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An Extended Generalized F-expansion Method for Solving Coupled MKDV Equations

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Abstract: An extended generalized F-expansion method is proposed and applied to the coupled MKDV equations. Many new and more general exact travelling wave solutions are obtained including trigonometric function solutions, hyperbolic function solutions and rational solutions.

Key words: extended generlized F-expansion method; exact solution; coupled MKDV equations, Mathematica
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1 Introduction

It is well known that nonlinear complex physical phenomena are related to nonlinear partial differential equations (NLPDEs) which are involved in many fields from physics to biology, chemistry, mechanics, etc. As mathematical models of the phenomena, the investigation of exact solutions of NLPDEs will help one to understand these phenomenon better. Various methods for obtaining exact solutions of NLPDEs have been presented, such as variational iteration method[1], homogeneous balance method[2], Sine-Gordon expansion method[3], truncated Painleve expansion[4], Adomian decomposition method[5,6], Hirota methods[7], inverse scattering method[8], algebraic method[9], and so on. Recently, F-expansion method[10] was proposed to construct periodic wave solutions of NLPDEs, which can be thought of as an overall generalization of Jacobi elliptic function expansion method. F-expansion method was later further extended in different manners, for example, the modified F-expansion method[11,12], the generalized F-expansion method[13] etc.

In this Letter, we propose an extended generlized F-expansion method to construct more general exact solutions of NLPDEs. In order to illustrate the convenience of the method, we will consider the coupled MKDV Eqs:

$$\mathrm{u}_t = \frac{1}{2}u_{xxx} - 3u^2\mathrm{u}_x + \frac{3}{2}v_{xx} + 3(uv)_x - 3\lambda\mathrm{u}_x \tag{1}$$

$$v_t = -v_{xxx} - 3vv_x - 3u_xv_x + 3u^2v_x + 3\lambda v_x \tag{2}$$

where λ is constant. The solutions of coupled MKDV Eqs. possess their actual physical application; this is the reason why so many methods, such as trigonometric function transform method[14], extended tanh-function method[15,16], algebraic method[17], modified extended tanh-function (METF) method[18], a new generalized ansatz in homogeneous banlance method[19] have been applied to obtain exact solutions of the coupled MKDV Eqs., but what they gave are quite simple and little in quantity. In this letter, we obtain many more generalized exact solutions of the coupled MKDV Eqs., most of them are newly proposed.

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The rest of this letter is organized as follows: in Section 2, we give the description of the extended generalized F-expansion method; in Section 3, we apply this method to the coupled MKDV Eqs.; in Section 4, some conclusions are given.

2 Description of the extended generalized F-expansion method

For a given NLPDE with independent variables $x = (t, x_1, x_2, \cdots x_n)$ and dependent variable u:

$$F(u, u_t, u_{x_1}, u_{x_2}, \cdots, u_{x_m}, u_{x_1 t}, u_{x_2 t}, \cdots, u_{x_m t}, u_{tt}, u_{x_1 x_1}, u_{x_2 x_2}, \cdots, u_{x_m x_m}, \cdots) = 0 \quad (3)$$

We seek the solutions in the new and more general form:

$$u = a_0 + \sum_{i=1}^{n} \{a_i F^i(\xi) + b_i F^{-i}(\xi) + c_i F^{i-1}(\xi)F'(\xi) + d_i F^{-i}(\xi)F'(\xi)\} \quad (4)$$

where ξ=k(x+ct), and k, c are both constants ($k \neq$0). Here k denotes value of waves, c denotes speed of waves, a_i(i=-n,..., 0,..., n), k, c are all constants to be determined, $F(\xi)$ and F'(ξ) in (4) satisfy

$$F'(\xi) = A + BF(\xi) + MF^2(\xi) \quad (5)$$

where A, B and M are all parameters, the prime denotes $d/d\xi$. Given different values of A, B and M, the different Riccati function solution $F(\xi)$ can be obtained from equation (5) (see Table 1). To determine u explicitly, we take the following four steps:

Step 1. Determine the integer n by balancing the highest order nonlinear terms and the highest order partial derivative of u in Eq. (3).

Step 2. Substitute (4) along with (5) into Eq. (3) and collect coefficients of $F'^i(\xi)F^j(\xi)$ (i=1,..., n), then set each coefficient to zero to derive a set of over-determined partial differential Eqs. for a_0, a_i, b_i, c_i , d_i (i=1,2, ..., n) and ξ.

Step 3. Solve the system of over-determined partial differential Eqs. obtained in Step 2 for a_0, a_i, b_i, c_i , d_i (i=1,2, ..., n) and ξ by use of Mathematica.

Step 4. Select A, B, M and $F(\xi)$ from Table 1 and substitute them along with a_0, a_i, b_i, c_i , d_i (i=0,1; j=0, $\pm$1, ..., $\pm n$) and ξ into (4) to obtain Riccati function solutions of Eq. (3) (see Table 1 for $F'^i(\xi)$), from which hyperbolic function solutions trigonometric function solutions and rational solutions can be obtained.

Relations between values of (A, B, M) and corresponding $F(\xi)$ in Riccati Eq.

$$F'(\xi) = A + BF(\xi) + MF^2(\xi)$$

are listed in Table 1.

Table 1: The relations between values of (A, B, M) and corresponding $F(\xi)$ in Riccati Equation

A	B	M	F(ξ)
0	1	-1	$\frac{1}{2} + \frac{1}{2}\tanh(\frac{1}{2}\xi)$
0	-1	1	$\frac{1}{2} - \frac{1}{2}\coth(\frac{1}{2}\xi)$
$\frac{1}{2}$	0	$-\frac{1}{2}$	$coth(\xi) \pm csch(\xi)$, $tanh(\xi) \pm isech(\xi)$
1	0	-1	$tanh(\xi)$, $coth(\xi)$
$\frac{1}{2}$	0	$\frac{1}{2}$	$sec(\xi)+tan(\xi)$, $csc(\xi)-cot(\xi)$
$-\frac{1}{2}$	0	$-\frac{1}{2}$	$sec(\xi)-tan(\xi)$, $csc(\xi)+cot(\xi)$
1(-1)	0	1(-1)	$tan(\xi)$, $cot(\xi)$
0	0	$\neq 0$	-1/($C^*\xi$+ m) (m is an arbitrary constant)
arbitrary constants	0	0	$A\xi$
arbitrary constants	$\neq 0$	0	$\frac{\exp(B\xi)-A}{B}$

3 Exact solutions of the coupled MKDV Eqs.

Let us consider the coupled MKDV Eqs. (1), (2). We use the transformations $u(x,t) = U(\xi)$, $v(x,t) = V(\xi)$ with ξ=x+ct. Eqs. (1), (2) are transformed into the following form:

$$cU'(\xi) - \frac{1}{2}U'''(\xi) + 3U^2(\xi)U'(\xi) - \frac{3}{2}V''(\xi) - 3(U(\xi)V(\xi))' + 3\lambda U'(\xi) = 0 \quad (6)$$

$$cV'(\xi) + V'''(\xi) + 3V(\xi)V'(\xi) + 3U'(\xi)V'(\xi) - 3U^2(\xi)V'(\xi) - 3\lambda V'(\xi) = 0 \quad (7)$$

By balancing the order of $U^2(\xi)U'(\xi)$ with the order of $U'''(\xi)$ and the order of $U^2(\xi)V'(\xi)$ with $V'''(\xi)$ in Eqs. (6), (7), we get n=1. Thus the ansatz solutions of Eqs. (1), (2) can be expressed by

$$U(\xi) = \gamma_1 + a_1F(\xi) + b_1F(\xi)^{-1} + c_1F'(\xi) + d_1F(\xi)^{-1}F'(\xi) \quad (8)$$

$$V(\xi) = \gamma_2 + a_2F(\xi) + b_2F(\xi)^{-1} + c_2F'(\xi) + d_2F(\xi)^{-1}F'(\xi) \quad (9)$$

With the aid of Mathematica, substituting (8), (9) along with (4) and (5) into Eqs. (6)- (7), the left -hand side of Eqs. (6),(7) are converted into finite series of$F'^i(\xi)$ $F^j(\xi)$ (i=0,1; j=0, ±1,. . . ,$\pm n$), then setting each coefficient to zero, we derive a set of over-determined partial differential Eqs. for $\gamma_1, a_1, b_1, c_1, d_1, \gamma_2, a_2, b_2, c_2, d_2$ and c. Solving these over-determined partial differential Eqs. by use of Mathematica, we get the following results:

Case 1.

$$b_2 = -AB + \frac{Aa_2}{M} + 2ABd_1 + 2A\gamma_1, \gamma_2 = \frac{1}{2}(-B^2 + 2\lambda + \frac{2Ba_2}{M} + 2B^2d_1 + 2B\gamma_1), \quad d_2 = -\frac{a_2}{M},$$

$$c = \frac{1}{2}(B^2 + 2AM - 6B^2d_1 + 6B^2d_1^2 - 6B\gamma_1 + 12Bd_1\gamma_1 + 6\gamma_1^2), \quad a_1 = -Md_1, \quad b_1 = A - Ad_1,$$

where γ_1, c_1, d_1, a_2 and c_2 are arbitrary constants.

Case 2.

$$d_2 = -\frac{a_2}{M}, \quad \gamma_2 = \frac{B^2M + 4cM - 4AM^2 + 12M\lambda + 12Ba_2}{12M},$$

$$b_2 = \frac{Aa_2}{M}, \quad \gamma_1 = \frac{1}{2}(-B - 2Bd_1), \quad b_1 = -Ad_1, \quad a_1 = -M - Md_1,$$

where c_1, d_1, a_2, c_2 and c are arbitrary constants.

Case 3.

$$b_2 = \frac{Aa_2}{M}, \quad b_1 = -Ad_1, \quad a_1 = -Md_1, \quad d_2 = -\frac{a_2}{M},$$

where $\gamma_1, c_1, d_1, \gamma_2, a_2, c_2$and $c$are arbitrary constants.

Case 4.

$$\gamma_2 = \frac{1}{12M}(B^2M + 4cM - 4AM^2 + 12M\lambda + 12Ba_2), \quad b_2 = \frac{Aa_2}{M},$$

$$\gamma_1 = \frac{1}{2}(B - 2Bd_1), \quad b_1 = -Ad_1, \quad a_1 = M - Md_1, \quad d_2 = -\frac{a_2}{M},$$

where c_1, d_1, a_2, c_2 and c are arbitrary constants.

Case 5.

$$\gamma_2 = \frac{1}{12M}(B^2M + 4cM + 8AM^2 + 12M\lambda + 12Ba_2), \quad b_2 = \frac{Aa_2}{M},$$

$$\gamma_1 = \frac{1}{2}(-B - 2Bd_1), \quad b_1 = -A - Ad_1, \quad a_1 = -M - Md_1, \quad d_2 = -\frac{a_2}{M},$$

where c_1, d_1, a_2, c_2 and c are arbitrary constants.

Case 6.

$$\gamma_2=\frac{1}{12M}(B^2M+4cM+8AM^2+12M\lambda+12Ba_2),\quad b_2=\frac{Aa_2}{M},\quad \gamma_1=\frac{1}{2}(B-2Bd_1),$$
$$b_1=A-Ad_1,\quad a_1=M-Md_1,\quad d_2=-\frac{a_2}{M},$$

where c_1,d_1,a_2,c_2 and c are arbitrary constants.

Case 7.

$$\gamma_2=\frac{1}{12}(B^2+4c-4AM+12\lambda+\frac{12Ba_2}{M}),\quad b_2=\frac{Aa_2}{M},$$
$$\gamma_1=\frac{1}{2}(-B-2Bd_1),\quad b_1=-A-Ad_1,a_1=-Md_1,\quad d_2=-\frac{a_2}{M},$$

where c_1,d_1,a_2,c_2 and c are arbitrary constants.

Case 8.

$$\gamma_2=\frac{1}{12}(B^2+4c-4AM+12\lambda+\frac{12Ba_2}{M}),\quad b_2=\frac{Aa_2}{M},\quad \gamma_1=\frac{1}{2}(B-2Bd_1),$$
$$a_1=-Md_1,\quad b_1=A-Ad_1,\quad d_2=-\frac{a_2}{M},\quad a_1=M-Md_1,$$

where c_1,d_1,a_2,c_2 and c are arbitrary constants.

Case 9.

$$\gamma_2=\tfrac{1}{12}(B^2+4c+8AM+12\lambda+\tfrac{12Ba_2}{M}),\quad b_2=\tfrac{Aa_2}{M},$$
$$\gamma_1=\tfrac{1}{2}(B-2Bd_1),\quad b_1=A-Ad_1,\quad d_2=-\tfrac{a_2}{M},\quad b_1=A-Ad_1,\quad d_2=-\tfrac{a_2}{M},$$

where c_1,d_1,a_2,c_2 and c are arbitrary constants.

Substituting Cases 1-9 into (8), (9) respectively, we have nine kinds of formal solutions of Eq. (8), (9). Taking case 1 for example, we get the following exact solutions of the coupled MKDV Eqs.:

Now we give the formal solutions of the coupled MKDV Eqs. in case 1:

$$\begin{cases} U(\xi)=-MF(\xi)d_1+\frac{A-Ad_1}{F(\xi)}+\gamma_1+c_1F'(\xi)+\frac{d_1F'(\xi)}{F(\xi)} \\ V(\xi)=F(\xi)a_2+\frac{-AB+\frac{Aa_2}{M}+2ABd_1+2A\gamma_1}{F(\xi)}+\frac{1}{2}(-B^2+2\lambda+\frac{2Ba_2}{M}+2B^2d_1+2B\gamma_1) \end{cases} \quad (a)$$

where ξ=x+ct, $c=\frac{1}{2}(B^2+2AM-6B^2d_1+6B^2d_1^2-6B\gamma_1+12Bd_1\gamma_1+6\gamma_1^2)$.

1) From Table 1, choosing A=0, B=1, M=-1, $F(\xi)=\frac{1}{2}+\frac{1}{2}tanh(\frac{\xi}{2})$, inserting them into ($a$), we obtain

$$\begin{cases} u_1(x,t)=\frac{c_1}{4}sech^2(\frac{\xi}{2})+\gamma_1+\frac{d_1sech^2(\frac{\xi}{2})}{(2+2\tanh(\frac{\xi}{2}))}+d_1(\frac{1}{2}+\frac{1}{2}\tanh(\frac{\xi}{2})) \\ v_1(x,t)=\frac{c_2}{4}sech^2(\frac{\xi}{2})+\frac{1}{2}(-1+2\lambda-2a_2+2d_1+2\gamma_1)+\frac{a_2sech^2(\frac{\xi}{2})}{(2+2\tanh(\frac{\xi}{2}))}+a_2(\frac{1}{2}+\frac{1}{2}\tanh(\frac{\xi}{2})) \end{cases}$$

where ξ=x+ct, $c=\frac{1}{2}(1-6d_1+6d_1^2-6\gamma_1+12d_1\gamma_1+6\gamma_1^2)$,$a_2$, d_1, γ_1,c_2, λ, $c_1$are arbitrary constants.

2) From Table 1, choosing A=0, B=-1, M=1, $F(\xi)$ =1/2-(coth (ξ/2)) /2, inserting them into (a), we obtain

$$\begin{cases} u_2(x,t)=\frac{c_1}{4}\csc h^2(\frac{\xi}{2})-d_1(\frac{1}{2}-\frac{1}{2}\coth(\frac{\xi}{2}))+\frac{d_1\csc h^2(\frac{\xi}{2})}{2-2\coth(\frac{\xi}{2})}+\gamma_1 \\ v_2(x,t)=a_2(\frac{1}{2}-\frac{1}{2}\coth(\frac{\xi}{2}))-\frac{a_2\csc h^2(\frac{\xi}{2})}{2-2\coth(\frac{\xi}{2})}+\frac{c_2}{4}\csc h^2(\frac{\xi}{2})+\frac{1}{2}(-1+2\lambda-2a_2+2d_1-2\gamma_1) \end{cases}$$

where ξ=x+ct, $c=\frac{1}{2}(1-6d_1+6d_1^2+6\gamma_1-12d_1\gamma_1+6\gamma_1^2)$,$a_2$, d_1, γ_1,c_2, λ, $c_1$are arbitrary constants.

3) From Table 1, choosing A=1/2, B=0, M=-1/2, $F(\xi)$=$coth(\xi)\pm csch(\xi)$ or $tanh(\xi)\pm isech(\xi)$, inserting them into (a), we obtain

$$\begin{cases} u_{3(1)}(x,t)=\frac{d_1(-\coth(\xi)\csc h(\xi)-\csc h^2(\xi))}{\coth(\xi)+\csc h(\xi)}+\frac{\frac{1}{2}-\frac{d_1}{2}}{\coth(\xi)+\csc h(\xi)}+\frac{d_1}{2}(\coth(\xi)+\csc h(\xi)) \\ \qquad +c_1(-\coth(\xi)\csc h(\xi)-\csc h^2(\xi))+\gamma_1 \\ v_{3(1)}(x,t)=\frac{-a_2+\gamma_1}{\coth(\xi)+\csc h(\xi)}+\frac{2a_2(-\coth(\xi)\csc h(\xi)-\csc h^2(\xi))}{\coth(\xi)+\csc h(\xi)}+a_2(\coth(\xi)+\csc h(\xi)) \\ \qquad +c_2(-\coth(\xi)\csc h(\xi)-\csc h^2(\xi))+\lambda \end{cases}$$

$$\begin{cases} u_{3(2)}(x,t)=\frac{d_1(\coth(\xi)\csc h(\xi)-\csc h^2(\xi))}{\coth(\xi)-\csc h(\xi)}+\frac{\frac{1}{2}-\frac{d_1}{2}}{\coth(\xi)-\csc h(\xi)}+\frac{d_1}{2}(\coth(\xi)-\csc h(\xi)) \\ \qquad +c_1(\coth(\xi)\csc h(\xi)-\csc h^2(\xi))+\gamma_1 \\ v_{3(2)}(x,t)=\frac{-a_2+\gamma_1}{\coth(\xi)-\csc h(\xi)}+\frac{2a_2(\coth(\xi)\csc h(\xi)-\csc h^2(\xi))}{\coth(\xi)-\csc h(\xi)}+a_2(\coth(\xi)-\csc h(\xi)) \\ \qquad +c_2(\coth(\xi)\csc h(\xi)-\csc h^2(\xi))+\lambda \end{cases}$$

$$\begin{cases} u_{3(3)}(x,t)=\frac{\frac{1}{2}-\frac{d_1}{2}}{i\sec h(\xi)+\tanh(\xi)}+\frac{d_1}{2}(i\sec h(\xi)+\tanh(\xi))+\frac{d_1(\sec h^2(\xi)-i\sec h(\xi)\tanh(\xi))}{i\sec h(\xi)+\tanh(\xi)} \\ \qquad +c_1(\sec h^2(\xi)-i\sec h(\xi)\tanh(\xi))+\gamma_1 \\ v_{3(3)}(x,t)=\frac{-a_2+\gamma_1}{i\sec h(\xi)+\tanh(\xi)}+a_2(i\sec h(\xi)+\tanh(\xi))+\frac{2a_2(\sec h^2(\xi)-i\sec h(\xi)\tanh(\xi))}{i\sec h(\xi)+\tanh(\xi)} \\ \qquad +c_2(\sec h^2(\xi)-i\sec h(\xi)\tanh(\xi))+\lambda \end{cases}$$

$$\begin{cases} u_{3(4)}(x,t)=\frac{\frac{1}{2}-\frac{d_1}{2}}{-i\sec h(\xi)+\tanh(\xi)}+\frac{d_1}{2}(-i\sec h(\xi)+\tanh(\xi))+\frac{d_1(\sec h^2(\xi)+i\sec h(\xi)\tanh(\xi))}{-i\sec h(\xi)+\tanh(\xi)} \\ \qquad +c_1(\sec h^2(\xi)+i\sec h(\xi)\tanh(\xi))+\gamma_1 \\ v_{3(4)}(x,t)=\frac{-a_2+\gamma_1}{-i\sec h(\xi)+\tanh(\xi)}+a_2(-i\sec h(\xi)+\tanh(\xi))+\frac{2a_2(\sec h^2(\xi)+i\sec h(\xi)\tanh(\xi))}{-i\sec h(\xi)+\tanh(\xi)} \\ \qquad +c_2(\sec h^2(\xi)+i\sec h(\xi)\tanh(\xi))+\lambda \end{cases}$$

where $\xi=x+ct$, $c=\frac{1}{2}(-\frac{1}{2}+6\gamma_1^2)$, a_2, d_1, γ_1,c_2, λ, $c_1$are arbitrary constants.

4) From Table 1, choosing A=1, B=0, M=-1, $F(\xi)$ =*tanh*(ξ) or *coth*(ξ), inserting them into (a), we obtain

$$\begin{cases} u_{4(1)}(x,t)=c_1\sec h^2(\xi)+(1-d_1)coth(\xi)+d_1\csc h(\xi)\sec h(\xi)+\gamma_1+d_1tanh(\xi) \\ v_{4(1)}(x,t)=\lambda+a_2\csc h(\xi)\sec h(\xi)+c_2\sec h^2(\xi)+(2\gamma_1-a_2)coth(\xi)+a_2tanh(\xi) \end{cases}$$

$$\begin{cases} u_{4(2)}(x,t)=-c_1csch^2(\xi)+d_1\coth(\xi)-d_1\csc h(\xi)\sec h(\xi)+\gamma_1+(1-d_1)\tanh(\xi) \\ v_{4(2)}(x,t)=\lambda+a_2\coth(\xi)-a_2csch(\xi)sech(\xi)-c_2csch(\xi)+(2\gamma_1-a_2)coth^2(\xi)+(2\gamma_1-a_2)tanh(\xi) \end{cases}$$

where $\xi=x+ct$, $c=\frac{1}{2}(-2+6\gamma_1^2)$, a_2, d_1, γ_1,c_2, λ, $c_1$are arbitrary constants.

5) From Table 1, choosing A=1/2, B=0, M=1/2, $F(\xi)$=*sec*(ξ)+*tan*(ξ) or *csc*(ξ)-*cot*(ξ), inserting them into (a), we obtain

$$\begin{cases} u_{5(1)}(x,t)=\gamma_1+\frac{\frac{1}{2}-\frac{d_1}{2}}{sec(\xi)+tan(\xi)}-\frac{d_1}{2}(sec(\xi)+tan(\xi))+c_1(sec^2(\xi)+sec(\xi)tan(\xi))+\frac{d_1(sec^2(\xi)+sec(\xi)tan(\xi))}{(sec(\xi)+tan(\xi))} \\ v_{5(1)}(x,t)=\lambda+\frac{a_2+\gamma_1}{sec(\xi)+tan(\xi)}+a_2(sec(\xi)+tan(\xi))+c_2(sec^2(\xi)+sec(\xi)tan(\xi))-\frac{2a_2(sec^2(\xi)+sec(\xi)tan(\xi))}{sec(\xi)+tan(\xi)} \end{cases}$$

$$\begin{cases} u_{5(2)}(x,t)=c_1(csc^2(\xi)-cot(\xi)csc(\xi))+\frac{\frac{1}{2}-\frac{d_1}{2}}{csc(\xi)-cot(\xi)}-\frac{d_1}{2}(csc(\xi)-cot(\xi))+\frac{d_1(csc^2(\xi)-cot(\xi)csc(\xi))}{csc(\xi)-cot(\xi)}+\gamma_1 \\ v_{5(2)}(x,t)=\lambda+c_2(csc^2(\xi)-csc(\xi)cot(\xi))-\frac{2a_2(csc^2(\xi)-csc(\xi)cot(\xi))}{csc(\xi)-cot(\xi)}+\frac{a_2+\gamma_1}{csc(\xi)-cot(\xi)}+a_2(csc(\xi)-cot(\xi)) \end{cases}$$

where $\xi=x+ct$, $c=\frac{1}{2}(\frac{1}{2}+6\gamma_1^2)$, a_2, d_1, γ_1,c_2, λ, $c_1$are arbitrary constants.

6) From Table 1, choosing A=-1/2, B=0, M=-1/2, $F(\xi)$ =*sec*(ξ)-*tan*(ξ) or *csc*(ξ)+*cot*(ξ), inserting them into (a), we obtain

$$\begin{cases} u_{6(1)}(x,t)=\gamma_1+\frac{d_1(-sec^2(\xi)+sec(\xi)tan(\xi))}{sec(\xi)-tan(\xi)}+\frac{d_1}{2}(sec(\xi)-tan(\xi))+\frac{-\frac{1}{2}+\frac{d_1}{2}}{sec(\xi)-tan(\xi)} \\ \qquad +c_1(-sec^2(\xi)+sec(\xi)tan(\xi)) \\ v_{6(1)}(x,t)=\lambda+\frac{2a_2(-sec^2(\xi)+sec(\xi)tan(\xi))}{sec(\xi)-tan(\xi)}+a_2(sec(\xi)-tan(\xi))+\frac{a_2-\gamma_1}{sec(\xi)-tan(\xi)} \\ \qquad +c_2(-sec^2(\xi)+sec(\xi)tan(\xi)) \end{cases}$$

$$\begin{cases} u_{6(2)}(x,t)=\frac{-\frac{1}{2}+\frac{d_1}{2}}{csc(\xi)+cot(\xi)}+\frac{d_1(-\csc^2(\xi)-cot(\xi)\csc(\xi))}{csc(\xi)+cot(\xi)}+\frac{d_1}{2}(csc(\xi)+cot(\xi)) \\ \qquad +c_1(-\csc^2(\xi)-cot(\xi)\csc(\xi))+\gamma_1 \\ v_{6(2)}(x,t)=\frac{a_2-\gamma_1}{csc(\xi)+cot(\xi)}+c_2(-csc^2(\xi)-csc(\xi)cot(\xi))+\frac{2a_2(-csc^2(\xi)-csc(\xi)cot(\xi))}{csc(\xi)+cot(\xi)} \\ \qquad +\lambda+a_2(csc(\xi)+cot(\xi)) \end{cases}$$

where $\xi=x+ct$, c=(1+12γ_1^2)/4, a_2, d_1, γ_1,c_2, λ, $c_1$are arbitrary constants.

7) From Table 1, choosing A=1(-1), B=0, M =1(-1), $F(\xi)$ =*tan*(ξ) or *cot*(ξ) inserting them into (a), we obtain

$$\left\{\begin{array}{l} u_{7(1)}(x,t) = c_1 sec^2(\xi) + (1-d_1)cot(\xi) + d_1 csc(\xi)sec(\xi) + \gamma_1 - d_1 tan(\xi) \\ v_{7(1)}(x,t) = \lambda - a_2 csc(\xi)sec(\xi) + c_2 sec^2(\xi) + (a_2 + 2\gamma_1)cot(\xi) + a_2 tan(\xi) \end{array}\right.$$

$$\left\{\begin{array}{l} u_{7(2)}(x,t) = -c_1 csc^2(\xi) - d_1 cot(\xi) - d_1 csc(\xi)sec(\xi) + (1-d_1)tan(\xi) + \gamma_1 \\ v_{7(2)}(x,t) = \lambda + a_2 cot(\xi) + a_2 csc(\xi)sec(\xi) - c_2 csc^2(\xi) + (a_2 + 2\gamma_1)tan(\xi) \end{array}\right.$$

where ξ=*x*+*ct*, c=1+3γ_1^2, a_2, d_1, $\gamma_{1,}c_2$, λ, $c_1$are arbitrary constants.

8) From Table 1, choosing A=0, B=0, M $\neq$0, $F(\xi)$=-1/($C^*\xi$+ m) (m is an arbitrary constant), inserting them into (a), we obtain

$$\left\{\begin{array}{l} u_8(x,t) = \frac{C^* c_1}{(C^*\xi+m)^2} + \frac{C^*(-m-C^*\xi)d_1}{(C^*\xi+m)^2} + \frac{Md_1}{C^*\xi+m} + \gamma_1 \\ v_8(x,t) = \lambda + \frac{c_2 C^*}{(m+\xi C^*)^2} - \frac{a_2 C^*(-m-\xi C^*)}{(m+\xi C^*)^2} - \frac{a_2}{(m+\xi C^*)} \end{array}\right.$$

where ξ=*x*+*ct*, $c = 3\gamma_1^2$, a_2, d_1, $\gamma_{1,}c_2$, λ, $c_{1,}$ C^*, m, are arbitrary constants.

9) From Table 1, choosing A as an arbitrary constant, $B = M$=0, $F(\xi)$ =$A\xi$, inserting them into (a), we obtain

$$\left\{\begin{array}{l} u_9(x,t) = A\xi a_1 + \frac{b_1}{A\xi} + Ac_1 + \frac{d_1}{\xi} + \gamma_1 \\ v_9(x,t) = A\xi a_2 + \frac{b_2}{A\xi} + Ac_2 + \frac{d_2}{\xi} + \gamma_2 \end{array}\right.$$

where ξ=*x*+*ct*, c=3γ_1^2, A, a_1, b_1, c_1, d_1, a_2, b_2, c_2, d_2, γ_1, γ_2, are arbitrary constants.

10) From Table 1, choosing A as an arbitrary constant, B $\neq$0, M=0, $F(\xi)$=$\frac{exp(B\xi)-A}{B}$,inserting them into (a), we obtain

$$\left\{\begin{array}{l} u_{10}(x,t) = \frac{(-A+e^{B\xi})a_1}{B} + \frac{Bb_1}{-A+e^{B\xi}} + c_1 e^{B\xi} + \frac{Be^{B\xi}d_1}{-A+e^{B\xi}} + \gamma_1 \\ v_{10}(x,t) = \frac{(-A+e^{B\xi})a_2}{B} + \frac{Bb_2}{-A+e^{B\xi}} + c_2 e^{B\xi} + \frac{Be^{B\xi}d_2}{-A+e^{B\xi}} + \gamma_2 \end{array}\right.$$

where ξ=*x*+*ct*, $c = \frac{1}{2}(B^2 - 6B^2 d_1 + 6B^2 d_1^2 - 6B\gamma_1 + 12Bd_1\gamma_1 + 6\gamma_1^2)$, A, B, a_1, b_1, c_1, d_1, a_2, b_2, c_2,d_2, γ_1, γ_2, are arbitrary constants. We may give the other explicit exact solutions of the coupled MKDV Eqs. for cases 2 to 9 in the same way. It is clear that we may get many new and more explicit exact solutions for Eqs. (1), (2).

4 Conclusion

In this letter, we have proposed an extended generalized F-expansion method to construct more exact solutions of NLPDEs. The advantages of the method is that it can be used to obtain more exact solutions which cannot be fully obtained by trigonometric function transform method[m14], extended tanh-function method[15,16], algebraic method[17], modified extended tanh-function (METF) method[18], a new generalized ansatz in homogeneous banlance method[19]. With the aid of Mathematica, the method provides a powerful mathematical tool to obtain more general exact solutions of a great many NLPDEs in mathematical physics. Applying the method to the coupled MKDV Eqs., we have successfully obtained many new and more general combined Riccati function solutions, including hyperbolic function solutions, trigonometric function solutions and rational solutions. Most of those results we obtained are newly found in present papers.

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