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A Descent Algorithm for The Extended Semidefinite Linear Complementarity Problem *

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Abstract: In this paper, we consider the extended semidefinite linear complementarity problem(XSDLCP). We formulate the problem as an optimization problem with semidefnite constraints and give conditions for any stationary point of the optimization problem to be a solution of the XSDLCP. Furthermore, we give a descent algorithm for the optimization problem and obtain its global convergence.

Key words: The extended semidefinite linear complementarity, Optimization reformulation, Descent algorithm, Global Convergence
Mathematics Subject Classification 65K05, 90C30

1 Introduction

Let $\mathcal{S}^m$ denote the linear space of $m \times m$ real symmetric matrices and $\mathcal{K}^m \subset \mathcal{S}^m$ denote the closed convex cone of $m \times m$ semidefinite positive matrices, $\langle \cdot, \cdot \rangle$ is the inner product in $\mathcal{S}^m$ defined by $\langle x, y \rangle = tr[x^T y]$ for $x,\ y \in \mathcal{S}^m$ and $tr[\cdot]$ denotes the matrix trace. The extended semidefinite linear complementarity problem (XSDLCP) is to find a pair $(x, y) \in \mathcal{S}^m \times \mathcal{S}^m$ such that

$$Mx - Ny \in C,\ x \in \mathcal{K}^m,\ y \in \mathcal{K}^m,\ \langle x, y \rangle = 0. \tag{1}$$

where $M : \mathcal{S}^m \to \mathcal{S}^n$ and $N : \mathcal{S}^m \to \mathcal{S}^n$ are linear mappings and $C = \{u \in \mathcal{S}^n | Au - b \in \mathcal{K}^k\}$ with $A : \mathcal{S}^n \to \mathcal{S}^k$ being a linear mapping and $b \in \mathcal{S}^k$. Throughout this paper, we assume the feasible set $\{(x, y)|Mx - Ny \in C,\ x \in \mathcal{K}^m,\ y \in \mathcal{K}^m\}$ is nonempty.

XSDLCP can be seen as an extension of the extended linear complementarity problem (XLCP) and the semidefinite complementarity problem (SDCP) and it was first studied by M.Shibata, N.Yamashita and M.Fukushima [11]. The XLCP was introduced by Magasarian and pang [9] and it had a variety of applications in such problems as linear and quadratic programming problems, bimatrix game problems, market equilibritm problem and network equilibrium problem [2, 4, 9, 10, 12]. While the SDCP is closely related to the optimality conditions for the SDP which has recently draw growing interesting in control theory and combinatorial optimization [1, 5, 8, 16].

As an extension of both XLCP and SDCP, it is natural to expect that XSDLCP can be solved by applying the solution methods developed for XLCP and SDCP. Recently, various reformulation of XLCP and SDCP to minimization problem have been proposed and studied vigorously [2, 3, 6, 12, 13, 15]. The objective function of an equivalent minimization problem is called a merit function. For the SDCP, P.Tseng [13] first studied such reformulation approaches, he showed that some well-known merit functions such as the regularized gap function, the implicit Lagrangian and the squared Fisher- Bumeister function can be

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extended to SDLCP. For the XLCP, Solodov [12] proposed some merit functions which are extension of the Fisher-Bumeister function and the implicit Lagrangian augumented function by a penalty term associated with $Mx - Ny \in C$.

Motivated by the above mentioned development of reformulation for XLCP and SDLCP, M.Shibata, N.Yamashita and M.Fukushima [11] considered the unconstrained optimization reformulation of XSDLCP and gave conditions for any stationary point of the optimization problem to be a solution of the original problem. We note that no solution method was given in [11].

In this paper, we consider the semidefinite constrained optimization reformulation for XSDLCP by extending the results obtained by Solodov [12] and R.Andreani and J.M.Martinez [2]. Besides giving conditions for any stationary point of the optimization problem to be a solution of the original problem, we also propose a descent algorithm for the semidefinite constrained optimization problem.

The paper is organized as follows: In Section 2, we review some basic results; In Section 3, we show conditions under which any stationary point of the optimization problem solves XSDLCP. In Section 4, we propose the descent algorithm and prove its global convergence; Some conclusions are given in Section 5.

Notations: For any matrix $x \in \mathcal{S}^m$, $[x]_+$ and $[x]_-$ denote the orthogonal projections of x onto $\mathcal{K}^m$ and $-\mathcal{K}^m$ respectively. If p is the orthogonal matrix such that $p^T xp = diag(\lambda_1, \lambda_2, \cdots, \lambda_m)$, then

$$[x]_+ = p^T diag([\lambda_1]_+, [\lambda_2]_+, \cdots, [\lambda_m]_+)p$$

and

$$[x]_- = p^T diag([\lambda_1]_-, [\lambda_2]_-, \cdots, [\lambda_m]_-)p,$$

where $[\lambda_i]_+ = \max\{0, \lambda_i\}$ and $[\lambda_i]_- = \min\{0, \lambda_i\}$ for $i = 1, 2, \cdots, m$. Therefore, for any $x \in \mathcal{S}^m$, we have $x = [x]_+ + [x]_-$ and $[x]_+[x]_- = 0$.

2 Basic results

In this section, we review some basic results that will play an important role in the subsequent analysis. These notations are taken from [13].

Definition 2.1 *(a) For any $m \times m$ matrix x, $sym[x]$ is defined by $sym[x] := x + x^T$, where x^T denotes the transpose of x.*
(b) *For any $c \in \mathcal{K}^m$, the set $\mathcal{S}_c^m \subset \mathcal{S}^m$ denotes the subspace comprising those $x \in \mathcal{S}^m$ whose null space contains the null space of c.*
(c) *For any $c \in \mathcal{K}^m$, the linear mapping $L_c : \mathcal{S}_c^m \to \mathcal{S}_c^m$ is defined by $L_c[x] := cx + xc$.*
(d) *For any subspace $\mathcal{T} \subset \mathcal{S}^m$, a mapping $\mathcal{H} : \mathcal{T} \to \mathcal{T}$ is said to be positive definite on $\mathcal{T}$ if*

$$\langle x, \mathcal{H}[x]\rangle > 0, \quad for\ any\ x \in \mathcal{T}\ (x \neq 0).$$

Lemma 2.1 **(a)** *For any $a, b \in \mathcal{S}^m$, let $c := (a^2 + b^2)^{\frac{1}{2}}$. Then $a, b \in \mathcal{S}_c^m$.*
(b) *For any $c \in \mathcal{K}^m$, the linear mapping L_c is positive definite and hence invertible on $\mathcal{S}_c^m$. Specifically, for any $x \in \mathcal{S}_c^m$, $L_c^{-1}[x]$ is the unique matrix $y \in \mathcal{S}_c^m$, satisfying $cy + yc = x$.*
(c) *For any $c \in \mathcal{K}^m$ and $x, y \in \mathcal{S}_c^m$, we have $\langle y, L_c[x]\rangle = \langle L_c[y], x\rangle, xL_c^{-1}[c] = L_c^{-1}[c]x = \frac{1}{2}x$ and $xL_c^{-1}[x] = 0 \Rightarrow x = 0$.*

Next we review the squared Fisher-Burmeister merit function studied by Tseng [13] for SDCP and its some properties, the merit function is defined by:

$$\Psi(x, y) = \frac{1}{2}\|\phi(x, y)\|^2,$$

where $\phi : \mathcal{S}^m \times \mathcal{S}^m \to \mathcal{S}^m$ is defined $\phi(a, b) = (a^2 + b^2)^{\frac{1}{2}} - a - b$.

Lemma 2.2 **(a)** *$\Psi(a,b) \geq 0$ for all $(a,b) \in \mathcal{S}^m \times \mathcal{S}^m$, $\Psi(a,b) = 0$ and $(a,b) \in \mathcal{S}^m \times \mathcal{S}^m$ if and only if $a \in \mathcal{K}^m$, $b \in \mathcal{K}^m$ and $\langle a,b\rangle = 0$.*
(b) *Ψ is differentiable on $(a,b) \in \mathcal{S}^m \times \mathcal{S}^m$ and*

$$\nabla_a \Psi(a,b) = sym[L_c^{-1}[c-a-b](a-c)] \in \mathcal{S}^m,$$

$$\nabla_b \Psi(a,b) = sym[L_c^{-1}[c-a-b](b-c)] \in \mathcal{S}^m,$$

where $c = (a^2+b^2)^{\frac{1}{2}} \in \mathcal{K}^m$.
(c) *For every $(a,b) \in \mathcal{S}^m \times \mathcal{S}^m$, we have*

$$\langle \nabla_a \Psi(a,b), \nabla_b \Psi(a,b)\rangle \geq \|(c-a-b)g\|^2,$$

where $c = (a^2+b^2)^{\frac{1}{2}} \in \mathcal{K}^m$ and $g = L_c^{-1}[c-a-b] \in \mathcal{S}^m$.
(d) *For every $(a,b) \in \mathcal{S}^m \times \mathcal{S}^m$, we have*

$$\langle a, \nabla_a \Psi(a,b)\rangle + \langle b, \nabla_b \Psi(a,b)\rangle = \|c-a-b\|^2,$$

where $c = (a^2+b^2)^{\frac{1}{2}} \in \mathcal{K}^m$.

We introduce some concepts about linear mapping on $\mathcal{S}^m$.

Definition 2.2 *(a) A linear mapping $M : \mathcal{S}^m \to \mathcal{S}^m$ is positive semidefinite if*

$$\langle y, My\rangle \geq 0, \ \ for \ \ all \ \ y \in \mathcal{S}^m.$$

(b) A linear mapping $M : \mathcal{S}^m \to \mathcal{S}^m$ is said to be copositive on the cone $\mathcal{T} \subset \mathcal{S}^m$ if the following holds:

$$\langle x, Mx\rangle \geq 0 \ \ for \ \ all \ \ x \in \mathcal{T}, \ x \neq 0.$$

(c) The adjoint of a linear mapping $M : \mathcal{S}^m \to \mathcal{S}^k$ is the linear mapping $M^* : \mathcal{S}^k \to \mathcal{S}^m$ such that $\langle Mu, \mu\rangle = \langle u, M^*\mu\rangle$ for all $u \in \mathcal{S}^m$ and $\mu \in \mathcal{S}^k$.

Definition 2.3 *The recession cone 0^+C of the set $C = \{u \in \mathcal{S}^n | Au - b \in \mathcal{K}^k\}$ is given by $0^+C = \{u \in \mathcal{S}^n | Au \in \mathcal{K}^k\}$, and its dual cone $(0^+C)^*$ is given by*

$$(0^+C)^* := \{v \in \mathcal{S}^n | \langle u, v\rangle \geq 0 \ for \ all \ u \in 0^+C\} = \{v \in \mathcal{S}^n | v = A^*\mu, \mu \in \mathcal{K}^k\}.$$

3 Semidefinite constrained optimization reformulation for XSDLCP

In this section, we show the equivalence between XSDLCP and the minimization problem

$$\min f(x,y), \ \ (x,y) \in \mathcal{K}^m \times \mathcal{K}^m, \tag{2}$$

where f is defined by $f(x,y) = \frac{1}{2}\|[-AMx + ANy + b]_+\|^2 + \Psi(x,y)$.

The following result implies the equivalence between XSDLCP and the minimization problem (2), it can be easily proved from Lemma 2.2 and Lemma 2.3.

Theorem 3.1 *Let $f(x,y)$ be defined by (2), then $f(x,y)$ is nonnegative on $\mathcal{S}^m \times \mathcal{S}^m$. Moreover, $f(x,y) = 0$ and $(x,y) \in \mathcal{K}^m \times \mathcal{K}^m$ if and only if (x,y) solves XSDLCP.*

The following theorem gives the conditions that any stationary point of (2) solves XSDLCP.

Theorem 3.2 *Suppose that one of the following assumptions is satisfied:*
(i): For all $v \in (0^+C)^*$, it hods that $-M^*v \in \mathcal{K}^m$ and $N^*v \in \mathcal{K}^m$.
(ii): $(0,0) \in C$.
Then every stationary point of (2) solves XSDLCP.

Proof. Let $v = A^*[-AMx + ANy + b]_+$, then $v \in (0^+C)^*$ since $[-AMx + ANy + b]_+ \in \mathcal{K}^m$. Let $(x, y) \in \mathcal{S}^m \times \mathcal{S}^m$ be a stationary point of (2), then there exist two matrices s, t such that

$$-M^*v + \nabla_x \Psi(x, y) - t = 0, \tag{3}$$

$$N^*v + \nabla_y \Psi(x, y) - s = 0, \tag{4}$$

and $0 = \langle x, t\rangle = \langle y, s\rangle, \ \ x, y, s, t \in \mathcal{K}^m$. Suppose that assumption (i) is satisfied, using (3)(4) and Lemma 2.2(d), we have

$$\begin{aligned} 0 &= \langle x, t\rangle + \langle y, s\rangle \\ &= \langle x, \nabla_x\Psi(x, y)\rangle + \langle y, \nabla_y\Psi(x, y))\rangle - \langle x, M^*v\rangle + \langle y, N^*v\rangle \\ &= \|c - x - y\|^2 - \langle x, M^*v\rangle + \langle y, N^*v\rangle \end{aligned}$$

where $c = (x^2 + y^2)^{\frac{1}{2}}$.
Since for all $v \in (0^+C)^*, \ -M^*v \in \mathcal{K}^m$ and $N^*v \in \mathcal{K}^m$ and $x \in \mathcal{K}^m, \ y \in \mathcal{K}^m$, we have

$$\|c - x - y\|^2 = 0, \ \ \langle x, M^*v\rangle = 0, \ \ \langle y, N^*v\rangle = 0.$$

Hence $(c - x - y) = 0$, it follows from Lemma 2.2(a) that $(x^2 + y^2)^{\frac{1}{2}} = x + y$, hence

$$x \in \mathcal{K}^m, \ y \in \mathcal{K}^m$$

and

$$\langle x, y\rangle = 0.$$

by Lemma 2.2(b) we have that $\nabla_x\Psi(x, y) = \nabla_y\Psi(x, y) = 0$, hence the stationary point of (2) implies that

$$-M^*v - t = 0, \ \ N^*v - s = 0,$$

which means (x, y) is a stationary point of the following convex programming

$$\min \ \tfrac{1}{2}\|[-AMx + ANy + b]_+\|^2 \ \text{ s.t } \ x \in \mathcal{K}^m, \ y \in \mathcal{K}^m,$$

it follows that (x, y) is a global minimization of (2). Since the set $\{(x, y)|Mx - Ny \in C, \ x \in \mathcal{K}^m, \ y \in \mathcal{K}^m\}$ is nonempty, the global minimization (x, y) actually satisfies $Mx - Ny \in C$, thus (x, y) solves XS-DLCP.

Suppose now that the condition (ii) holds, by (3)(4) we have

$$\begin{aligned} 0 &= \|c - x - y\|^2 + \langle -Mx + Ny, v\rangle \\ &= \|c - x - y\|^2 + \langle -AMx + Ny + b, [-AMx + ANy + b]_+\rangle - \langle b, [-AMx + ANy + b]_+\rangle \\ &= \|c - x - y\|^2 + \|[-AMx + ANy + b]_+\|^2 - \langle b, [-AMx + ANy + b]_+\rangle \end{aligned}$$

Since $(0, 0) \in C$, we have $-b \in \mathcal{K}^k$, therefore we have

$$\|c - x - y\|^2 = 0 \text{ and } \|[-AMx + ANy + b]_+\| = 0.$$

which means (x, y) solves XSDLCP.

The merit function $f(x, y)$ defined by (2) includes the projection and the square root of matrix, this means one should make matrix decompose during the course of computation, which is a hard work. To overcome this drawback, we introduce another merit function which does not include the computation of matrix decompose, this merit function is motivated by the merit function for XSLCP proposed by R.Andreani, J.M.Martinez [2]. To define the merit function, we introduce a slack matrix $z \in \mathcal{K}^k$ and rewrite the set $C = \{u \in \mathcal{S}^n|Au - b - z = 0\}$, then follows from the idea in [2], we propose the associated semidefinite-constrained problem given by

$$\begin{aligned} &\min \ \langle x, y\rangle^2 + \gamma\|AMx - ANy - b - z\|^2, \\ &\text{s.t} \ \ x \succeq 0, \ y \succeq 0, \ z \succeq 0. \end{aligned} \tag{5}$$

where $\gamma > 0$ is an arbitrary constant, and $x \succeq 0, \ y \succeq 0$ mean $x \in \mathcal{K}^m, \ y \in \mathcal{K}^m$, while $z \succeq 0$ means $z \in \mathcal{K}^k$. ■

Theorem 3.3 *Assume that MN^* is copositive on $(0^+C)^*$, then every stationary point of (5) solves XSDLCP.*

Proof. Let (x, y, z) be a stationary point of (5) and $v = AMx - ANy - b - z$. Then (x, y, z, v) satisfies

$$2\gamma M^*(A^*v) + 2\langle x, y\rangle y - v_1 = 0, \tag{6}$$

$$-2\gamma N^*(A^*v) + 2\langle x, y\rangle x - v_2 = 0, \tag{7}$$

$$-2\gamma v - v_3 = 0, \tag{8}$$

$$\langle x, v_1\rangle = 0, \ \ \langle y, v_2\rangle = 0, \ \ \langle z, v_3\rangle = 0, \tag{9}$$

$$x \succeq 0, \ y \succeq 0, \ z \succeq 0, \ v_1 \succeq 0, \ v_2 \succeq 0, \ v_3 \succeq 0, \tag{10}$$

Rewrite (6) and (7) as

$$2\gamma M^*(A^*v) = -2\langle x, y\rangle y + v_1, \tag{11}$$

$$2\gamma N^*(A^*v) = 2\langle x, y\rangle x - v_2, \tag{12}$$

then by (9)(11)(12) we have

$$4\gamma^2\langle M^*(A^*(-v)), N^*(A^*(-v))\rangle = -4\langle x, y\rangle^2 - \langle v_1, v_2\rangle \le 0, \tag{13}$$

by (8) we know that $-v = \frac{v_3}{2\gamma} \succeq 0$, which implies that $A^*(-v) \in (0^+C)^*$. Since MN^* is copositive on $(0^+C)^*$, (10) and (13) imply that

$$\langle M^*(A^*(-v)), N^*(A^*(-v))\rangle = \langle MN^*(A^*(-v)), A^*(-v)\rangle = 0. \tag{14}$$

Therefore by (13) we get

$$\langle x, y\rangle = 0. \tag{15}$$

Thus (6)-(10) now equivalent to

$$2\gamma M^*(A^*v) - v_1 = 0, \tag{16}$$

$$-2\gamma N^*(A^*v) - v_2 = 0, \tag{17}$$

$$-2\gamma v - v_3 = 0, \tag{18}$$

$$\langle x, v_1\rangle = 0, \ \ \langle y, v_2\rangle = 0, \ \ \langle z, v_3\rangle = 0, \tag{19}$$

$$x \succeq 0, \ y \succeq 0, \ z \succeq 0, \ v_1 \succeq 0, \ v_2 \succeq 0, \ v_3 \succeq 0, \tag{20}$$

Equations (16)-(20) mean that (x, y, z) is a global minimization of the following convex programming problem:

$$\begin{array}{l} \min \ \gamma\|AMx - ANy - b - z\|^2, \\ \text{s.t} \ \ x \succeq 0, \ y \succeq 0, \ z \succeq 0. \end{array}$$

Since $C \neq \varnothing$ it turns out that (x, y, z) is a global solution of (5) with minimum value zero, that is

$$AM^*x - AN^*y - b - z = 0,$$

which together with (15) imply that (x, y, z) solves XSDLCP. ■

4 Descent algorithm for XSDLCP

In this section, we give a descent algorithm for the equivalent reformulation problem of XSDLCP and analyze it global convergence. We choose the model (2) in our algorithm since the form of (2) and (5) are very similar.

For convenience, we write $z = (x, y)$ and denote $z \succeq 0$ if $x \succeq 0$ and $y \succeq 0$, the algorithm is described as follows:

Algorithm 1

(S.1) Choose $z_0 \succeq 0, \alpha \in (0, 1),\ \rho \in (0, 1), k := 0.$

(S.2) Find the solution $d_k \in \mathcal{S}^m \times \mathcal{S}^m$ of the problem:

$$\min\langle \nabla f_k, d\rangle + \frac{1}{2}\|d\|^2,\ s.t.\ z_k + d_k \succeq 0. \tag{21}$$

If $d_k = 0$ stop.
(S.3) Compute $\alpha_k = \max\{1, \alpha, \alpha^2, \cdots\}$ such that

$$f[z_k + \alpha_k d_k] \le f(z_k) + \rho\alpha_k\langle \nabla f(z_k), d_k\rangle.$$

(S.4) Set $z_{k+1} = z_k + d_k,\ k := k + 1$, go to (S.2).

Lemma 4.1 *Let $z_k = (x_k, y_k)$ be a given iteration point and $d_k = (d_k^x, d_k^y)$ be the solution of problem (21). Then*

$$\langle \nabla f_k, d_k\rangle \le -\frac{1}{2}\|d\|^2.$$

Proof. Since $z_k \succeq 0$ and $d = 0$ is feasible for problem (21), while d_k is a solution of this problem, so we have

$$\langle \nabla f_k, d\rangle + \frac{1}{2}\|d\|^2 \le 0,$$

and therefore we have the desired result.

■

Lemma 4.2 *Let $z_k = (x_k, y_k)$ be a given iteration point and $d_k = (d_k^x, d_k^y)$ be the solution of problem (21). If $d_k = 0$, then z_k is a stationary of problem (2).*

Proof. Since d_k is a solution of problem (21), then there exist two matrices p_k, q_k such that

$$\nabla_x f(x_k, y_k) + d_k^x - p_k = 0,$$

$$\nabla_y f(x_k, y_k) + d_k^y - q_k = 0,$$

$$x_k + d_k^x \succeq 0,\ y_k + d_k^y \succeq 0,$$

$$\langle x_k + d_k^x, p_k\rangle = 0,\ \langle y_k + d_k^y, q_k\rangle = 0.$$

If $d_k = 0$ then the above equations is

$$\nabla_x f(x_k, y_k) - p_k = 0,$$

$$\nabla_y f(x_k, y_k) - q_k = 0,$$

$$x_k \succeq 0,\ y_k \succeq 0,$$

$$\langle x_k, p_k\rangle = 0,\ \langle y_k, q_k\rangle = 0.$$

which means (x_k, y_k) is a stationary point of problem (2). ■

Theorem 4.1 *Let $z_k = \{(x_k, y_k)\}$ be a sequence generated by Algorithm 1 and $\{z_k\}_{k\in K}$ be a subsequence converging to $z^\star = (x^\star, y^\star)$, then $z^\star$ is a stationary point of problem (2).*

Proof. Since $\{z_k\}_{k\in K} \to z^\star$, by the continuity of $f(z)$, we have $f(z_k)_{k\in K} \to f(z^\star)$ and $(d_k)_{k\in K} \to d^\star$.

(1) If $\underset{k\in K, k\to\infty}{lim} \alpha_k = 0$, then by S.3, we have

$$f(z_k + \frac{\alpha_k}{\alpha} d_k) > f(z_k) + \rho \frac{\alpha_k}{\alpha} \langle \nabla f_k, d_k \rangle.$$

Let $k \in K, k \to \infty$ we have

$$\langle \nabla f(x^\star, y^\star), d^\star \rangle \geq \rho \langle \nabla f(x^\star, y^\star), d^\star \rangle.$$

By Lemma 4.1, we have $\frac{1}{2}(1-\rho)\|d^\star\| \leq 0$, thus $d^\star = 0$, hence by Lemma 4.1, we have $z^\star$ is a stationary point of problem (2).

(2) $\lim_{k\in K, k\to\infty} inf \ \alpha_k > 0$, then by S.3 and Lemma 1, for $k \in K$ we have

$$f(z_{k+1}) - f(z_k) \leq \rho\alpha_k \langle \nabla f(z_k), d_k \rangle \leq \rho\alpha_k \|d_k\|^2.$$

Since $\{f(x_k, y_k)\}$ is descent monotonically and bounded below from zero, let $k \in K, k \to \infty$ we have $\{d_k\}_{k\in K} \to d^\star = 0$, which implies that $z^\star$ is a stationary point of problem (2). ■

5 Conclusion

This paper we proposed a semidefinite-constrained optimization reformulation for the XSDLCP and established sufficient conditions which guarantee that any stationary point of the optimization problem is a solution of XSDLCP. Furthermore, we gave a descent algorithm and analyzed its global convergence. What conditions can guarantee the level set to be bounded is an interesting topic for further research. Moreover, designing a smoothing method or trust region method such as [7] and [14] also deserves studying.

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