The Bend Stiffness of Crane beam Strengthened with CFRP under Monotonic and Fatigue Load Condition

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Abstract: The change of the beam’s stiffness can reflect the working ability of the beam. Based on the technical specification for strengthening structures with CFRP and correlative conclusions from other studies and tests, the flexural stiffness of the damaged crane beam was calculated, and the development of the stiffness under fatigue condition was analyzed as the parameter of cycle time. By comparing the analytical results with the experimental results, it can be concluded that the damage curve of the beam follows the three-phase distortion. Analytically calculated results versus experimental results for all cross sections are listed, which shows that the analytical method is valid when used to compute the flexural stiffness of RC beams and beams strengthened with CFRP.

Key words: Structure strengthening; Stiffness; Fatigue stiffness; CFRP

1 Introduction

The bend stiffness is one of the most important parameters in evaluating the working ability of the crane beam. But many crane beams are excessively deformed or not capable enough as the time lapses so that they need to be strengthened for further use. External steel plate bonding and carbon fiber reinforced plastic (CFRP) bonding are the most popular ways to repair and rehabilitate of existing structures. Although external steel plate bonding can provide a satisfactory solution in many cases, it does have some disadvantages: manipulating heavy plates on site, potential steel corrosion, and necessity of on-going maintenance. Yet the carbon fiber reinforced plastic plates (CFRP) offer several advantages over steel because of the ease and speed of installation, the structural efficiency of the repair, the corrosion resistance of the materials, and the minimal effect that these materials have on structural dimensions aesthetics and versatility [1]. Many tests about strengthened member under monotonic load have been performed and have gained many conclusions. Although some specifications [2, 3] have been used in engineering, the stiffness of the strengthened member, especially when structures working under fatigue condition, is not involved. In this article, the stiffness of the T-shaped RC beams strengthened with CFRP is analyzed as the parameter of cycle time.

The stiffness of the beam is influenced by many factors, such as the performance of the materials (steel, concrete and CFRP), the bonding between the different material and the load condition etc. So the concrete specification [4] doesn’t offer a formulary concluding these parameters when structures work in cycle load condition.

In the concrete specification, the RC members, working under cycle load, are designed without considering the fatigue load directly but checking the stress of the material (concrete and steel) to insure the working ability. And the fatigue stiffness is analyzed by replacing the elastic modulus with that of deformation for

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concrete. So the development of the stiffness can not be reflected, and the result is conservative. In the second part of this article, the fatigue stiffness of the CFRP strengthened crane beams is analyzed as the parameter of cycle time. A new calculating method of the fatigue stiffness is given.

2 Analysis of the CFRP strengthened beam’s deformation regulation

Many tests have verified the conclusion that the three-phase deformation regulation of the RC beams and the CFRP strengthened beams are coincident. The three-phase can be distinguished by the crack of concrete and the yield of the steel. The majority of the beams are working in the second phase, when the beam cracked but not yielded. Beams need to be repaired when achieve the end of the second phase soon. After being strengthened, the stress of the steel are reduced, which extends the life of steel before yield. So, the performance of the beam when in the second phase need to be deep researched.

Repeated loading may lead to internal cracking of a member that influences its stiffness and load-carrying characteristics, which need to be repaired. There is, however, a paucity of research [5–9] investigating the fatigue stiffness of RC beams strengthened with CFRP. Results of tests carried out by many research teams have proved the three-phase deformation rules corresponding to the three-phase damage of the beam. In the article, the fatigue stiffness during the whole life is analyzed from the damage of the material.

3 Instantaneous rigidity of CFRP strengthened beam

3.1 Constitutional properties of material and the main assumptions

The stress-strain response of CFRP, steel and concrete is assumed to be elastic-perfectly, as the beam’s working condition is lower than the yield load. The main assumptions employed in the follow analysis are:

Plane sections are considered to remain plane after bending. It is generally accepted that this assumption is reasonable even well into the inelastic range. Measurements of strains along the height show that this assumption is good for beams.

Perfect bond and no slipping is assumed between concrete and other materials (steel reinforcement and CFRP laminates).

The tensile action of concrete is not accounted for. The tensile reinforcement and CFRP provide the tensile.

3.2 Theoretical analysis

The stress-strain relationship of the control section is shown in Fig.1.

![Figure 1: Stress-stain of the control section. (left)Shape of the section.(middle)Strain of the section.(right)Stress of the section.](http://www.nonlinearscience.org.uk/)
From the Fig.1, we can get the balance equation as follow:

\[ \alpha E_A(h_0 - x) + \alpha_{cf} A_{cf}(h - x) = \alpha E_A'(x - a'_0) + \begin{cases} \frac{bf x^2}{2} - (bf - b)(x - h_f)^2/2, & if \ x > h_f \\ \frac{bf x^2}{2}, & if \ x \leq h_f \end{cases} \]  

(1)

By Eq. (1), we can compute the neutral axis of the beam, and then the bending moment can be calculated as follow:

\[ I_0 = \begin{cases} \frac{1}{3} [bf x^3 - (bf - b)(x - h_f)^3] + E_A A'_s(x - a'_0)^2 + E_A A_s(h_0 - x)^2 + E_{cf} A_{cf}(h - x)^2, & if \ x > h_f \\ \frac{1}{3} bf x^3 + E_A A'_s(x - a'_0)^2 + E_A A_s(h_0 - x)^2 + E_{cf} A_{cf}(h - x)^2, & if \ x \leq h_f \end{cases} \]  

(2)

And the stiffness and deformation of the beam are:

\[ B_0 = E_c I_0 \]  

(3)

\[ f = K_0 S \frac{M}{B_0} I_0^2 \]  

(4)

where \( S23/432 \) is the deformation parameter in the test, and the \( K_0 \) is a modified parameter in according to the uneven stress distribution along the CFRP. In the test, \( K_0 \) can be got by statistic as 1.732.

4 Fatigue stiffness of CFRP strengthened beam

4.1 Constitutional properties of material

Most of the time, the stress condition of material is not similar to the bearing test when beams working under cycle load, which cause the different failure mechanism. So, the constitutional properties of material are assumed as follow:

Experimental results presented in many studies[10] suggest that the modulus of elasticity for steel remain unchanged until just before failure by high cycle fatigue. In the following analysis, the modulus of elasticity for steel is assumed to remain unchanged during cyclic loading.

The CFRP fabric utilized in the tests was composed of unidirectional dry carbon material formed by weaving individual yarns into a fabric. The fracture of one yarns can not influence the neighboring yarns, which causes the good performance on fatigue resist. The fatigue strength of CFRP can reach 70 or 80 percent of its intensity ability. Test data suggest that the behavior of CFRP is virtually unaffected by fatigue loading. Hence, the modulus of elasticity for CFRP remains constant.

The fatigue behavior of concrete is researched by many institutions[11,12,13,14,15]. Test results indicate the nonlinear deformation of concrete when working under fatigue loading, and many fatigue models were gained. Most scholars proposed that the elastic-plastic deformation of concrete is the sum of two components: elastic and residual deformation, which are denoted as \( \varepsilon'_e \) and \( \varepsilon'_r \). In other words

\[ \varepsilon'_f = \varepsilon'_e + \varepsilon'_r = \sigma'_c / E_c + \varepsilon'_f \]  

(5)

So the modulus of deformation of concrete can be expressed as

\[ E'_{c,N} = \sigma'_c / \varepsilon'_f = \sigma'_c / (\sigma'_c / E_c + \varepsilon'_f) \]  

(6)

Balaguru and Shah [9] established the residual deformation of concrete concerning the stress range, stress level and cycle times as

\[ \varepsilon'_f = 129 * S_m * t^{1/3} + 17.8 * S_m * \Delta * N^{1/3} \]  

(7)

where \( \Delta = S_{max} - S_{min} = (\sigma_{c,max} - \sigma_{c,min}) / f_c \) is the stress range; \( S_m = (S_{max} + S_{min}) / 2 = (\sigma_{c,max} + \sigma_{c,min}) / 2f_c \) is the average stress level; \( \sigma_{c,max} = M_{max}x / I_0 \) and \( \sigma_{c,min} = M_{max}x / I_0 \) are the maximal and minimal stress of concrete; \( t \) is the duration of alternation load in hour; \( N \) is the cycle times.
The isolated-body diagram of concrete is not even but with stress gradient when bending. So we adjust the Eq. (7) as

$$\varepsilon_f = K_N(129 * S_m * t^{1/3} + 17.8 * S_m * \Delta * N^{1/3})$$

Where the $K_N=8.7$ is a modified parameter obtained from a regression analysis of the test date.

Concrete under tension is assumed to have no significant tensile strength during cyclic fatigue calculations.

Furthermore, the epoxy between the CFRP laminates and concrete is assumed to be rigid and unaffected by cyclic loading. This is a reasonable assumption for beams in which failure initiates in the high moment zone, where shear stresses in the epoxy are low.

### 4.2 Main assumptions

The main assumptions employed in the following analysis are:

1. Plane sections are considered to remain plane after bending.
2. Perfect bond and no slipping is assumed between concrete and other materials (steel reinforcement and CFRP laminates).
3. The tensile action of concrete is not accounted for. The tensile reinforcement and CFRP provide the tensile. The stress diagram of concrete in compression is triangle.
4. The constitutional properties of material are stated as above.

### 4.3 Theoretical analysis

After certain $(N)$ cycle, the neutral axis can be calculated by

$$b_f h_f (x_N - h_f/2) + b(x_N - h_f)^2 / 2 + \alpha_{E_f} A_s(x_N - a_0) = \alpha_{E_f} A_s(h_0 - x_N) + \alpha_{E_f} A_{cf}(h - x_N)$$

where $\alpha_{cf} = E_{cf}/E_f$, $\alpha_{E_f} = E_{c,N}/E_{c,cf}$, $\alpha_{E_f} = E_s/E_{c,f}$. $I_N$ is the converted section moment of CFRP, steel in compression and steel in tensile; $E_{c,f}$ is the deformation modulus of concrete introduced above.

So the bending moment can be calculated as follows:

$$I_N^f = \begin{cases} 
\frac{1}{3}[b_f x_N^3 - (b_f - b)(x_N - h_f)^3] + \alpha_{E_f} A_s(x_N - a_0)^2 \\
+ \alpha_{E_f} A_s(h_0 - x_N)^2 + \alpha_{E_f} A_{cf}(h - x_N)^2, \text{ if } x > h_f \\
\frac{1}{3} b_f x_N^3 + \alpha_{E_f} A_s(x_N - a_0)^2 + \alpha_{E_f} A_s(h_0 - x_N)^2 \\
+ \alpha_{E_f} A_{cf}(h - x_N)^2, \text{ if } x \leq h_f 
\end{cases}$$

The neutral axis of the beam at given time can be got from Eq. (6)to (9), and then we can set up the formula for fatigue stiffness as the parameter of cycle time.

$$B_N = f(N) = E_{c,N} I_N^f$$

And the deformation of the beam is

$$f = S M B_N I_0^2$$

where $S23/432$ is the deformation parameter in the test.

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5 Check of the theoretical analysis

Three CFRP strengthened beams and one control beam were tested to improve the accuracy of these technical way of stiffness computation. Something particularly interesting in this study is the effect of scale. The T-shaped beams used in this study are half scale models of crane beam from an old industrial mill factory, which is 3 meters long. The beams, whose details are shown in Fig.2, are 300 mm deep, having a 50 mm thick by 120 mm wide range and a 60 mm wide stem.

Grade HRB335 deformed reinforcing bars were used as tensile steel, whose yield and tensile strengths are 409.5 and 578.7 MPa, respectively. The erection bar, compression bar and stirrup use the steel of grade HPB235, whose yield and tensile strengths are 338.3 and 509.1 MPa, respectively. The concrete of grade C30 was used for all beams. The beams were cast together and the actual 28-day strengths obtained by standard specimen are 20.64 MPa, and the average elastic modulus is 39,500 MPa. The thickness of CFRP is 0.111 mm with the actual strengths and elastic modulus of 4192.5 and 243149.2 MPa, respectively. The actual strengths of adhesives is 3134 MPa, with the elastic modulus being 2,350 GPa and fracture elongation being 1.5%. The whole strengthening proceeding was done by professional works. The beams were cured more than 28-days, and the adhesive was cured more than one week. All of the steel are binded rather than tack-welding to reduce the stress-riser. Furthermore, four steel plates were installed in the beams before casting to prevent local destroy of concrete.

Three beams were retrofit with CFRP materials on the soffit of the T-beam stem. Two different CFRP systems were used. The retrofit extended the length of the beam but did not extend under the supports. No additional anchorage, apart from the adhesive system, was used in B1a. But longitudinal CFRP of B1b and B1c were anchorage by U-shaped CFRP at two shear span. B0 was left unretrofit and used as control specimen. Scheme of test is shown in Table 1.

<table>
<thead>
<tr>
<th>Beam symbol</th>
<th>$P_{\text{min}}$</th>
<th>$P_{\text{max}}$</th>
<th>Strengthening project</th>
</tr>
</thead>
<tbody>
<tr>
<td>B0</td>
<td>5kN</td>
<td>30kN</td>
<td>No strengthened</td>
</tr>
<tr>
<td>B1a</td>
<td>5kN</td>
<td>70kN</td>
<td>One layer of CFRP</td>
</tr>
<tr>
<td>B1b</td>
<td>5kN</td>
<td>70kN</td>
<td>One layer of CFRP and U-shaped anchorage</td>
</tr>
<tr>
<td>B1c</td>
<td>5kN</td>
<td>55kN</td>
<td>One layer of CFRP and U-shaped anchorage</td>
</tr>
</tbody>
</table>

In order to simulate the real condition in engineering, all the beams were preloaded to make sure that the width of primary cracks reach the maximum width allowed by specification (0.2 mm). After that, B1a, B1b, B1c were strengthened and tested under fatigue conditions. Many technical specification [16] and correlative tests [5–9] were used for reference during the test.

The fatigue load is applied by means of electronically controlled hydraulic pulsator from structural lab in Southeast University. The beam is loaded, via a spreader steel beam, at two points, each located 900 mm from the support. The beam is loaded at a frequency from 4 to 6 Hz.

During each test, the following values are measured:
(1)number of cycles;(2)load force measured in dynamometer;(3)deflections and dynamic deflections measured by the linear voltage displacement transducers located at the midspan of the beam and two supports;(4)steel strains and dynamic strains measured by the gauges;(5)strains of the compressive concrete zone measured at the top edge of the beam and on side faces;(6)strains of CFRP laminates.

The static data were collected by DH3818, and the dynamic data were collected by DHDAS. The apparatus were linked by computer, and the data were automatically recorded by computer. Tests are carried out continuously up to beam failure or to obtain 2 million of cycles.

The relationship between strain of the control section and cycle time is given in Fig.3. The relative life is used, which is the ratio of actual cycle time to real life. Conclusions can be got from Fig.3 that the average strain of the control section satisfies the plane section assumption basically. The strain of steel is higher for the reason that the steel gages were displaced near the crack, and the uneven distribution of stress near the cracks influence the measure of steel strain.
Figure 2: Details of specimens and test setup

Figure 3: Stress of the control section versus life

Figure 4: Contrast of the deformation
Fig. 4 represents the deformation curves of strengthened beams before and after strengthening. In Fig. 4, “U” and “S” mean unstrengthened and strengthened, respectively. The stiffness of plated beams are more stiff, as might be expected. The average flexural stiffness (EI) of the CFRP plated beams was 14.9%–16.1% greater than that for the unplated beams, which improved the validity of CFRP, effectively. Analytically calculated results versus experimental results for all cross sections are listed in Table 2. It is clear from the figures that the analytical response correlates well with the experimental data at all stages of behavior up to failure.

### Table 2: Contrast of theoretical results and test results

<table>
<thead>
<tr>
<th>Beam symbol</th>
<th>Before strengthen</th>
<th>After strengthen</th>
<th>0.1N</th>
<th>0.3N</th>
<th>0.5N</th>
<th>0.7N</th>
<th>0.9N</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measured date</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B0</td>
<td>3.18</td>
<td>DDD</td>
<td>3.65</td>
<td>3.69</td>
<td>3.79</td>
<td>3.82</td>
<td>3.91</td>
</tr>
<tr>
<td>B1a</td>
<td>5.21</td>
<td>4.53</td>
<td>7.88</td>
<td>8.34</td>
<td>8.42</td>
<td>8.51</td>
<td>8.81</td>
</tr>
<tr>
<td>B1b</td>
<td>5.14</td>
<td>4.31</td>
<td>8.21</td>
<td>8.77</td>
<td>9.23</td>
<td>9.81</td>
<td>10.21</td>
</tr>
<tr>
<td>B1c</td>
<td>5.05</td>
<td>4.44</td>
<td>6.11</td>
<td>6.33</td>
<td>6.61</td>
<td>6.91</td>
<td>7.28</td>
</tr>
<tr>
<td>Analyzed date</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B0</td>
<td>3.08</td>
<td>DDD</td>
<td>3.71</td>
<td>3.76</td>
<td>3.82</td>
<td>3.88</td>
<td>3.93</td>
</tr>
<tr>
<td>B1a</td>
<td>5.51</td>
<td>4.11</td>
<td>7.99</td>
<td>8.28</td>
<td>8.55</td>
<td>8.83</td>
<td>9.02</td>
</tr>
<tr>
<td>B1b</td>
<td>6.91</td>
<td>5.17</td>
<td>7.99</td>
<td>8.28</td>
<td>8.55</td>
<td>8.83</td>
<td>9.02</td>
</tr>
<tr>
<td>B1c</td>
<td>6.21</td>
<td>4.59</td>
<td>6.22</td>
<td>6.48</td>
<td>6.75</td>
<td>7.02</td>
<td>7.41</td>
</tr>
</tbody>
</table>

(Where the load corresponding to deformation for beams are 30, 70, 70 and 55kN, respectively.)

The residual strains of concrete in compression are illustrated in Fig. 5. It was observed from Fig. 5 that strain development follows three distinct phases: a rapid increase from 0 to about 10% of the total fatigue life; a uniform increase from 10 to about 80%, then a rapid increase until failure.

### Figure 5: Residual stain of the beam

![Residual stain of the beam](image)

### Figure 6: Change of deferment versus cycle times

![Change of deferment versus cycle times](image)

Fig. 6 shows plots of deformation versus life of every beam under high level of fatigue load. It is clear from the figure that the development of deformation is similar to the strain of concrete during test. It can be clearly found that the U-shaped CFRP can well prevent the desquamation of CFRP by contrasting the failure mechanism of B1a and B1b, for the reason of sliding control of CFRP.

### 6 Conclusion

The study demonstrates the feasibility of using unidirectional CFRP fabric in the rehabilitation and strengthening of RC structures with respect to both static and fatigue performance. The following conclusions are drawn from this study:

1. The development of deformation for CFRP strengthened beam follows the three-phase distorting rule whether in usual or fatigue condition.

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(2) A model for computing the static and accelerated fatigue stiffness of reinforced concrete beams strengthened with CFRP is presented. The model is implemented in a computer program and is verified and exercised by comparing analytical results to data from experimental investigations. The accuracy of this model illustrates that this model can be used in engineering. But as for the limited test in our program, the parameter needs to be further researched.

(3) The stiffness is enhanced after being strengthened by CFRP, which has an increase of 14.9% to 16.1%. The observed increase in fatigue life, however, is limited by the quality of the bond between the CFRP and the concrete substrate. Once debonding has progressed, stresses are no longer transferred to the CFRP and the fatigue behavior of the beam reverts to that of an unretrofit beam.

References


