ISSN 1479-3889 (print), 1479-3897 (online)
International Journal of Nonlinear Science
Vol.3(2007) No.3,pp.228-234

Chaos Synchronization of the Energy Resource Chaotic System with Active Control

Lixin Tian *, Jianting Xu, Mei Sun
Nonlinear Scientific Research Center, Jiangsu University
Zhenjiang, Jiangsu,212013,P.R.China
(Received 11 April 2007, accepted 18 May 2007)

Abstract: This paper is devoted to study the problem of chaos synchronization of the energy resource chaotic system. Active control is used to achieve chaos synchronization of energy resource chaotic system.Sufficient conditions for achieving synchronization of energy resource chaotic system are derived by using Lyapunov stability theory. Then the use of the stability of autonomy system proves the stability of the solution of the response system. Numerical simulations are carried out using Matlab and numerical simulations are presented to demonstrate the effectiveness of the proposed chaos synchronization schemes.

Key words: the energy resource chaotic system; active control; synchronization; stability

1 Introduction

Chaos synchronization is a very important subject in the nonlinear science. Chaos synchronization,has been widely investigated in a variety of fields, such as physical ,chemical and ecological science , secure communications ,etc .There are a wide variety of approaches of the synchronization of chaotic systems[1–3, 13]

In this paper, we study the synchronization of energy resource system using active control. Energy resource system[4] is a kind of complex nonlinear system ,which is made up of a lot of elements and is exoteric and far from equilibrium state .In this nonlinear system, it is quite a universal phenomenon that the motion state of the system appears chaos because of destabilization with a variety of parameters.

Sun and Tian [4, 12] established a three-dimension energy resource chaotic system and discovered an energy resource attractor of three-dimension energy resource chaotic system.This energy resource attractor is different from the Lorenz attractor, the Chen attractor and the $L\ddot{u}$ attractor. This shows that the energy resource system is a fully new chaotic system, and reflects the nonlinearity characteristic between demand and supply of energy resource. Achievements have already been obtained from the energy resource chaotic system , for example adaptive synchronization [3], linear feedback control[5], non-autonomous feedback control[5], adaptive control[5] and time-delay feedback control[6].In this paper we analyze energy resource chaotic system and achieve chaos synchronization using active control[1, 7–9].

From[4], we fix the parameters of the energy resource system and obtain

$$\begin{cases} \frac{dx}{dt} = ax\left(1-\frac{x}{1.8}\right) - 0.15(y+z) \\ \frac{dy}{dt} = -by - cz + 0.07x\left[1-(x-z)\right] \\ \frac{dz}{dt} = 0.2z(0.5x-0.4) \end{cases}$$

An energy resource chaotic attractor can be found in paper[4]. In this paper,active control is used to achieve chaos synchronization of energy resource chaotic system .Then the stability of response system is

*Corresponding author. *E-mail address*: tianlx@ujs.edu.cn

analyzed. Furthermore, numerical simulations are presented to demonstrate the effectiveness of the proposed chaos synchronization schemes.

2 Active control synchronization of the energy resource system

When drive and response systems have known and same parameters, it is convenient and valid to achieve synchronization using active control.Assume that there exists a drive system

$$\dot{x} = Ax + f(x) \tag{2.1}$$

where $x = (x_1, x_2, \dots, x_n)^T \in R^n, A \in R^n \times R^n$, A is a constant system matrix and $f(x)$ is nonlinear sequence function.

The response system is

$$\dot{y} = Ay + f(y) + u(t) \tag{2.2}$$

where $\dot{y} = (y_1, y_2, \dots, y_n)^T \in R^n, u(t) = (u_1(t), u_2(t), ..., u_n(t))^T \in R^n$

Definition 1 *([10]): If there exists appropriate controller $u(t)$ satisfying $\forall x, y, e \in R^n$, $\lim\limits_{t\to 0} \|e\| = \lim\limits_{t\to 0} \|y - x\| = 0$, then drive and response systems are supposed to achieve synchronization.*

The error is defined as $e = y - x$. Then the error system is

$$\dot{e} = \dot{y} - \dot{x} = Ae + F(x, y) + u(t) \tag{2.3}$$

where $F(x, y) = f(y) - f(x)$. Controller $u(t)$ may eliminate nonlinear section without e of system (2.3), i.e.

$$u(t) = V(t) - F(x, y) \tag{2.4}$$

where $V(t) = Ke$ is a linear section with e. Bring (2.4) into (2.3).

$$\dot{e} = Ae + V(t) \tag{2.5}$$

Since $V(t)$ is a linear section with e and $V(t) = ke$ is a constant diagonal matrix, Eq.(2.5) becomes

$$\dot{e} = (A + K)e \tag{2.6}$$

Proposition 1 *([8]) If the following conditions are satisfied for diagonal matrix: $\lambda_i \le 0$, where λ_i is the eigenvalue of matrix $(A+K)$, (2.6) state vectors asymptotically converge to zero, i.e.drive system (2.1) and response system (2.2) asymptotically synchronize.*

In the case, we define the drive and response systems as follows:

$$\begin{cases} \frac{dx_1}{dt} = ax_1\left(1 - \frac{x_1}{1.8}\right) - 0.15(y_1 + z_1) \\ \frac{dy_1}{dt} = -by_1 - cz_1 + 0.07x_1\left[1 - (x_1 - z_1)\right] \\ \frac{dz_1}{dt} = 0.2z_1(0.5x_1 - 0.4) \end{cases} \tag{2.7}$$

and

$$\begin{cases} \frac{dx_2}{dt} = ax_2\left(1 - \frac{x_2}{1.8}\right) - 0.15(y_2 + z_2) + u_1(t) \\ \frac{dy_2}{dt} = -by_2 - cz_2 + 0.07x_2\left[1 - (x_2 - z_2)\right] + u_2(t) \\ \frac{dz_2}{dt} = 0.2z_2(0.5x_2 - 0.4) + u_3(t) \end{cases} \tag{2.8}$$

We have introduced three control functions $u_1(t), u_2(t), u_3(t)$ in (2.8). Our goal is to determine the control functions $u_1(t), u_2(t), u_3(t)$. In order to estimate the control functions, we subtract (2.7) from (2.8). Let us define the state errors between the response system (2.8) that is to be controlled and the controlling system (2.7) as

$$e_1 = x_2 - x_1, e_2 = y_2 - y_1, e_3 = z_2 - z_1 \tag{2.9}$$

Subtracting (2.7) from (2.8) and using the notation (2.9) yields

$$\begin{cases} \dot{e}_1 = ae_1 - 0.15e_2 - 0.15e_3 - ax_2^2/1.8 + ax_1^2/1.8 + u_1(t) \\ \dot{e}_2 = -be_2 - ce_3 + 0.07e_1 - 0.07x_2^2 + 0.07x_2z_2 + 0.07x_1^2 - 0.07x_1z_1 + u_2(t) \\ \dot{e}_3 = -0.08e_3 + 0.1x_2z_2 - 0.1x_1z_1 + u_3(t) \end{cases} \tag{2.10}$$

We define the active control functions $u_1(t), u_2(t), u_3(t)$ as follows:

$$\begin{cases} u_1(t) = V_1(t) + ax_2^2/1.8 - ax_1^2/1.8 \\ u_2(t) = V_2(t) + 0.07x_2^2 - 0.07x_2z_2 - 0.07x_1^2 + 0.07x_1z_1 \\ u_3(t) = V_3(t) - 0.1x_2z_2 + 0.1x_1z_1 \end{cases} \tag{2.11}$$

Bring (2.11)into (2.10), and the error system (2.10) becomes

$$\begin{cases} \dot{e}_1 = ae_1 - 0.15e_2 - 0.15e_3 + V_1(t) \\ \dot{e}_2 = -be_2 - ce_3 + 0.07e_1 + V_2(t) \\ \dot{e}_3 = -0.08e_3 + V_3(t) \end{cases} \tag{2.12}$$

The error system (2.12) to be controlled is a linear system with a control input $V_1(t), V_2(t), V_3(t)$ as function of the error states e_1, e_2, e_3. As long as these feedbacks stabilize the system, e_1, e_2, e_3 converge to zero as time t tends to infinity. This implies that drive and response systems are synchronized with active control. There are many possible choices for the control $V_1(t), V_2(t), V_3(t)$. We choose:

$$\begin{bmatrix} V_1(t) \\ V_2(t) \\ V_2(t) \end{bmatrix} = A \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} \tag{2.13}$$

where A is a 3x3 constant matrix. In order to make the closed loop system stable, the proper choice of the elements of the matrix A is such that the feedback system must have all eigenvalues with negative real parts. Let the matrix A be chosen in the following form:

$$A = \begin{pmatrix} -2a & 0.15 & 0.15 \\ -0.07 & -b & c \\ 0 & 0 & -0.02 \end{pmatrix}$$

Bring (2.13) into (2.12), we may obtain

$$\dot{e} = Be$$

$$\begin{pmatrix} \dot{e}_1 \\ \dot{e}_2 \\ \dot{e}_3 \end{pmatrix} = \begin{pmatrix} a & -0.15 & -0.15 \\ 0.07 & -b & -c \\ 0 & 0 & -0.08 \end{pmatrix} \begin{pmatrix} e_1 \\ e_2 \\ e_3 \end{pmatrix} + A \begin{pmatrix} e_1 \\ e_2 \\ e_3 \end{pmatrix}$$

$$B = \begin{pmatrix} -a & 0 & 0 \\ 0 & -2b & 0 \\ 0 & 0 & -0.1 \end{pmatrix}$$

In this particular choice, the closed loop system (2.12) has the eigenvalues $-a, -2b, -0.1$. Due to stability theory of linearity system, this choice will lead to the convergence of error states e_1, e_2, e_3 to zero as time t tends to infinity and hence the synchronization of the two system is achieved.

3 The stability analysis of response system

This response system is autonomous system.Bringing $u_1(t), u_2(t), u_3(t)$ into(2.8) ,we obtain (3.1)

$$\begin{cases} \dfrac{dx_2}{dt} = -ax_2 + 2ax_1 - 0.15z_1 - 0.15y_1 - ax_1^2/1.8 \\ \dfrac{dy_2}{dt} = -2by_2 + 0.07x_1 + by_1 - cz_1 + 0.07x_1z_1 - 0.07x_1^2 \\ \dfrac{dz_2}{dt} = -0.1z_2 + 0.1x_1z_1 + 0.02z_1 \end{cases} \tag{3.1}$$

From [11], one can obtain that $A = \begin{pmatrix} -a & 0 & 0 \\ 0 & -2b & 0 \\ 0 & 0 & -0.1 \end{pmatrix}$ is stable.

Function $V(x)$ is:

$$V = x_2^2 + y_2^2 + z_2^2$$

$$\begin{aligned}
\frac{dV}{dt} &= 2x_2\frac{dx_2}{dt} + 2y_2\frac{dy_2}{dt} + 2z_2\frac{dz_2}{dt} \\
&= 2x_2(-ax_2 + 2ax_1 - 0.15z_1 - 0.15y_1 - ax_1^2/1.8) \\
&+ 2y_2(-2by_2 + 0.07x_1 + by_1 - cz_1 + 0.07x_1z_1 - 0.07x_1^2) \\
&+ 2z_2(-0.1z_2 + 0.1x_1z_1 + 0.02z_1) \\
&= -2ax_2^2 - 4by_2^2 - 0.2z_2^2 + 2x_2(2ax_1 - 0.15z_1 - 0.15y_1 - ax_1^2/1.8) \\
&+ 2y_2(0.07x_1 + by_1 - cz_1 + 0.07x_1z_1 - 0.07x_1^2) \\
&+ 2z_2(0.1x_1z_1 + 0.02z_1) \\
&\leq 2(x_2 + y_2 + z_2)M
\end{aligned}$$

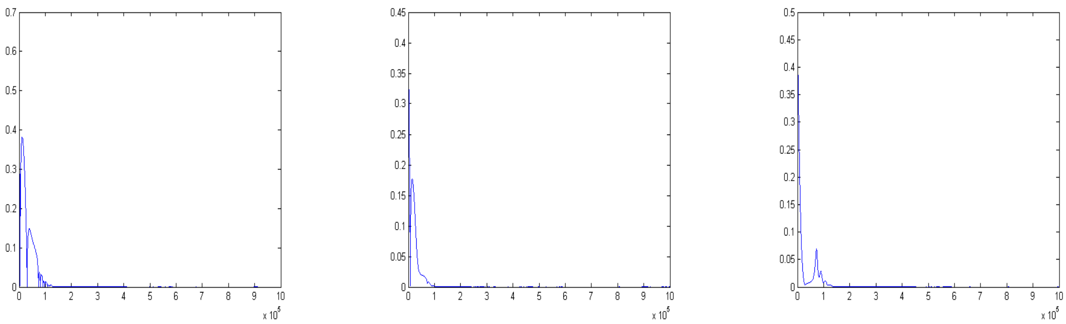
Therefore $\frac{dV}{dt}$ is negative definite about $\Omega = \{X, x_2 + y_2 + z_2 = 0\}$. From [11] we know zero solution is utterly stable.

4 Numerical simulations

In this section, numerical experiments are carried out by using the Matlab. We select the parameters as $a_1 = 0.09, a_2 = 0.15, b_1 = 0.07, b_2 = 0.082, b_3 = 0.07, c_1 = 0.2$, $c_2 = 0.5, c_3 = 0.4, M = 1.8, N = 1$, so that energy resource chaotic system exhibits a chaotic behavior.The initial values of the drive and response systems are $(x_1(0), y_1(0), z_1(0)) = (0.3, 0.4, 0.5)$ and $(x_2(0), y_2(0), z_2(0)) = (0.6, 0.6, 0.2)$ respectively,while the initial states of the error system (2.9) are $e_1 = 0.3, e_2 = 0.2, e_3 = -0.3$. Fig.1 shows synchronization error (e_x, e_y, e_z) of the drive and response systems achieve synchronization respectively when t approaches 1000s.

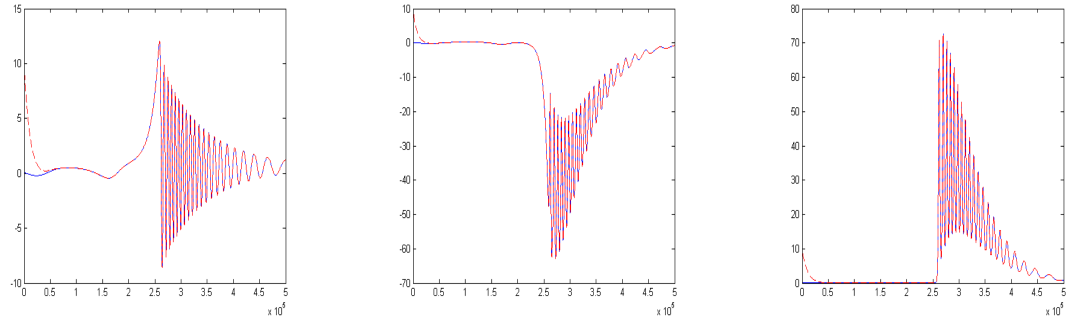
To verify the synchronization capability of the energy resource chaotic system, we get different initial values. The initial values of the drive and response systems are $(x_1(0), y_1(0), z_1(0)) = (0.1, 0.1, 0.1)$ and $(x_2(0), y_2(0), z_2(0)) = (10, 10, 10)$ respectively.We get the time response of states for drive system and response system using active control.(Fig 2)

Select different parameters as $a_1 = 0.09, a_2 = 0.15, b_1 = 0.07, b_2 = 0.09, b_3 = 0.07$, $c_1 = 0.2, c_2 = 0.5, c_3 = 0.4, M = 1.8$. The initial values of the drive and response systems are $(x_1(0), y_1(0), z_1(0)) = (0.3, 0.2, 5)$ and $(x_2(0), y_2(0), z_2(0)) = (4, -1.5, 60)$ respectively, while the initial states of the error system (2.12) are $e_1 = 3.7, e_2 = -1.7, e_3 = 55$. We get another synchronization graph (see Fig. 4) and the time response of states graph (see Fig. 5).



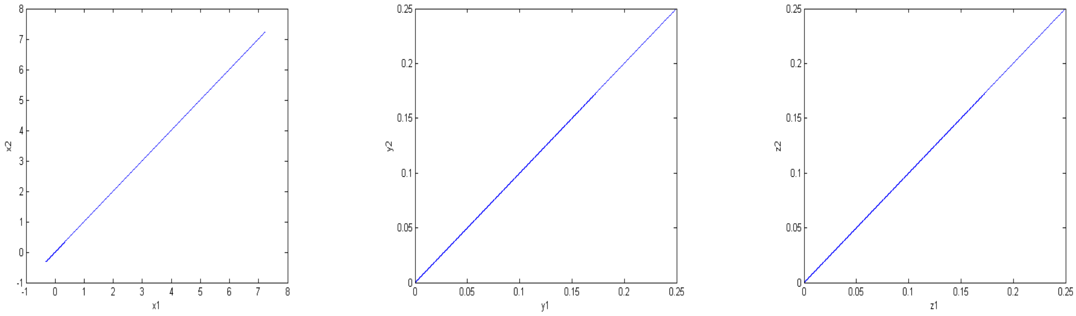
(a) The signal of e_x (b) The signal of e_y (c) The signal of e_z

Figure 1: Synchronization error (*ex*, *ey*, *ez*) for drive system and response system with time $t > 1000s$



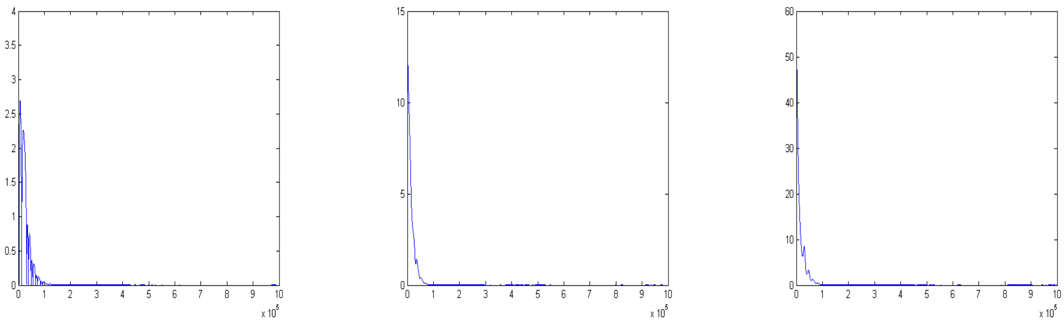
(a) The time series of signals x_1 and x_2 (b) The signals y_1 and y_2 (c) The signals z_1 and z_2

Figure 2: The time response of states for drive system (x_1, y_1, z_1)(dashed line) and response system (x_2, y_2, z_2)(full line)



(a) (b) (c)

Figure 3: The synchronization of drive system and response system

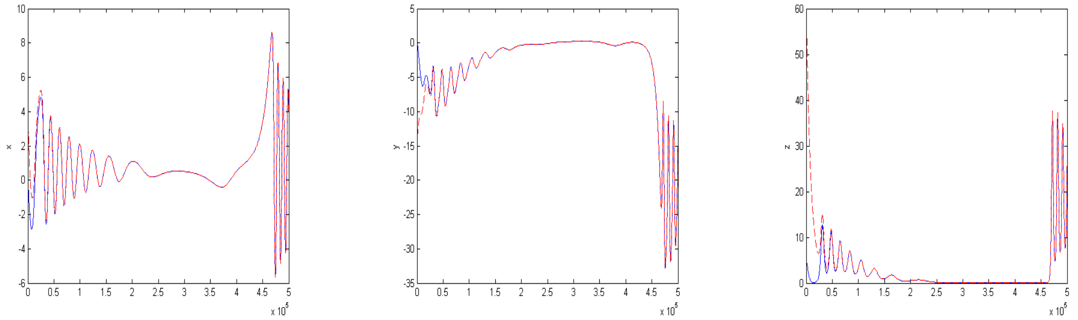


(a) The signal of e_x (b) The signal of e_y (c) The signal of e_z

Figure 4: Synchronization error (*ex*, *ey*, *ez*) for drive system and response system with time $t > 1000s$

5 Conclusion

This paper is devoted to study the problem of chaos synchronization of the energy resource chaotic system.Above all active control is used to achieve chaos synchronization of energy resource chaotic system.



(a) The time series of signals x_1 and x_2 (b) The time series of signals y_1 and y_2 (c) The time series of signals z_1 and z_2

Figure 5: Time response of states for drive system (x_1, y_1, z_1) (dashed line) and response system (x_2, y_2, z_2) (full line)).

The key of the active control is to choose nonlinear function for achieving synchronization .Sufficient conditions to achieve synchronization of energy resource chaotic system are derived by using Lyapunov stability theory. We use the stability theory of autonomy system to prove the stability of the response system. Numerical simulations are presented to demonstrate the effectiveness of the proposed chaos synchronization schemes. Since there are much more left out for in-depth study about energy resource chaotic system in terms of dynamics and complexity, the synchronization procedure in this paper may have practical applications in the future.

Acknowledgements

Research was supported by Outstanding Personnel Program in Six Fields of Jiangsu Province (No: 6-A-029) and the Teaching and Research Award Program for Outstanding Young Teachers in Higher Education Institutions of MOE, P. R. C. (No: 2002-383)

References

[1] Ahmet Ucar,Karl E.Lonngren,Er-Wei Bai : Chaos synchronization in RCL-shunted Josephson junction via active control.*Chaos Solitons and Fractals.* 31, 105-111(2007)

[2] ThongchaiBotmart,PiyapongNiamsup : Adaptive control and synchronization of the perturbed Chua's system.*Mathematics and Computers in Simulation.* In press

[3] Mei Sun,Lixin Tian,Ying Fu,Wei Qian : Dynamics and adaptive synchronization of the energy resource system.*Chaos Solitons and Fractals.* 31, 879-888(2007)

[4] Mei Sun,Lixin Tian,Ying Fu : An energy resources demand-supply system and its dynamical analysis.*Chaos Solitons and Fractals.* 32, 168-180(2007)

[5] Mei Sun,Lixin Tian,Shumin Jiang,Jun Xu : Feedback control and adapitive control of the energy resource chaotic system.*Chaos Solitons and Fractals.*32,1725-1734(2007)

[6] Mei Sun,Lixin Tian,Jun Xu : Time-delayed feedback control of the energy resource chaotic system. *International Journal of Nonlinear Science.*3(3),(2006)

[7] M.T.Yassen : Chaos synchronization between two different chaotic systems using active control.*Chaos Solitons and Fractals.*23,131-140(2005)

[8] Youming Lei, Wei Xu, Wenxian Xie : Synchronization of two chaotic four-dimensional systems using active control.*Chaos Solitons and Fractals.*32,1823-1829(2007)

[9] U.E.Vincent,J.A.Laoye : Synchronization and control of directed transport in chaotic ratchets via active control.*Physics Letters A* .363,91-95(2007)

[10] Zengrong Liu : Several academic problem about synchronization.*Science forum Ziran Zazhi.*26(5)

[11] Xiaoxi Liao : The theory,method and application of stability.*Publishing company of natural science and technology university.* (1994)

[12] Mei Sun , Lixin Tian, Jian Yin: Hopf bifurcation analysis of the energy resource chaotic system. *International Journal of Nonlinear Science.*1(1):49-53(2006)

[13] Hua Chen,Mei Sun:Generalized projective synchronization of the energy resource system. *International Journal of Nonlinear Science.*2(3):166-170(2006)