

Controlling and Tracking of Newton–Leipnik System via Backstepping Design

Xuedi Wang *, Chao Ge

Nonlinear Scientific Research Center, Faculty of Science, Jiangsu University
Zhenjiang, Jiangsu, 212013, P.R. China

(Received 10 November 2007, accepted 11 January 2008)

Abstract: This paper firstly introduces the chaotic system (Newton–Leipnik system) which possesses two strange attractors. And then the backstepping design control approach is used to control Newton–Leipnik system to a steady state as well as tracking of a desire trajectory. The proposed controller is obtained by a systematic design approach and consists in recursive procedures that interlace the choice of a Lyapunov function according to the design of active control. Numerical simulations are provided to verify the feasibility and effectiveness, so the result of the control is mutually verified with the theoretical analyses and numerical simulations.

Key words: controlling; tracking; backstepping design

1 Introduction

Since the seminal work of Ott, Grebogi and Yorke (OGY)[1], there has been an increasing interest in the study of controlling chaotic systems in physics, mathematics and engineering community, etc. in recent years. Different techniques and methods[2-7] have been proposed over the last decade to achieve the control, stabilization and synchronization of chaotic systems. Moreover, some of these methods have been successfully applied to experimental systems. Therefore, controlling chaos and synchronization have been very important goals and subjects of current researches.

In recent years, backstepping design[8-9] and active control [10-13] have been widely recognized as two powerful design methods to control and synchronize chaos. Active control technique gives the flexibility to construct a control law so that it can be used widely to control and synchronize various nonlinear systems, including chaotic systems.

In this paper, Newton–Leipnik system is controlled with backstepping design method. At the same time, we use the same method to enable stabilization of chaotic motion to a steady state as well as tracking of a desired trajectory to be achieved in a systematic way. Computer simulation is also given for the purpose of illustration and verification.

2 Controlling Newton–Leipnik system

The Newton–Leipnik system is described by

$$\begin{cases} \dot{x} = -ax + y + 10yz \\ \dot{y} = -x - 0.4y + 5xz \\ \dot{z} = bz - 5xy \end{cases} \quad (1)$$

*Corresponding author. E-mail address: wxd959@ujs.edu.cn

where a, b are positive parameters. The Newton–Leipnik system is a chaotic system with two strange attractors. For the system parameter $a = 0.4$, $b = 0.175$, and initial states $(0.349, 0, -0.160)$ and $(0.349, 0, -0.180)$, we can obtain the two strange attractors which are demonstrated in Fig.1 and Fig.2.

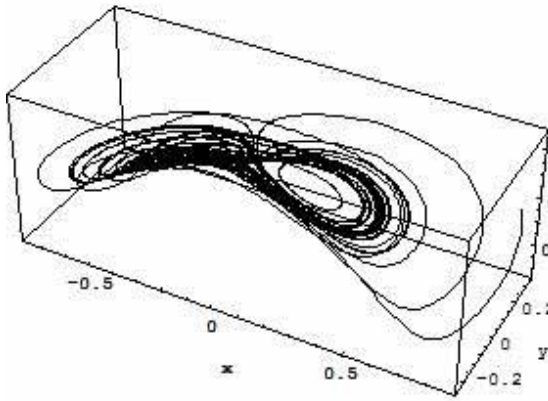


Figure 1: Chaotic attractor of Newton-Leipnik system, the initial point in $(0.349, 0, -0.160)$.

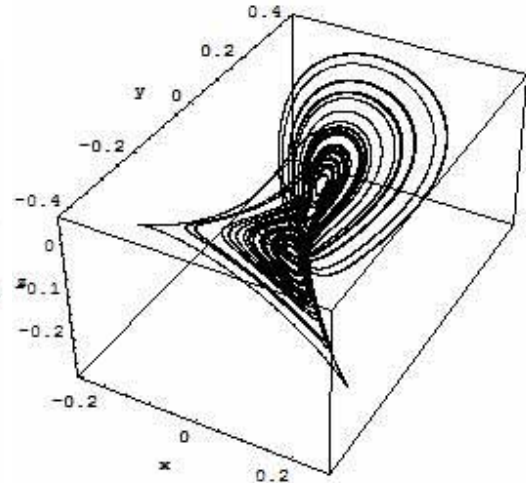


Figure 2: Chaotic attractor of Newton-Leipnik system, the initial point in $(0.349, 0, -0.180)$.

We will use backstepping design to construct a controller. In order to control Newton–Leipnik system, we add a control input u_1 to the third equation of system (1). Then the controlled Newton–Leipnik system is

$$\begin{cases} \dot{x} = -ax + y + 10yz \\ \dot{y} = -x - 0.4y + 5xz \\ \dot{z} = bz - 5xy + u_1 \end{cases} \quad (2)$$

Our objective is to find a control law u_1 for stabilizing the state of the controlled system (2) at a bounded point.

Starting from the first equation, a stabilizing function has to be designed for the virtual control y in order to make the derivative of $V_1 = \frac{x^2}{2}$, i.e., $\dot{V}_1(x) = -ax^2 + xy + 10xyz$ be negative definite. Assume that $\alpha_1(x) = 0$, and define an error variable

$$\bar{y} = y - \alpha_1(x) \quad (3)$$

Then we obtain the (x, \bar{y}) -subsystem

$$\begin{cases} \dot{x} = -ax + \bar{y} + 10\bar{y}z \\ \dot{\bar{y}} = -x - 0.4\bar{y} + 5xz \end{cases} \quad (4)$$

We can construct a Lyapunov function as follows:

$$V_2(x, \bar{y}) = V_1(x) + \frac{1}{2}\bar{y}^2$$

Calculating the time derivative of $V_2(x, \bar{y})$ along with system (4), we have

$$\dot{V}_2 = -ax^2 - 0.4\bar{y}^2 + 15zxy$$

We can choose $z = \alpha_2(x, \bar{y}) = 0$

Apparently, \dot{V}_2 is negative definite. Similarly, let

$$\bar{z} = z - \alpha_2(x, \bar{y}) \quad (5)$$

Then we get the following system in the (x, \bar{y}, \bar{z}) coordinates

$$\begin{cases} \dot{x} = -ax + \bar{y} + 10\bar{y}\bar{z} \\ \dot{\bar{y}} = -x - 0.4\bar{y} + 5x\bar{z} \\ \dot{\bar{z}} = b\bar{z} - 5x\bar{y} + u_1 \end{cases} \quad (6)$$

We can construct a Lyapunov function as follows:

$$V_3(x, \bar{y}, \bar{z}) = V_2(x, \bar{y}) + \frac{1}{2}\bar{z}^2$$

Calculating the time derivative of $V_3 = (x, \bar{y}, \bar{z})$ along with system (2), we have

$$\dot{V}_3 = -ax^2 - 0.4\bar{y}^2 + (u_1 + 5x\bar{y} + b\bar{z})\bar{z}$$

It is clear that \dot{V}_3 becomes negative definite by choosing the control input u_1 as follows:

$$u_1 = -5x\bar{y} - b\bar{z} \tag{7}$$

Therefore we have proved that the system (2) has been stabilized at the origin point $(0, 0, 0)$. The system (2) has been stabilized at the point $(0, 0, \alpha_2)$.

In order to control Newton–Leipnik to the origin point $(0, 0, 0)$, we add a control input u_2 to the second equation of system (1). Thus the controlled system becomes:

$$\begin{cases} \dot{x} = -ax + y + 10yz \\ \dot{y} = -x - 0.4y + 5xz + u_2 \\ \dot{z} = bz - 5xy \end{cases} \tag{8}$$

For the virtual control y , we design a stabilizing function $\alpha_1(x)$ to make the derivative of $V_1(x) = x^2/2$, i.e. $\dot{v}_1(x) = -ax^2 + xy + 10xyz$ be negative definite as $y = \alpha_1(x)$. We can choose $\alpha_1(x) = 0$ and define an error variable

$$\bar{y} = y - \alpha_1(x) \tag{9}$$

Then we can obtain the following (x, \bar{y}) -subsystem:

$$\begin{cases} \dot{x} = -ax + \bar{y} + 10\bar{y}z \\ \dot{\bar{y}} = -x - 0.4\bar{y} + 5xz + u_2 \end{cases} \tag{10}$$

We can construct a Lyapunov function as follows:

$$V_2(x, \bar{y}) = V_1(x) + \frac{1}{2}\bar{y}^2$$

Calculating the time derivative of $V_2(x, \bar{y})$ along with system (6), we have

$$\dot{V}_2 = -ax^2 - 0.4\bar{y}^2 + (u_2 + 15xz)\bar{y} \tag{11}$$

In order to make (7) negative definite, choose

$$u_2 = -15xz$$

Therefore we have proved that in the (x, \bar{y}) coordinates the equilibrium $(0, 0)$ of the subsystem (6) is asymptotically stable. According to (5), $\alpha_1(x) = 0$, $x \rightarrow 0$, $\bar{y} \rightarrow 0$, and the third equation of system (4), we get that (x, y, z) in controlled system (4) tends to $(0, 0, 0)$ as $t \rightarrow \infty$ when choosing the control input $u_2 = -15xz$.

3 Tracking any desired trajectory

In this section, we will find a control law $u(t)$ so that a scalar output $x(t)$ of Newton–Leipnik system can track the desired trajectory $r(t) = \sin(t)$.

Step 1. Assume that $\bar{x} = x - r$, then we can obtain its derivative

$$\dot{\bar{x}} = -a(\bar{x} + r) + y + 10yz + u_1 - \dot{r} \tag{12}$$

Where $y = \alpha_1(\bar{x})$ is regarded as a virtual control input.

For the design of $\alpha_1(\bar{x})$ to stabilize \bar{x} -subsystem (7), we can choose the Lyapunov function

$$V_1 = \frac{\bar{x}^2}{2}$$

The derivative of V_1 is

$$\dot{V}_1 = -a\bar{x}_2 + (-ar + \alpha_1 + 10\alpha_1 z + u_1 - \dot{r})\bar{x}$$

Here we choose

$$\alpha_1 = 0 \text{ and } u_1 = ar + \dot{r} \quad (13)$$

Then the derivative \dot{V}_1 is negative definite. This implies that the \bar{x} -subsystem (7) is asymptotically stable. Since the virtual control function $\alpha_1(\bar{x})$ is estimative, define an error variable

$$\bar{y} = y - \alpha_1$$

We can obtain the following (\bar{x}, \bar{y}) -subsystem:

$$\begin{cases} \dot{\bar{x}} = -a\bar{x} + \bar{y} + 10\bar{y}z \\ \dot{\bar{y}} = -(\bar{x} + r) - 0.4\bar{y} + 5(\bar{x} + r)z + u_2 \end{cases} \quad (14)$$

Where $z = \alpha_2(\bar{x}, \bar{y})$ is regarded as a virtual controller.

Step2. In order to stabilize the (\bar{x}, \bar{y}) -subsystem (9), we can choose a Lyapunov function V_2 as follows

$$V_2 = V_1 + \frac{1}{2}\bar{y}_2$$

Its derivative is given by

$$\dot{V}_2 = -a\bar{x}_2 - 0.4\bar{y}_2^2 + (u_2 - r)\bar{y} + (15\bar{x} + 5\bar{y})\bar{y}\alpha_2 \quad (15)$$

If $\alpha_2 = 0$ and $u_2 = r$ then $\dot{V}_2 = -a\bar{x}_2 - 0.4\bar{y}_2^2 < 0$ makes (\bar{x}, \bar{y}) -subsystem (9) is asymptotically stable. Similarly, assume that $\bar{z} = z - \alpha_2$

$$\begin{cases} \dot{\bar{x}} = -a\bar{x} + \bar{y} + 10\bar{y}\bar{z} \\ \dot{\bar{y}} = -\dot{\bar{x}} - 0.4\bar{y} + 5(\bar{x} + r)z \\ \dot{\bar{z}} = b\bar{z} - 5(\bar{x} + r)\bar{y} + u_3 \end{cases} \quad (16)$$

In order to stabilize the $(\bar{x}, \bar{y}, \bar{z})$ -subsystem (11), we can choose a Lyapunov function V_3 as follows

$$V_3 = V_2 + \frac{1}{2}\bar{z}^2$$

Its derivative is given by

$$\dot{V}_3 = -a\bar{x}_2 - 0.4\bar{y}_2^2 + (10\bar{x}\bar{y} + b\bar{z} + u_3)\bar{z}$$

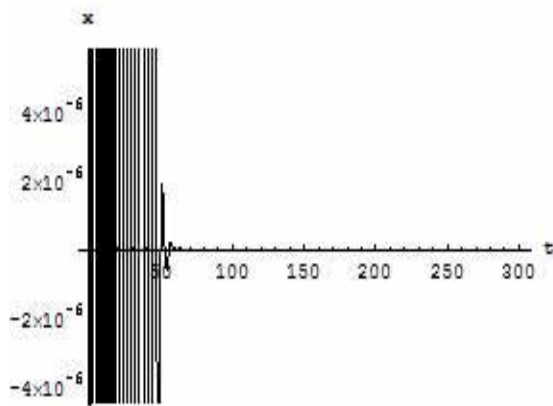
If we choose

$$u_3 = -b\bar{z} - 10\bar{x}\bar{y} \quad (17)$$

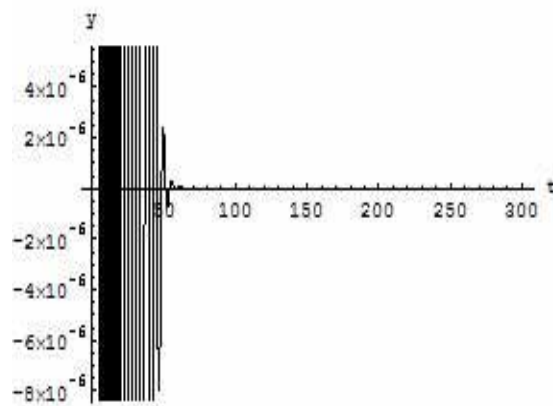
then $\dot{V}_3 = -a\bar{x}_2 - 0.4\bar{y}_2^2 < 0$, therefore $(\bar{x}, \bar{y}, \bar{z})$ -subsystem (13) is asymptotically stable. Since \dot{V}_3 is negative definite, then the equilibrium $(0, 0, 0)$ is asymptotically stable. In view of $\bar{x} = x - r$, $\bar{y} = y - \alpha_1$ and $\bar{z} = z - \alpha_2$, this implies that the output trajectory $x(t)$ of the controlled $u(t)$ asymptotically approaches the target trajectory $r(t)$.

4 Numerical simulations

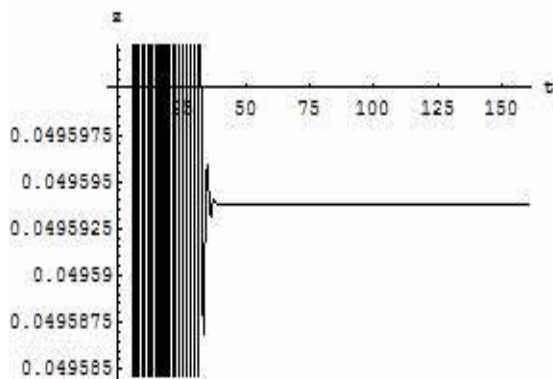
We have introduced the novel active backstepping control approach for controlling and tracking of Newton–Leipnik system. As follows, numerical simulations are given to verify the effectiveness of the control approach. In addition, a time step size 0.01 is employed. We select the parameters of Newton–Leipnik system as $a = 0.4$, $b = 0.175$ so that Newton–Leipnik system exhibits a chaotic behavior if no control is applied. The initial states of the controlled Newton–Leipnik system (2) and (4) are $x_0(0) = 10$, $y_0(0) = -10$, $z_0(0) = 10$.Fig.3 shows that Newton–Leipnik system can be stabilized with the control u_1 to the point $(0, 0, \alpha_2)$. Fig.4 shows that the Newton–Leipnik system can be stabilized with the control law u_2 to the origin point $(0, 0, 0)$.Fig.5 shows that the scalar output $x(t)$ can track the desired trajectory $r(t) = \sin(t)$ with the control input $u_3 = -b\bar{z} - 10\bar{x}\bar{y}$.



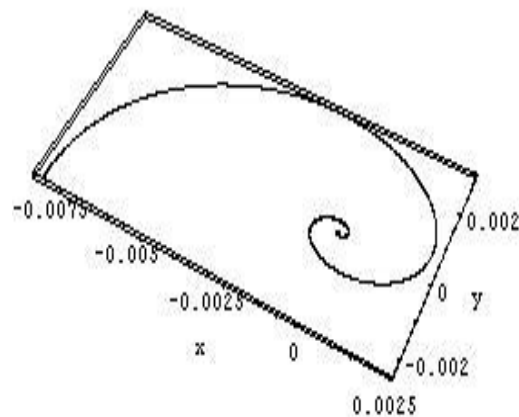
3(a) The time series of $x(t)$



3(b) The time series of $y(t)$

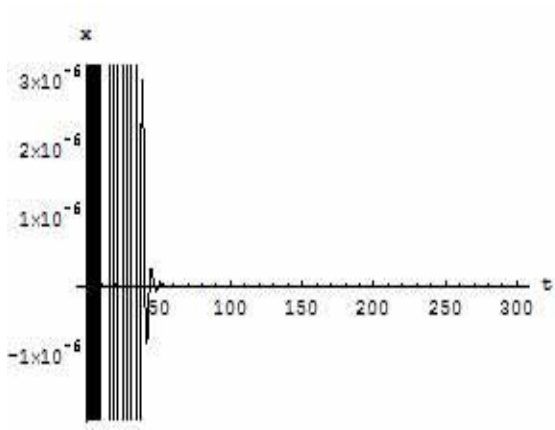


3(c) The time series of $z(t)$

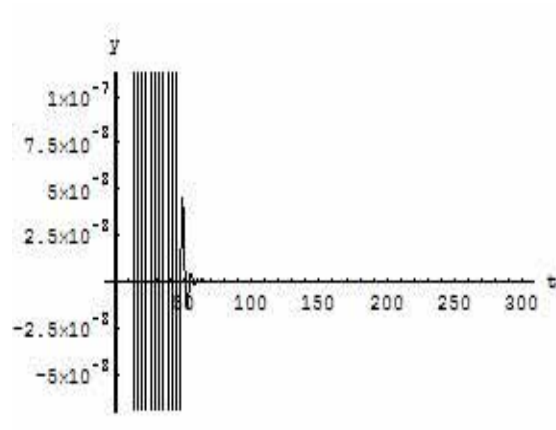


3(d) Stable fixed point $(0,0,\alpha_2)$

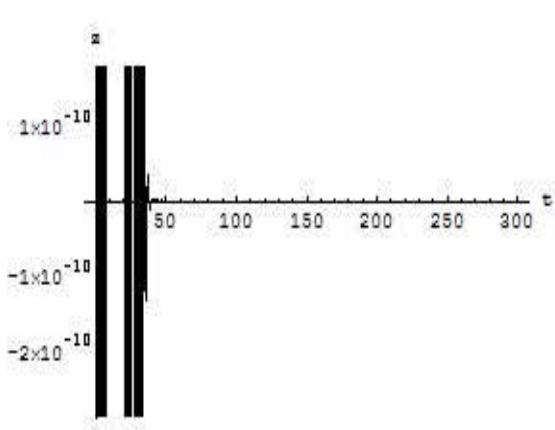
Figure 3: The Newton–Leipnik system is stabilized with the control to the point



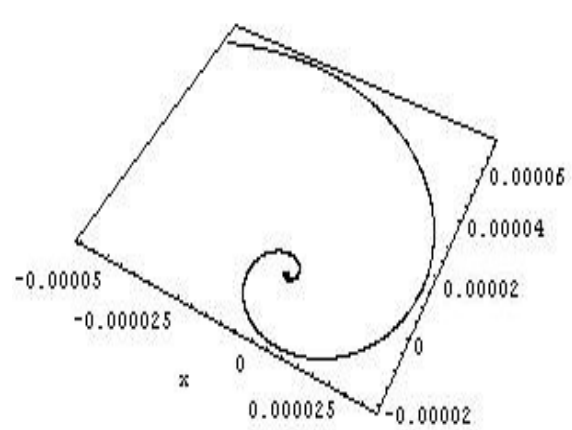
4(a) The time series of $x(t)$



4(b) The time series of $y(t)$



4(c) The time series of $z(t)$



4(d) Stable fixed point $(0, 0, \alpha_2)$

Figure 4: The Newton–Leipnik system is stabilized with the control law to the origin point

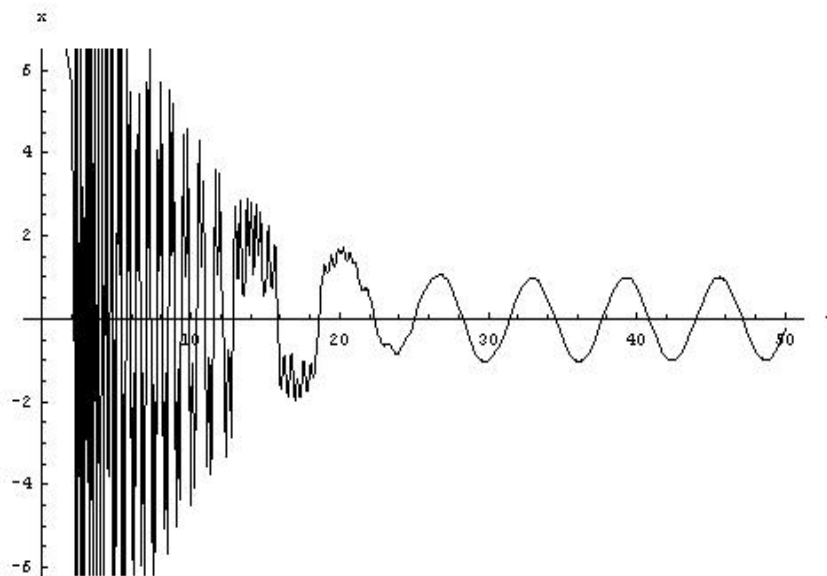


Figure 5: Output $x(t)$ of the Newton–Leipnik system track the trajectory $r(t) = \sin(t)$. The control is activated at $t = 20$

5 Conclusion

This work demonstrates that chaos control of Newton–Leipnik systems using backstepping design is achieved. At the same time we used the same method to enable stabilization of chaotic motion to a steady state as well as tracking of a desired trajectory to be achieved in a systematic way. The Newton–Leipnik is stabilized to a bounded point and to the origin point. Numerical simulations are used to verify the effectiveness of the proposed chaos control technique.

Acknowledgements

Research was supported by the Education Foundation of Jiangsu Province (No: 05KJB110018) and Foundation for Advanced Researchers of Jiangsu University (No: 1283000247)

References

- [1] E. Ott, C. Grebogi, J.A. Yorke: Controlling chaos. *Phys. Rev. Lett.*.64:1196-1199(1990)
- [2] G. Chen, X. Dong: On feedback control of chaotic continuous time systems. *IEEE Trans. Circuit Systems*.40:591(1993)
- [3] Yassen MT.:Chaos control of Chen chaotic dynamical system. *Chaos, Solitons & Fractals*.15:271-283(2003)
- [4] Wang Xuedi, Tian Lixin, Yu Liqin: Linear feedback controlling and synchronization of the Chen's chaotic system. *International Journal of Nonlinear Science*.2(1):43-49(2006)
- [5] Mei Sun, Lixin Tian, Jun Xu: Time-delayed feedback control of the energy resource chaotic system. *International Journal of the Nonlinear Science*.1(3):172-177(2006)
- [6] Yassen MT.: Adaptive control and synchronization of a modified Chua's circuit system. *Appl. Math. Comp.*.135:113-128(2003)
- [7] Liao T-L, Lin S-H.: Adaptive control and synchronization of Lorenz systems. *J. Franklin Inst.*.336:925-937(1999)
- [8] Ervin Goldfain: Chaotic dynamics of the renormalization group flow and standard model parameters. *International Journal of the Nonlinear Science*.3(3):170-180(2007)
- [9] Dianchen Lu, Caixia Chen: Global Synchronization for time-delay of HCSA system. *International Journal of the Nonlinear Science*.3(3):200-207(2007)
- [10] Vanecek A, Celikovsky S.: Control systems: from linear analysis to synthesis of chaos. *London: Prentice-Hall*.4:1331-1332(1996)
- [11] Lu J., Chen G.: A new chaotic attractor coined. *Int. J. Bifurcat. Chaos*.12:659-661(2002)
- [12] Kaplan JL, York YA.: A regime observed in a fluid flow model of Lorenz. *Comm. Math. Phys.*67:93-108(1979)
- [13] Agiza HN.: Controlling chaos for the dynamical system of coupled dynamos. *Chaos, Solitons & Fractals*.12:341-347(2002)