

## Electrostatic Charged Two-Phase Turbulent Flow Model

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(Received 1 May 2007, accepted 15 November 2007)

**Abstract:** Two-phase electrostatic charged flows have been applied in electrostatic spray, crop-dusting, fuel spray and so on. Electrostatic charged spray can improve desulfurization efficiency, decrease water usage. There exist interactions between non-uniform electric and flow fields, and phase interaction between charged particle and continuous phases, which makes the flow more complex. Based on the Reynolds transport equation, equations for the volume averaging and instantaneous state of the electrostatic charged two-phase flow have been obtained in this study, thus the  $k - \varepsilon - k_p$  model is closed.

**Key words:** Electrostatic charged two-phase flow; Turbulence; Phase interaction; Reynolds-averaging equation;  $k - \varepsilon - k_p$  model

### 1 Introduction

The research of electrostatic charged spray has developed abroad in crop-dusting, atomization, fuel spray and so on. Electrostatic charged two-phase turbulent flow is very complex[1]. There exist interaction between spatial non-uniform electric field and flow field, and phase interaction between charged particle phase and continuous phase, which make the flow more complex. From the point of electric field[2], there exist exterior electric field and derivational electric field by the charged droplets, which could influence the force on the droplets and the continuous phase via interactions between phases. Moreover, ripple of charged droplets could bring the ripple of electric field. So the structure of the charged two-phase turbulent flows of gas and liquid is very complex.

### 2 Volume-averaging conservation equations of electrostatic charged two-phase flows

#### 2.1 The volume-averaging approach

The diameter difference of solid particles in the pesticide insufflations or liquid droplets in the spray is very little in the charged two-phase turbulent flow. Furthermore, the experiments indicate that when the droplets are charged, atomization uniformity can be observed increasingly. So dispersed particles can be regarded as one phase with the same physics characteristics[3].

According to the description of the two-phase flow, the dispersed phase and continuous phase occupy the same space with interaction between them[4]. As shown in Fig. 1, a control volume  $dV$  is chosen in the mixture, and its surface area is represented by  $dA$ . Within the control volume, the volume of  $k$ -phase is  $dV_k$ , and the surface is  $dA_k$ .

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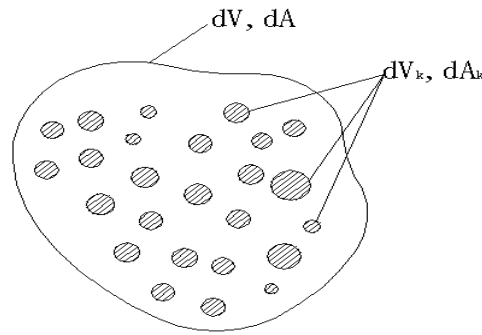


Figure 1: The control volume in two-phase flows

Volume averaging approach is applied. Videlicet, to macro flow field, and each variable value in liquid tiny unit is the average value in the control volume. The average value of parameter  $\varphi$  is defined as[5]:

$$\langle \varphi \rangle = \frac{1}{V} \int_V \varphi_k dV$$

The sign of  $\langle \rangle$  denotes the volume averaging.  $V = V_p + V_c$  is the control volume. The subscripts  $p$  and  $c$  denote particle and continuous phase, respectively. Three useful volume averaging expressions are summarized as follows.

1) Gauss formula

$$\int_V \frac{\partial a_j}{\partial x_j} dV = \int_{A_k} a_j n_j dA$$

2) Transport principle

$$\left\langle \frac{\partial a_i}{\partial t} \right\rangle = \frac{\partial}{\partial t} \langle a_i \rangle - \frac{1}{V} \int_A a_i v_j n_j dA$$

3) Average principle

$$\left\langle \frac{\partial a_j}{\partial x_j} \right\rangle = \frac{\partial}{\partial x_j} \langle a_j \rangle + \frac{1}{V} \int_A a_j n_j dA$$

## 2.2 The phase of microcosmic conservation equations

Dispersed phase would be regarded, for example, as liquid in this study. Taking the basic equation inside  $k$ -phase and adopting the apparent density, the conservation equations can be written as:

1) Continuity equation

$$\frac{\partial \rho_k}{\partial t} + \frac{\partial}{\partial x_j} (\rho_k v_{kj}) = 0$$

2) Momentum equation

$$\frac{\partial}{\partial t} (\rho_k v_{kj}) + \frac{\partial}{\partial x_j} (\rho_k v_{kj} v_{ki}) = \rho_k f_{ki} - \frac{\partial p_k}{\partial x_i} + \frac{\partial \tau_{k,ji}}{\partial x_j}$$

3) Energy equation

$$\frac{\partial}{\partial t} (\rho_k C_{vk} T_k) + \frac{\partial}{\partial x_j} (\rho_k C_{vk} T_k v_{kj}) = -p \frac{\partial v_{kj}}{\partial x_j} + \tau_{k,ij} s_{k,ij} + \frac{\partial}{\partial x_j} (\lambda_k \frac{\partial T_k}{\partial x_j}) + \rho_k q_k$$

where  $\rho_k$  is the apparent density of  $k$ -phase,  $v_{kj}$  is the velocity of  $k$ -phase,  $f_{ki}$  is the force acting on  $k$ -phase of unit mass,  $\tau_{k,ij}$  is tangency stress tensor;  $C_{vk}$  is the definite volume specific heat,  $\lambda_k$  is the heat exchange coefficient,  $s_{k,ij}$  is the deformation speed tensor and  $q_k$  is the gross of quantity of heat in  $k$ -phase.

### 2.3 Volume averaging conservation equation

Applying the average principle to continuity equation, we get:

$$\begin{aligned} \left\langle \frac{\partial \rho_k}{\partial t} \right\rangle &= \frac{\partial}{\partial t} \langle \rho_k \rangle - \frac{1}{V} \int_{A_k} \rho_k \vec{v}_s \cdot d\vec{A} \\ \langle \nabla \cdot \rho_k \vec{v}_k \rangle &= \nabla \cdot \langle \rho_k \vec{v}_k \rangle + \frac{1}{V} \int_{A_k} \rho_k \vec{v}_k \cdot d\vec{A}_k \\ \frac{\partial \langle \rho_k \rangle}{\partial t} + \nabla \cdot \langle \rho_k \vec{v}_k \rangle &= -\frac{1}{V} \int_{A_k} \rho_k (\vec{v}_k - \vec{v}_s) \cdot d\vec{A}_k \end{aligned}$$

According to physics meanings,  $\vec{v}_s$  is the displacement velocity at phase interface due to phase change, and  $\vec{v}_k - \vec{v}_s$  is the displacement velocity of  $k$ -phase relative to interface. So the right sides of the above equations are source terms related to the phase change in the control volume of two-phase flows.

$$\begin{aligned} &-\frac{1}{V} \int_{A_k} \rho_k (\vec{v}_k - \vec{v}_s) \cdot d\vec{A}_k \\ &= \frac{1}{V} \int_{A_k} \vec{n} \cdot [\rho_k (\vec{v}_k - \vec{v}_s)] \cdot dA_k \\ &= \frac{1}{V} \int_{V_k} \nabla \cdot [\rho_k (\vec{v}_k - \vec{v}_s)] \cdot dV_k \\ &= \frac{1}{V} \int_{V_k} S_k dV_k \\ &= \langle S_k \rangle \end{aligned}$$

where  $S_k$  is the unit volume averaging source term due to phase change.

Thus  $k$ -phase volume averaging continuity equation is:

$$\frac{\partial \langle \rho_k \rangle}{\partial t} + \nabla \cdot \langle \rho_k \vec{v}_k \rangle = \langle S_k \rangle$$

Removing  $\langle \quad \rangle$ , macro  $k$ -phase volume averaging continuity equation is obtained:

$$\frac{\partial \rho_k}{\partial t} + \nabla \cdot (\rho_k \vec{v}_k) = S_k$$

In the same way, the continuity equations of particle phase and continuous phase can be given by:

$$\left. \begin{aligned} \frac{\partial \rho_p}{\partial t} + \frac{\partial}{\partial x_j} (\rho_p v_{pj}) &= S \\ \frac{\partial \rho_c}{\partial t} + \frac{\partial}{\partial x_j} (\rho_c v_{cj}) &= -S \end{aligned} \right\}$$

where  $S = n_p \frac{dm_p}{dt} n_p$  is the particle number density of  $p$ -phase, and  $m_p$  is the mass of one particle.

The momentum and energy equations can also be obtained by the similar method. These equations are:

1) Particle phase

$$\left. \begin{aligned} \frac{\partial \rho_p}{\partial t} + \frac{\partial}{\partial x_j} (\rho_p v_{pj}) &= S \\ \frac{\partial}{\partial t} (\rho_p v_{pi}) + \frac{\partial}{\partial x_j} (\rho_p v_{pj} v_{pi}) &= \rho A_q E_i + \rho_p g_i - \frac{\partial p}{\partial x_i} + \rho_p \frac{v_{pi} - v_{ci}}{\tau_v} + v_{ci} S \\ \frac{\partial}{\partial t} (\rho_p C_{vp} T_p) + \frac{\partial}{\partial x_j} (\rho_p C_{vp} T_p v_{pj}) &= \frac{\partial}{\partial x_j} (\lambda \frac{\partial T_p}{\partial x_j}) + \rho_p q_p - C_{vp} T_p S \end{aligned} \right\}$$

## 2) Continuous phase

$$\left. \begin{aligned} \frac{\partial \rho_c}{\partial t} + \frac{\partial}{\partial x_j}(\rho_c v_{cj}) &= -S \\ \frac{\partial}{\partial t}(\rho_c v_{ci}) + \frac{\partial}{\partial x_j}(\rho_c v_{cj} v_{ci}) &= \rho_c g_i - \frac{\partial p}{\partial x_i} + \frac{\partial \tau_{ji}}{\partial x_j} - \rho_c \frac{v_{pi} - v_{ci}}{\tau_v} - v_{ci} S \\ \frac{\partial}{\partial t}(\rho_c C_{vc} T_c) + \frac{\partial}{\partial x_j}(\rho_c C_{vc} T_c v_{cj}) &= -p \frac{\partial v_{cj}}{\partial x_j} + \tau_{ij} s_{ij} + \frac{\partial}{\partial x_j} \left( \lambda \frac{\partial T_c}{\partial x_j} \right) + \rho_c q_c + C_{vc} T_p S \end{aligned} \right\}$$

where  $A_q$  is the ratio of charge to mass,  $\vec{E}$  is the electric field intensity,  $v_{ci} S$  is the momentum of phase change matter sources,  $C_{vp} T_p S$  and  $C_{vc} T_p S$  are energy of particles and continuous phase in the unit volume due to phase change, respectively.

### 3 Time averaging conservation equations of charged two-phase flows

The instantaneous state of the electrostatic charged two-phase flow has been obtained previously. For the charged two-phase turbulent flows, with the similarity to the single phase turbulent flow equations, the relevant Reynolds-averaging equations can be given by:

## 1) Electrostatic dispersed phase

$$\left. \begin{aligned} \frac{\partial \rho_p}{\partial t} + \frac{\partial}{\partial x_j}(\rho_p v_{pj}) &= S \\ \frac{\partial}{\partial t}(\rho_p v_{pi}) + \frac{\partial}{\partial x_j}(\rho_p v_{pj} v_{pi}) &= \rho A_q E_i + \rho_p g_i \\ -\frac{\partial p}{\partial x_i} + \rho_p \frac{v_{pi} - v_{ci}}{\tau_v} + v_{ci} S + \frac{\partial}{\partial x_j}(\overline{-\rho_p v'_{pj} v'_{pi}}) + \overline{\rho_p A'_q E'_i} \\ \frac{\partial}{\partial t}(\rho_p C_{vp} T_p) + \frac{\partial}{\partial x_j}(\rho_p C_{vp} T_p v_{pj}) &= \frac{\partial}{\partial x_j} \left( \lambda \frac{\partial T_p}{\partial x_j} \right) + \\ \rho_p q_p - C_{vp} T_p S - \frac{\partial}{\partial x_j}(\rho_p C_{vp} \overline{T'_p v'_{pj}}) & \end{aligned} \right\}$$

## 2) Continuous phase

$$\left. \begin{aligned} \frac{\partial \rho_c}{\partial t} + \frac{\partial}{\partial x_j}(\rho_c v_{cj}) &= -S \\ \frac{\partial}{\partial t}(\rho_c v_{ci}) + \frac{\partial}{\partial x_j}(\rho_c v_{cj} v_{ci}) &= \rho_c g_i - \frac{\partial p}{\partial x_i} + \frac{\partial \tau_{ij}}{\partial x_j} \\ &\quad - \rho_c \frac{v_{pi} - v_{ci}}{\tau_v} - v_{ci} S + \frac{\partial}{\partial x_j}(\overline{-\rho_c v'_{cj} v'_{ci}}) \\ \frac{\partial}{\partial t}(\rho_c C_{vc} T_c) + \frac{\partial}{\partial x_j}(\rho_c C_{vc} T_c v_{cj}) &= -p \frac{\partial v_{cj}}{\partial x_j} + \Phi + \\ \frac{\partial}{\partial x_j} \left( \lambda \frac{\partial T_c}{\partial x_j} \right) + \rho_c q_c + C_{vc} T_p S - \frac{\partial}{\partial x_j}(\rho_c C_{vc} \overline{T'_c v'_{cj}}) &+ \varepsilon \end{aligned} \right\}$$

where  $\Phi$  is the dissipation function and  $\varepsilon$  is the dissipation rate of turbulent energy.

### 4 The model of charged two-phase turbulent flows

Based on the time averaging conservation equations and under the supposition that particles can be regarded as continuous phase, equation groups are closed.

### 4.1 The simulation of continuous phase

Relevant terms of turbulence fluctuating in the equation groups are:

$$-\overline{\rho_c v'_{ci} v'_{cj}}, \quad \overline{\tau'_{ij} s'_{ij}}, \quad -\overline{\rho_c v_c T'_c v'_{cj}}$$

1) Turbulence stress term  $\tau_t = -\overline{\rho_c v'_{ci} v'_{cj}}$  and turbulent energy dissipation term  $\overline{\tau'_{ij} s'_{ij}}$  can be solved by the single phase  $k - \varepsilon$  two-equation model.

2) The term of temperature and velocity fluctuating is simulated by the temperature gradients:

$$-\overline{T'_c v'_{cj}} = \frac{k_T}{\rho C_v} \frac{\partial T}{\partial x_j} = \frac{\alpha_T}{\sigma_T} \frac{\partial T}{\partial x_i}$$

where  $\alpha_T = \frac{\nu_t}{\sigma_T}$  is defined as turbulent Prandtl number.

### 4.2 The simulation of charged dispersed phase

Relevant terms of turbulence fluctuating in the dispersed equation group are:

$$-\overline{\rho_p v'_{pi} v'_{pj}}, \quad \overline{\rho_p A'_q E'_i}, \quad -\overline{T'_p v'_{pj}}$$

1) Reynolds stress term of charged dispersed phase

$$-\overline{\rho_p v'_{pj} v'_{pi}} = \mu_p \left( \frac{\partial v_{pj}}{\partial x_i} + \frac{\partial v_{pi}}{\partial x_j} \right) - \frac{2}{3} \rho_p k_p \delta_{ij}$$

where  $\mu_p = \rho_p \nu_p$ , and

$$\nu_p = C_{\mu p} \frac{k_p^2}{\varepsilon_p}$$

Equations for solving the dispersed phase turbulent energy  $k_p$  are:

$$\frac{\partial k_p}{\partial t} + v_{pj} \frac{\partial k_p}{\partial x_j} = \frac{\partial}{\partial x_j} \left( \frac{\nu_p}{\sigma_p} \frac{\partial k_p}{\partial x_j} \right) + v_p \left( \frac{\partial v_{pi}}{\partial x_j} + \frac{\partial v_{pj}}{\partial x_i} \right) \frac{\partial v_{pi}}{\partial x_j} + \varepsilon_p$$

$$\varepsilon_p = v_{pi} \frac{v_{ci} - v_{pi}}{\tau_r}$$

2) The term of interaction between charged dispersed phase and electric field is:

$$\overline{\rho_p A'_q E'_i} = \frac{\nu_E}{\sigma_E} A_q \frac{\partial V}{\partial x_i}$$

where  $V$  is distributing function of voltage in the electric field and  $\sigma_E$  is Schmidt number of field fluctuating.

3) The term of temperature and velocity fluctuating is:

$$-\overline{T'_p v'_{pj}} = \frac{\nu_{pT}}{\sigma_{pT}} \frac{\partial T_p}{\partial x_j} = \alpha_{pT} \frac{\partial T_p}{\partial x_j}$$

## 5 Conclusions

Based on the analysis of forces in the charged two-phase flow and Reynolds transport equation, and under the assumption that particle phase is regarded as continuous phase, the sequence, momentum and energy equations for the charged dispersed phase and continuous phase have been obtained. Three-dimension basic equation group for the charged two-phase flows is established[6–8].

Based on the characteristics of charged two-phase turbulent flows, the dispersed phase has been regarded as one phase with the same physics property. The  $k - \varepsilon - k_p$  model for the charged two turbulent flow is deduced. In addition, the two phases are simulated separately in the model. So the problem is simplified. The comparison between  $k - \varepsilon - k_p$  model numerical results and experimental PIV data has shown that the model of the electrostatic charged two-phase flows is capable for the engineering simulation. The conclusion will be helpful for the development of the electrostatic charged two-phase flows.

## Acknowledgements

The authors would like to thank Dr. Li Changfeng for his helpful suggestions and comments.

## References

- [1] Wang Ze, Jin Hanhui: Numerical simulation of charged gas-liquid two-phase jet flows in electrostatic spraying.*Chinese Journal of Mechanical Engineering. (English Edition)*14(3): 266-270(2001)
- [2] Wang Junfeng, Wen Jianlong: Numerical Modeling of Charged Two-Phase Jet Flow in Electrostatic Spray.*Journal of JiangSu University.* 25(1): 13-16(2004)
- [3] Miller, J. A.Biblarz, O: Electrostatic spray modification in gas turbine combustion.*Journal of Propulsion and Power.*3(2): 187-192(1987)
- [4] Zhou Lixing: Recent advance in studies on multiphase and reacting flows in China.*Acta Mechanica Sinica.*18(2): 97-113(2002)
- [5] Lixin Tian, Guilong Gui, Yue Liu: On the well-posedness problem and the scattering problem for DGH equation.*Comm. Math. Phys.* .257(3):667-701(2005)
- [6] Lixin Tian, Qing Shi: Boundary control of viscous Dullin-Gottwald equation.*International Journal of Nonlinear Science.* 4(1):66-75(2007)
- [7] Jinlou Shi, Yongchui Zheng: The human resource flow problem.*International Journal of Nonlinear Science.* 3(3):235-240(2007)
- [8] Hua Chen, Mei Sun: Generalized projective synchronization of the energy resource system.*International Journal of Nonlinear Science.* 2(3):166-170(2006)