alpha y - 4(b + b\alpha^3)t]$$

$$u_{14}(x, y, t) = -\frac{2b\alpha}{3} + 2b^2\alpha\sqrt{b}\tan^2\sqrt{b}[x + \alpha y - 4(b + b\alpha^3)t]$$

$$u_{15}(x, y, t) = -2b\alpha - 2b^2\alpha\sqrt{b}\cot^2\sqrt{b}[x + \alpha y + 4(b + b\alpha^3)t]$$

$$u_{16}(x, y, t) = -2b\alpha + 2b^2\alpha\sqrt{b}\tan^2\sqrt{b}[x + \alpha y + 4(b + b\alpha^3)t]$$

$$u_{17}(x, y, t) = -2b^2\alpha\sqrt{b}\cot^2\sqrt{b}[x + \alpha y - 16(b + b\alpha^3)t] + \frac{4b\alpha}{3} - 2\alpha\sqrt{b}\tan^2\sqrt{b}[x + \alpha y - 16(b + b\alpha^3)t]$$

$$u_{18}(x, y, t) = 2b^2\alpha\sqrt{b}\tan^2\sqrt{b}[x + \alpha y - 16(b + b\alpha^3)t] + \frac{4b\alpha}{3} + 2\alpha\sqrt{b}\cot^2\sqrt{b}[x + \alpha y - 16(b + b\alpha^3)t]$$

$$u_{19}(x,y,t) = -\frac{2b\alpha}{3} - 2\alpha\sqrt{b}\tan^2\sqrt{b}[x+\alpha y-4(b+b\alpha^3)t]$$

$$u_{20}(x,y,t) = -\frac{2b\alpha}{3} + 2\alpha\sqrt{b}\cot^2\sqrt{b}[x+\alpha y-4(b+b\alpha^3)t]$$

$$u_{21}(x,y,t) = -2b\alpha - 2\alpha\sqrt{b}\tan^2\sqrt{b}[x+\alpha y+4(b+b\alpha^3)t]$$

$$u_{22}(x,y,t) = -2b\alpha + 2\alpha\sqrt{b}\cot^2\sqrt{b}[x+\alpha y+4(b+b\alpha^3)t]$$

$$u_{23}(x,y,t) = -2b^2\alpha\sqrt{b}\cot^2\sqrt{b}[x+\alpha y+16(b+b\alpha^3)t] - 4b\alpha - 2\alpha\sqrt{b}\tan^2\sqrt{b}[x+\alpha y+16(b+b\alpha^3)t]$$

$$u_{24}(x,y,t) = 2b^2\alpha\sqrt{b}\tan^2\sqrt{b}[x+\alpha y+16(b+b\alpha^3)t] - 4b\alpha + 2\alpha\sqrt{b}\cot^2\sqrt{b}[x+\alpha y+16(b+b\alpha^3)t]$$

For $b=0$, the MNV equation has the following rational solution

$$u_{25}(x,y,t) = -2\alpha\frac{1}{(x+\alpha y)^2}$$

4 Conclusions

The extended-$tanh$ method was applied to conduct an analytic study for the MNV equation. We derive abundant solitary wave solutions and periodic solutions for the MNV equation. The work also confirms the power of the extended-$tanh$ method in handling nonlinear equations in general.

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