

The Grey Complex Networks and Design of Some Topological Indexes of Grey Indirect Networks

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Abstract: Many theoretical and applied results about complex networks have been obtained. For the expand scale, it is difficult to get the sums of the vertices and the edges between them. The grey systems theory aims at the thing whose information is inadequate, and discovers the developing regularity by using the known information. The complexity of network derives the inadequate information, so it is actually a grey one. For the inadequate information, we could not get the sums of the vertices and the edges in a complex network, then the quantities of them are grey numbers. The topological indexes to describe network are grey ones at the same time. We proposed the grey complex networks and obtained the computational methods of some grey topological indexes for the grey indirect networks. The approach can make us to approximately discover the objective networks. And it has more operativeness.

Key words: Complex network; Grey system theory; Topological index; Grey computational formula; Grey number

1 Introduction

The network problems always exist in many fields, such as the social relationship, biological systems, railway network, Internet, and so on. Since the small-world networks [1] and the scale-free networks [2] appeared, many theoretical and applied results about complex networks were obtained (see [3]-[11]). Many useful review articles on complex networks have also appeared (see [12] and [13], etc.).

The complex networks shows its complexity as follows (see [3]):

- Structural complexity: the wiring diagram could be an intricate.
- Network evolution: the wiring diagram could change over time.
- Connection diversity: the links (edges) between nodes (vertices) could have different weights, directions and signs.
- Dynamical complexity: the nodes (vertices) could be nonlinear dynamical systems.
- Node (vertex) diversity: there could be many different kinds of nodes (vertices).
- Meta-complication: the various complications can influence each other.

The grey systems theory is a system methodology which is proposed by Deng Julong, a Chinese scholar. It aims at a complex system whose information is partially knowable and partially unknowable (see [14])

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and [15], etc.). Since the grey systems theory was proposed, many theoretic and applied results have been obtained (see [15]-[22] etc.).

For the complex system, people often confront the information-missing situation, then the statistic datum are uncertain [15]. But under the correct investigation methods, we can get an interval which the true data belongs to. The uncertain datum are grey numbers and the intervals are their number-covered sets (see [15] and [23], etc.).

On the other hand, the complex system often derives the nonlinearity and there are many methods to resolve it(see [24]-[27]). The nonlinearity also make information-missing situation. The complexity and the nonlinearity of systems make them be grey. Then it may be more practical to propose the grey complex networks.

The paper is organized as follows: we propose the grey complex networks in section II. For an instance, we get some topological indexes for the grey indirect networks in section III. At last, we have a conclusion and point out our future works.

2 The grey complex networks

The complex networks always own numerous vertices and edges. Generally, the sums of the vertices and the edges for a network are denoted as N and K , respectively. For the weighted networks, the weight of the edge l_k is w_k (see [28]-[30], etc.).

Just as the explanations for the complexity of complex networks in section I (also see [3]), it can induces that the researchers could not get complete information, i.e., the obtained information is usually inadequate. The missing information make the statistical datum be uncertain when we research the complex networks. Then we get some grey numbers and their number-covered sets under correct investigation methods (see [15]).

Generally, there are three types of grey numbers when we research the complex networks:

- The sum of vertices is a grey number and we denote it as $N(\otimes)$. Under the correct investigation methods, we can get its number-covered set $[N]$, i.e., the true number N^* of the vertices satisfies $N^* \in [N]$.
- The sum of edges is a grey number and we denote it as $K(\otimes)$. Under the correct investigation methods, we can get its number-covered set $[K]$, i.e., the true number K^* of the edges satisfies $K^* \in [K]$.
- For the weighted complex networks, the weight of the edge l_k is a grey number and we denote it as $w_k(\otimes)$. Under the correct investigation methods, we can get its number-covered set $[w_k]$, i.e., the true weight w_k^* of the edge l_k satisfies $w_k^* \in [w_k]$.

In the process to research complex networks, maybe one or more types of grey numbers above appear, then the complex network is grey and we call it a grey complex network.

Generally, the three types of grey numbers above can make other statistical datum be grey.

3 Some topological indexes of grey indirect networks

For the grey indirect networks, it has two types of grey numbers for the missing information, i.e., the grey number of vertices $N(\otimes)$ and the other of edges $K(\otimes)$. Under the correct investigation methods, we get their number-covered sets $[N] = [N^-, N^+]$ and $[K] = [K^-, K^+]$, respectively. Then we have some topological grey indexes and their number-covered sets below:

- Denoting the grey sum of vertices whose degree is k as $n_k(\otimes)$ for the missing information, and the number-covered set of $n_k(\otimes)$ as $[n_k] = [n_k^-, n_k^+]$ under the correct investigation methods, we get the grey degree distribution

$$P_k(\otimes) = \frac{n_k(\otimes)}{N(\otimes)}$$

and its number-covered set

$$[P_k] = \frac{[n_k]}{[N]} = \left[\frac{n_k^-}{N^+ - \Delta n_k}, \frac{n_k^+}{N^- + \Delta n_k} \right]$$

where $\Delta n_k = n_k^+ - n_k^-$ is the chaos of $n_k(\otimes)$.

- Supposing that S_v is the set of vertices whose element is the neighbor of the given vertex v . Denote that $N_v(\otimes)$ and $E_v(\otimes)$ are the grey sums of the vertices and the existed edges in V_v for the missing information, respectively. Under the correct investigation methods, we get their number-covered sets and denote them as $[N_v] = [N_v^-, N_v^+]$ and $[E_v] = [E_v^-, E_v^+]$, respectively. Then we get the grey clustering coefficient of the vertex v

$$C_v(\otimes) = \frac{2E_v(\otimes)}{N_v(\otimes)(N_v(\otimes) - 1)}$$

and its number-covered set

$$[C_v] = \frac{2[E_v]}{[N_v]([N_v] - 1)}$$

Because the true number N_v^* often satisfies $N_v^* > 1$, we have

$$\begin{aligned} [C_v] &= \frac{2[E_v^-, E_v^+]}{[N_v^-, N_v^+]([N_v^-, N_v^+] - 1)} \\ &= \frac{[2E_v^-, 2E_v^+]}{[N_v^-, N_v^+]([N_v^- - 1, N_v^+ - 1])} \\ &= \left[\frac{2E_v^-}{N_v^+(N_v^+ - 1)}, \frac{2E_v^+}{N_v^-(N_v^- - 1)} \right] \end{aligned}$$

Denoting $[C_v] = [C_v^-, C_v^+]$, we have the average grey clustering degree of the grey network

$$C(\otimes) = \frac{1}{N(\otimes)} \sum_{v \in S} C_v(\otimes)$$

and its number-covered set

$$\begin{aligned} [C] &= \frac{1}{[N]} \sum_{v \in S} [C_v] \\ &= \frac{1}{[N^-, N^+]} \sum_{v \in S} [C_v^-, C_v^+] \\ &= \left[\frac{1}{N^+} \sum_{v \in S} C_v^-, \frac{1}{N^-} \sum_{v \in S} C_v^+ \right] \end{aligned}$$

where S is the set of vertices of the grey network.

- Supposing that $d_{ij}(\otimes)$ is the grey sum of edges of the shortest path which connects the vertices v_i and v_j , which i and j are the order numbers of vertices. Under the correct investigation methods, we get the number-covered set $[d_{ij}] = [d_{ij}^-, d_{ij}^+]$ of $d_{ij}(\otimes)$. Then the grey characteristic path length and its number-covered set are

$$L(\otimes) = \frac{2}{N(\otimes)(N(\otimes) - 1)} \sum_{i \geq j} d_{ij}(\otimes)$$

and

$$\begin{aligned} [L] &= \frac{2}{[N]([N] - 1)} \sum_{i \geq j} [d_{ij}] \\ &= \frac{2}{[N^-, N^+]([N^-, N^+] - 1)} \sum_{i \geq j} [d_{ij}^-, d_{ij}^+] \\ &= \left[\frac{2}{N^+(N^+ - 1)} \sum_{i \geq j} d_{ij}^-, \frac{2}{N^-(N^- - 1)} \sum_{i \geq j} d_{ij}^+ \right] \end{aligned}$$

respectively.

- Supposing that $n_{jk}(\otimes)$ is the grey sum of the shortest pathes which connect the vertices v_j and v_k , and $n_{jk}(i)(\otimes)$ is the grey sum of the shortest pathes which connect v_j and v_k through the vertex v_i , where i, j and k are the order numbers of vertices. Under the correct investigation methods, we get the number-covered sets $[n_{jk}] = [n_{jk}^-, n_{jk}^+]$ and $[n_{jk}(i)] = [n_{jk}^-(i), n_{jk}^+(i)]$ of $n_{jk}(\otimes)$ and $n_{jk}(i)(\otimes)$, respectively. Then we have the grey edge betweenness

$$B_i(\otimes) = \sum_{v_i, v_j, v_k \in V, j \neq k} \frac{n_{jk}(i)(\otimes)}{n_{jk}(\otimes)}$$

and its number-covered set

$$\begin{aligned} [B_i] &= \sum_{v_i, v_j, v_k \in V, j \neq k} \frac{[n_{jk}(i)]}{[n_{jk}]} \\ &= \sum_{v_i, v_j, v_k \in V, j \neq k} \frac{[n_{jk}^-(i), n_{jk}^+(i)]}{[n_{jk}^-, n_{jk}^+]} \\ &= \left[\sum_{v_i, v_j, v_k \in V, j \neq k} \frac{n_{jk}^-(i)}{n_{jk}^+}, \sum_{v_i, v_j, v_k \in V, j \neq k} \frac{n_{jk}^+(i)}{n_{jk}^-} \right] \end{aligned}$$

The computational rules about grey number, which have been applied above, can be seen in [15].

4 Conclusions and future works

4.1 Conclusions

The complexity of complex networks usually induces the information-missing situation, then we could not get the exact sums of the vertices and the edges. They are grey numbers. On the other hand, the weights are also grey. At the same time, the three types of grey numbers can make other statistical numbers be grey. But we can get their number-covered sets under the correct investigation methods. We proposed the idea of grey complex networks by using grey systems theory. It can make the research of complex networks be possible under the uncertain situation. Furthermore, we get some grey topological indexes for the grey indirect networks.

4.2 Future works

The grey systems theory has been an effective methodology to resolve the system problem with missing information. However, it has not been perfective. The grey mathematics should be further studied. The calculation methods of some grey functions should be established. Based on them, we have some further works for the grey complex networks and simply point out them below:

- The grey evolving networks.
When the connection probability is also a grey number, how do we get the topological indexes?
- The grey direct networks.
The main problem is also to get the topological indexes.
- The grey weighted networks.
When the weights are also grey, how can we get the topological indexes?
- The problems of chaos synchronization and network control.
When the network equation includes grey parameters, how can we justify the chaos synchronization and network control?
- The problem of network propagation.
The main task is the network propagation of grey complex networks.

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