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The Existence, Uniqueness and Stability of Almost Periodic Solutions for Riccati Differential Equation

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Abstract: In this paper, Riccati differential equation is studied, by using the fixed point theorem, the existence and uniqueness of almost periodic solution of this system are gained, by using second method of Liapunov, the uniformly asymptotical stability and instability of the almost periodic solutions are obtained.

Key words: Riccati differential equation; almost periodic solutions; stability.
AMS2000 subject classification: 34C15; 34C25

1 Introduction

In research works, certain equations are often considered by scholars. Tian and Li[1] study the local well-posedness of the Cauchy problem for the generalized Degasperis-Proces equation; Ding and Tian[2] studied dissipative Camassa-Holm Equation and obtained the existence and uniqueness of the stationary solution belonging to absorbing set; G.A.Afrouzi1 et al.[3] obtained the existence of solutions to a non-autonomous p-Laplacian equation; Fan et al.[4] investigated multiple compactons in a nonlinear atomic chain equation.

This paper deals with the following Riccati differential equation

$$\frac{dx}{dt} = a(t)x^2 + b(t)x + c(t), \tag{1.1}$$

where $a(t), b(t), c(t)$ are all continuous functions defined on R.

About Riccati differential equation, many scholars have studied the periodic solutions of the equation (1.1), see [5-10], but about its almost periodic solution, yet related papers are not many, in [11], He Chongyou used the fixed point theorem and obtain the existence and uniqueness of the almost periodic solutions for Riccati equation (1.1), moreover, he used first degree approximation theory to discuss the stability of the almost periodic solutions and derived the sufficient conditions to guarantee the instability and asymptotically stability of the almost periodic solutions for Riccati equation (1.1).

In this paper, we further study the almost periodic solutions of the system (1.1), first, by using variable change and the fixed point theorem, we obtain the existence and uniqueness of the negative and the positive almost periodic solutions for Riccati equation, then by using second method of Liapunov, we discuss the stability of the almost periodic solutions and establish the sufficient conditions which guarantee the instability and uniformly asymptotically stability of the almost periodic solutions for the Riccati equation (1.1), the results we obtain are new.

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2 Preliminary Definition and Lemmas

It is well known to us that if $f(t)$ is almost periodic, and $f(t) \neq 0$, then $\frac{1}{f(t)}$ is also almost periodic. To simplify our writing, we introduce the following notations $\bar{f}, \underline{f}$, and we set $\bar{f} = \sup_{t\in R} f(t)$, $\underline{f} = \inf_{t\in R} f(t)$ and $m(f(t)) = \lim_{T\to+\infty} \frac{1}{T}\int_0^T f(s)ds$.

Consider the following system

$$\frac{dx}{dt} = a(t)x + f(t), \tag{2.1}$$

where $a(t), f(t) \in AP(E)$.

Lemma 2.1 (Coppel[12]) *If* $Re\ m(a(t)) \neq 0$*, then the equation (2.1) exists a unique almost periodic solution* $\varphi(t)$*, and* $\mod(\varphi) \subset \mod(a, f)$*,* $\varphi(t)$ *can be given as follows*

$$\varphi(t) = \begin{cases} -\int_t^{+\infty} e^{\int_s^t a(\tau)d\tau} f(s)ds, \ Re\ m(a(t)) > 0 \\ \int_{-\infty}^t e^{\int_s^t a(\tau)d\tau} f(s)ds, \ Re\ m(a(t)) < 0 \end{cases} \tag{2.2}$$

Consider the following system

$$\frac{dx}{dt} = f(t, x), \tag{2.3}$$

assume $f(t, x) \in C(I \times \Omega, R^n)$ *can guarantee the uniqueness property of solution of the initial value condition for the system (2.3) and* $f(t, 0) \equiv 0$ *for every* $t \in I$ *.*

Lemma 2.2 ([13]) *Suppose that there exist certain* $\varepsilon > 0$*,* $t_0 \in I; B_\varepsilon \subset \Omega$*, and exist an open set* $\psi \in B_\varepsilon$*,* $V : [t_0, +\infty) \times B_\varepsilon \to R$ *belonging to* C^1 *and is positive definite and not decreasing , such that when* $(t, x) \in [t_0, +\infty) \times \psi$*, the following conditions hold:*
(I) $0 < V(t, x) \leq b(|x|)$*, where* $b(r)$ *is a continuous positive nondecreasing function;*
(II)$V'(t, x) \geq a(|x|)$ *where* $a(r)$ *is a continuous positive nondecreasing function;*
(III) $x = 0$ *is in* $\partial\psi$*;*
(IV) $V(t, x) = 0$*,* $(t, x) \in [t_0, +\infty) \times (\partial\psi \cap B_\varepsilon)$*;*
then the null solution of the system (2.3) is unstable.

Lemma 2.3 ([14]) *Suppose that on the certain region* $G_H = \{(t, x) : t \geq t_0, \|x\| < H\}$*, there exist a function* $V(t, x)$ *and functions* a, b *where* $a(r), b(r)$ *are both continuous positive nondecreasing functions, such that the following conditions hold:*
(I) $0 \leq V(t, x) \leq a(|x|)$*;*
(II)$V'(t, x)|_{(2.3)} \leq -b(|x|)$*;*
then the null solution of the system (2.3) is uniformly asymptotically stable.

3 Main Results

Theorem 3.1 *Consider the system (1.1), suppose that* $a(t), b(t), c(t)$ *are all almost periodic functions in* $t \in R$*,* $b(t) < 0, a(t) < 0, 0 \leq c(t) \leq \frac{\bar{b}^2}{\eta \underline{a}^2}(-a(t))$*, where* η *is a positive constant which satisfies* $2 < \eta < +\infty$*,then there exists a unique negative almost periodic solution* $\varphi^*(t)$ *of the equation (1.1), and* $\mod(\varphi^*) \subset \mod(a, b, c)$*; moreover, if* $2\bar{a}\bar{b} - \underline{ab} > 0$*, then* $\varphi^*(t)$ *is unstable.*

Proof. Taking $x(t) = \frac{1}{u(t)}$, then the system (1.1) is changed to the following form

$$\frac{du}{dt} = -b(t)u - a(t) - c(t)u^2(t). \tag{3.1}$$

Since $a(t) < 0$, from (3.1), we know $u(t) = 0$ is not a solution of the system (3.1), thus if $u(t)$ is an almost periodic solution of the system (3.1), it follows $\inf_{t\in R} |u(t)| > 0$, then $x(t) = \frac{1}{u(t)}$ is an almost periodic solution of the system (1.1). Following we show the proof of Theorem 3.1 in two steps.

(I) Show that the equation (1.1) exists a unique almost periodic solution.
Define $B = \{\varphi(t); \varphi$ is an almost periodic function, $-\frac{\underline{a}}{\bar{\bar{b}}} \leq \varphi(t) \leq -\frac{(\eta-1)\bar{a}}{\eta \underline{b}}$, and $\mod(\varphi) \subset \mod(a,b,c)\}$. Take norm $\|\varphi\| = \sup_{t\in R} |\varphi(t)|$ on B, then $(B, \|\cdot\|)$ is a Banach space.

For any function $\varphi(t) \in B$, let us consider the following equation

$$\frac{du}{dt} = -b(t)u - a(t) - c(t)\varphi^2(t), \tag{3.2}$$

since $b(t) < 0$, $-b(t) > 0$, obviously $Re\, m(-b(t)) > 0$, according to Lemma 2.1, the system (3.2) exists a unique almost periodic solution $T\varphi(t)$, and $\mod(T\varphi(t)) \subset \mod(a,b,c)$, from (2.2), we have

$$T\varphi(t) = -\int_t^{+\infty} e^{\int_s^t -b(\tau)d\tau}[-a(s) - c(s)\varphi^2(s)]ds,$$

hence

$$\begin{aligned}
T\varphi(t) &= -\int_t^{+\infty} e^{\int_t^s b(\tau)d\tau}[-a(s) - c(s)\varphi^2(s)]ds \\
&\leq -\int_t^{+\infty} e^{\underline{b}(s-t)}[-a(s) - c(s)\varphi^2(s)]ds \\
&\leq -\int_t^{+\infty} e^{\underline{b}(s-t)}[-a(s) + a(s)\frac{\bar{b}^2}{\eta \underline{a}^2}\frac{\underline{a}^2}{\bar{\bar{b}}^2}]ds \\
&= -\int_t^{+\infty} e^{\underline{b}(s-t)}[-\frac{\eta-1}{\eta}a(s)]ds \\
&\leq -\frac{\eta-1}{\eta}\frac{\bar{a}}{\underline{b}}
\end{aligned}$$

Also

$$\begin{aligned}
T\varphi(t) &= -\int_t^{+\infty} e^{\int_t^s b(\tau)d\tau}[-a(s) - c(s)\varphi^2(s)]ds \\
&\geq -\int_t^{+\infty} e^{\bar{b}(s-t)}[-a(s)]ds \\
&\geq -\frac{\underline{a}}{\bar{\bar{b}}}
\end{aligned}$$

Hence $T\varphi(t) \in B$, so $T: B \to B$, for any $\varphi(t), \psi(t) \in B$, we have

$$\begin{aligned}
|T\varphi(t) - T\psi(t)| &= |-\int_t^{+\infty} e^{\int_t^s b(\tau)d\tau}\{-c(s)[\varphi^2(s) - \psi^2(s)]\}ds| \\
&\leq \int_t^{+\infty} e^{\int_t^s b(\tau)d\tau}|c(s)||\varphi(s) + \psi(s)||\varphi(s) - \psi(s)|ds \\
&\leq \int_t^{+\infty} e^{\int_t^s b(\tau)d\tau}|\frac{\bar{b}^2}{\eta \underline{a}^2}(-a(s))||\frac{2\underline{a}}{\bar{\bar{b}}}||\varphi(s) - \psi(s)|ds \\
&\leq \int_t^{+\infty} e^{\bar{b}(s-t)}\frac{\bar{b}^2}{\eta \underline{a}^2}|\underline{a}||\frac{2\underline{a}}{\bar{\bar{b}}}||\varphi(s) - \psi(s)|ds \\
&\leq \int_t^{+\infty} e^{\bar{b}(s-t)}\frac{\bar{b}^2}{\eta |\underline{a}|}|\frac{2\underline{a}}{\bar{\bar{b}}}||\varphi(s) - \psi(s)|ds \\
&\leq \frac{1}{\bar{b}}e^{\bar{b}(s-t)}|_t^{+\infty}\frac{\bar{b}^2}{\eta |\underline{a}|}|\frac{2\underline{a}}{\bar{\bar{b}}}|\|\varphi - \psi\| \\
&= -e^{\bar{b}(s-t)}|_t^{+\infty}\frac{\bar{b}^2}{\eta |\underline{a}|}\frac{2|\underline{a}|}{\bar{b}^2}\|\varphi - \psi\| \\
&= \frac{2}{\eta}\|\varphi - \psi\|
\end{aligned}$$

thus $\|T\varphi - T\psi\| \leq \frac{2}{\eta}\|\varphi - \psi\|$. Note that the condition $\eta > 2$, thus $\frac{2}{\eta} < 1$, therefore, T is a contraction mapping on B, T exists a unique fixed point on B, which is the unique negative almost periodic solution $u^*(t)$ of the system (3.1), and $\mod (u^*) \subset \mod (a, b, c)$, notice that $x(t) = \frac{1}{u(t)}$, thus the system (1.1) exists a unique negative almost periodic solution $\varphi^*(t) = \frac{1}{u^*(t)}$, and $\mod (\varphi^*) \subset \mod (a, b, c)$.

(II)we shall show the almost periodic solution $\varphi^*(t)$ of the system (1.1) is unstable.
Define a Liapunov function as follows

$$V(t, x(t) - \varphi^*(t)) = (x(t) - \varphi^*(t))^2, \tag{3.3}$$

where $x(t)$ is the solution of the system (1.1) with initial value (t_0, x_0), where $(t_0, x_0) \in (R \times R)$, $\varphi^*(t)$ is the unique almost periodic solution, obviously $V(t, x(t) - \varphi^*(t))$ satisfies the conditions (I),(III) and (IV) of Lemma 2.2.
Differentiating both sides of (3.3) along the solution of the system (1.1) gives
$V'(t, x(t) - \varphi^*(t))|_{(1.1)}$

$$\begin{aligned}
&= 2(x(t) - \varphi^*(t))(x'(t) - {\varphi^*}'(t)) \\
&= 2(x(t) - \varphi^*(t))\{a(t)x^2(t) + b(t)x(t) + c(t) - [a(t)\varphi^*(t)^2 + b(t)\varphi^*(t) + c(t)]\} \\
&= 2(x(t) - \varphi^*(t))\{a(t)[x^2(t) - \varphi^*(t)^2] + b(t)[x(t) - \varphi^*(t)]\} \\
&= 2(x(t) - \varphi^*(t))\{a(t)[x(t) - \varphi^*(t) + 2\varphi^*(t)][x(t) - \varphi^*(t)] + b(t)[x(t) - \varphi^*(t)]\} \\
&= 2a(t)(x(t) - \varphi^*(t))^3 + 4a(t)\varphi^*(t)(x(t) - \varphi^*(t))^2 + 2b(t)(x(t) - \varphi^*(t))^2 \\
&= 2a(t)(x(t) - \varphi^*(t))^3 + [4a(t)\varphi^*(t) + 2b(t)](x(t) - \varphi^*(t))^2
\end{aligned}$$

Note that

$$-\frac{\underline{a}}{\bar{b}} \leq \varphi(t) \leq -\frac{(\eta - 1)\bar{a}}{\eta \underline{b}},$$

thus

$$-\frac{\eta \underline{b}}{(\eta - 1)\bar{a}} \leq \varphi^*(t) \leq -\frac{\bar{b}}{\underline{a}},$$

therefore

$$\begin{aligned}
V'(t, x(t) - \varphi^*(t))|_{(1.1)} &\geq 2a(t)(x(t) - \varphi^*(t))^3 + (4a(t)(-\frac{\bar{b}}{\underline{a}}) + 2b(t))(x(t) - \varphi^*(t))^2 \\
&\geq 2a(t)(x(t) - \varphi^*(t))^3 + (4\bar{a}(-\frac{\bar{b}}{\underline{a}}) + 2\underline{b})(x(t) - \varphi^*(t))^2.
\end{aligned}$$

Notice that the condition $2\bar{a}\bar{b} - \underline{a}\underline{b} > 0$, thus there exists a small positive number $\delta > 0$ such that $2\bar{a}\bar{b} - \underline{a}\underline{b} > \delta > 0$ holds, hence we have

$$V'(t, x(t) - \varphi^*(t))|_{(1.1)} \geq 2a(t)(x(t) - \varphi^*(t))^3 + \delta(x(t) - \varphi^*(t))^2, \tag{3.4}$$

note that $[x(t) - \varphi^*(t)]^3$ is infinitesimal of higher order of $[x(t) - \varphi^*(t)]^2$ as $x(t) \to \varphi^*(t)$, from (3.4), there exist a neighborhood $D \subset R^n$ of $\varphi^*(t)$ and a small positive constant ε such that

$$V'(t, x(t) - \varphi^*(t))|_{(1.1)} \geq \varepsilon(x(t) - \varphi^*(t))^2$$

holds when $(t, x) \in [t_0, +\infty) \times D$, by Lemma 2.2, the almost periodic solution $\varphi^*(t)$ of the system (1.1) is unstable. ■

Theorem 3.2 *Consider the system (1.1), suppose that $a(t), b(t), c(t)$ are all almost periodic functions in t, $b(t) > 0, a(t) < 0, 0 \leq c(t) \leq \frac{b^2}{\eta \underline{a}^2}(-a(t))$, where η is a positive constant which satisfies $2 < \eta < +\infty$,then there exists a unique positive almost periodic solution $\varphi^*(t)$ of the equation (1.1), and $\mod (\varphi^*) \subset \mod (a, b, c)$; moreover, if $-2\bar{a}\underline{b} + \underline{a}\bar{b} < 0$, then $\varphi^*(t)$ is uniformly asymptotically stable.*

Proof. Since $a(t) < 0$, from (3.1), we know $u(t) = 0$ is not a solution of the system (3.1), thus if $u(t)$ is an almost periodic solution of the system (3.1), it follows $\inf_{t\in R} |u(t)| > 0$, then $x(t) = \frac{1}{u(t)}$ is an almost periodic solution of the system (1.1). Following we show the proof of Theorem 3.2 in two steps.
(I) Show that the equation (1.1) exists a unique almost periodic solution.
Define $B = \{\varphi(t); \varphi$ is an almost periodic function, $\frac{\eta-1}{\eta}(\frac{-\bar{a}}{\bar{b}}) \leq \varphi(t) \leq -\frac{a}{\underline{b}}$, and $\mod(\varphi) \subset \mod(a,b,c)\}$. Take norm $\|\varphi\| = \sup_{t\in R} |\varphi(t)|$ on B, then $(B, \|\cdot\|)$ is a Banach space.

For any function $\varphi(t) \in B$, let us consider the following equation

$$\frac{du}{dt} = -b(t)u - a(t) - c(t)\varphi^2(t), \tag{3.5}$$

since $b(t) > 0$, $-b(t) < 0$, obviously $Re\, m(-b(t)) < 0$, according to Lemma 2.1, the system (3.5) exists a unique almost periodic solution $T\varphi(t)$, and $\mod(T\varphi(t)) \subset \mod(a,b,c)$, from (2.2), we have

$$T\varphi(t) = \int_{-\infty}^{t} e^{\int_s^t -b(\tau)d\tau}[-a(s) - c(s)\varphi^2(s)]ds,$$

hence

$$\begin{aligned} T\varphi(t) &\leq \int_{-\infty}^{t} e^{-\underline{b}(t-s)}[-a(s)]ds \\ &\leq -\frac{a}{\underline{b}} \end{aligned}$$

Also

$$\begin{aligned} T\varphi(t) &= \int_{-\infty}^{t} e^{\int_s^t -b(\tau)d\tau}[-a(s) - c(s)\varphi^2(s)]ds \\ &\geq \int_{-\infty}^{t} e^{-\bar{b}(t-s)}[-a(s) - \frac{\underline{b}^2}{\eta \underline{a}^2}(-a(s))(-\frac{a}{\underline{b}})^2]ds \\ &\geq \frac{\eta-1}{\eta}\int_{-\infty}^{t} e^{-\bar{b}(t-s)}[-a(s)]ds \\ &\geq \frac{\eta-1}{\eta}(\frac{-\bar{a}}{\bar{b}}) \end{aligned}$$

Hence $T\varphi(t) \in B$, so $T : B \to B$, for any $\varphi(t), \psi(t) \in B$, we have

$$\begin{aligned} |T\varphi(t) - T\psi(t)| &= |\int_{-\infty}^{t} e^{\int_s^t -b(\tau)d\tau}\{-c(s)[\varphi^2(s) - \psi^2(s)]\}ds| \\ &\leq \int_{-\infty}^{t} e^{\int_s^t -b(\tau)d\tau}|c(s)||\varphi(s) + \psi(s)||\varphi(s) - \psi(s)|ds \\ &\leq \int_{-\infty}^{t} e^{-\underline{b}(t-s)}|\frac{\underline{b}^2}{\eta \underline{a}^2}(-a(s))||2(-\frac{a}{\underline{b}})||\varphi(s) - \psi(s)|ds \\ &\leq \int_{-\infty}^{t} e^{-\underline{b}(t-s)}\frac{\underline{b}^2}{\eta \underline{a}^2}|\underline{a}||2(-\frac{a}{\underline{b}})||\varphi(s) - \psi(s)|ds \\ &\leq \int_{-\infty}^{t} e^{-\underline{b}(t-s)}\frac{\underline{b}^2}{\eta|\underline{a}|}2\frac{|\underline{a}|}{|\underline{b}|}||\varphi(s) - \psi(s)|ds \\ &\leq \frac{1}{\underline{b}}e^{-\underline{b}(t-s)}|_{-\infty}^{t}\frac{\underline{b}^2}{\eta|\underline{a}|}2\frac{|\underline{a}|}{|\underline{b}|}\|\varphi - \psi\| \\ &= e^{-\underline{b}(t-s)}|_{-\infty}^{t}\frac{\underline{b}^2}{\eta|\underline{a}|}2\frac{|\underline{a}|}{\underline{b}^2}\|\varphi - \psi\| \\ &= \frac{2}{\eta}\|\varphi - \psi\| \end{aligned}$$

thus

$$\|T\varphi - T\psi\| \leq \frac{2}{\eta}\|\varphi - \psi\|$$

Note that the condition $\eta > 2$, thus $\frac{2}{\eta} < 1$, therefore, T is a contraction mapping on B, T exists a unique fixed point on B, which is the unique positive almost periodic solution $u^*(t)$ of the system (3.1), and $\mod (u^*) \subset \mod (a, b, c)$, notice that $x(t) = \frac{1}{u(t)}$, thus the system (1.1) exists a unique positive almost periodic solution $\varphi^*(t) = \frac{1}{u^*(t)}$,and $\mod (\varphi^*) \subset \mod (a, b, c)$.

(II)we shall show the almost periodic solution $\varphi^*(t)$ of the system (1.1) is uniformly asymptotically stable

Define a Liapunov function as follows

$$V(t, x(t) - \varphi^*(t)) = (x(t) - \varphi^*(t))^2, \tag{3.6}$$

where $x(t)$ is the solution of the system (1.1) with initial value (t_0, x_0), where $(t_0, x_0) \in (R \times R)$, $\varphi^*(t)$ is the unique almost periodic solution of the system (1.1), obviously $V(t, x(t) - \varphi^*(t))$ satisfies the conditions (I) of Lemma 2.3.

Differentiating both sides of (3.6) along the solution of the system (1.1) gives

$V'(t, x(t) - \varphi^*(t))|_{(1.1)}$

$$\begin{aligned}
&= 2(x(t) - \varphi^*(t))(x'(t) - {\varphi^*}'(t)) \\
&= 2(x(t) - \varphi^*(t))\{a(t)x^2(t) + b(t)x(t) + c(t) - [a(t)\varphi^*(t)^2 + b(t)\varphi^*(t) + c(t)]\} \\
&= 2(x(t) - \varphi^*(t))\{a(t)[x^2(t) - \varphi^*(t)^2] + b(t)[x(t) - \varphi^*(t)]\} \\
&= 2(x(t) - \varphi^*(t))\{a(t)[x(t) - \varphi^*(t) + 2\varphi^*(t)][x(t) - \varphi^*(t)] + b(t)[x(t) - \varphi^*(t)]\} \\
&= 2a(t)(x(t) - \varphi^*(t))^3 + 4a(t)\varphi^*(t)(x(t) - \varphi^*(t))^2 + 2b(t)(x(t) - \varphi^*(t))^2 \\
&= 2a(t)(x(t) - \varphi^*(t))^3 + [4a(t)\varphi^*(t) + 2b(t)](x(t) - \varphi^*(t))^2
\end{aligned}$$

Note that

$$\frac{\eta - 1}{\eta}(\frac{-\bar{a}}{\bar{b}}) \leq \varphi(t) \leq -\frac{\underline{a}}{\underline{b}},$$

thus

$$-\frac{\underline{b}}{\underline{a}} \leq \varphi^*(t) \leq \frac{\eta}{\eta - 1}(-\frac{\bar{b}}{\bar{a}}),$$

therefore

$$\begin{aligned}
V'(t, x(t) - \varphi^*(t))|_{(1.1)} &\leq 2a(t)(x(t) - \varphi^*(t))^3 + (4a(t)(-\frac{\bar{b}}{\underline{a}}) + 2b(t))(x(t) - \varphi^*(t))^2 \\
&\leq 2a(t)(x(t) - \varphi^*(t))^3 + (4\bar{a}(-\frac{\underline{b}}{\underline{a}}) + 2\bar{b})(x(t) - \varphi^*(t))^2.
\end{aligned}$$

Notice that the condition $-2\bar{a}\underline{b} + \underline{a}\bar{b} < 0$, thus there exists a positive small number $\delta > 0$ such that $-2\bar{a}\underline{b} + \underline{a}\bar{b} < -\delta < 0$ holds, hence we have

$$V'(t, x(t) - \varphi^*(t))|_{(1.1)} \leq 2a(t)(x(t) - \varphi^*(t))^3 - \delta(x(t) - \varphi^*(t))^2, \tag{3.7}$$

note that $[x(t) - \varphi^*(t)]^3$ is infinitesimal of higher order of $[x(t) - \varphi^*(t)]^2$ as $x(t) \to \varphi^*(t)$, from (3.7), there exist a neighborhood $D \subset R^n$ of $\varphi^*(t)$ and a small positive constant ε such that

$$V'(t, x(t) - \varphi^*(t))|_{(1.1)} \leq -\varepsilon(x(t) - \varphi^*(t))^2$$

holds when $(t, x) \in [t_0, +\infty) \times D$, hence $V(t, x(t) - \varphi^*(t))$ also satisfies the conditions (II) of Lemma 2.3, by Lemma 2.3, the almost periodic solution $\varphi^*(t)$ of the system (1.1) is uniformly asymptotically stable. ■

4 Conclusion

Riccati Differential Equation is a famous differential equation, but the General Solutions of the Equation can not be expressed by elementary functions or integrations of elementary functions. In this paper, we obtain the existence and uniqueness of the negative and the positive almost periodic solutions for Riccati equation by using variable change and the fixed point theorem, and further discuss the stability of the almost periodic solutions for the Riccati equation, our results extend and improve some existing results .

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