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Global Synchronization for Time-delay of HCSA System

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Abstract: Considering a time-delay in the receiver as compared with the transmitter, a practical issue of global chaos synchronization is proposed to the HCSA system which is based on the Lyapunov stabilization theory and matrix measure. By choosing proper coupling parameters, the error system is globally asymptotical stable, such that the state of the slave system at time $t+\tau$ is asymptotically synchronizing with the master at time t . The Mathematical software is used to simulate. Both theoretical analysis and simulation result show the effectiveness and anti-interference ability of this method .

Key words:HCSA system, linear feedback functional method, channel time-delay, global chaos synchronization

1 Introduction

Chaos synchronization is one of the most interesting problems in nonlinear sciences [1]-[7]. It can be applied to many fields, such as secure communication, human heartbeat regulation and ecological systems. Paper [8]-[9] study the chaos synchronization, where the unavoidable signal propagation delays in the typical master-slave synchronization. In the practical environment with signal propagation delays, it is not reasonable to require the slave system to synchronize the master system at exactly the same time. It is just like the telephone communication systems, where one hears the voice at time $t+\tau$ on the receiver side, which was said from the transmitter side some time ago, at time t . For this reason, in paper [8], chaotic synchronization is re-defined in such a way that the state of the slave system at time $t+\tau$ is asymptotically synchronizing with the master at time t , namely,

$$\lim_{t\to\infty}\left\|X(t)-\tilde{X}(t+\tau)\right\|=0$$

where X and $\tilde{X}$ are the state of the master and slave systems, respectively. Consequently, the control signal uses the output error between the drive system at time t and the slave system at time $t+\tau$, i.e., $X\left(t\right)-\tilde{X}\left(t+\tau\right)$.

The HCSA system is a complex dynamical system, which exhibits a variety of dynamical states. Paper [10] describes the HCSA system. In this paper, we address a scheme on chaos synchronization of HCSA system where the unavoidable signal propagation delays. Using Lyapunov stabilization theory we study the synchronization of HCSA system, give the numerical results and disturb to the third variable of the system $z\left(t\right)$. Finally, conclusion remarks are given.

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2 Time-delay chaotic synchronization

Consider a chaotic continuous system described by

$$\dot{X}(t) = AX(t) + g(X(t)) \tag{1}$$

where $X(t) \in R^n$ is the state vector, $A \in R^{n\times n}$ is constant matrix, and $g(X)$ is a continuous nonlinear function. Assume that

$$g(X(t_1)) - g(\tilde{X}(t_2)) = M_{X,\tilde{X}}(X(t_1) - \tilde{X}(t_2)) \tag{2}$$

for a bounded matrix $M_{X,\tilde{X}}$ with elements depending on $X(t_1)$ and $\tilde{X}(t_2)$. At this point, it is important to note that most chaotic systems can be described by (1) and (2) in paper [8]. From the typical unidirectional linear error feedback coupling approach, and taking the time-delay into account, the slave system based on the chaotic system (1) is described by (see Fig.1)

$$\dot{\tilde{X}}(t+\tau) = A\tilde{X}(t+\tau) + g\left(\tilde{X}(t+\tau)\right) + L\left(X(t) - \tilde{X}(t+\tau)\right) \tag{3}$$

where τ is a finite time-delay which is an unknown constant, and $L \in R^{n\times m}$ is the coupling matrix to be designed to achieve synchronization.

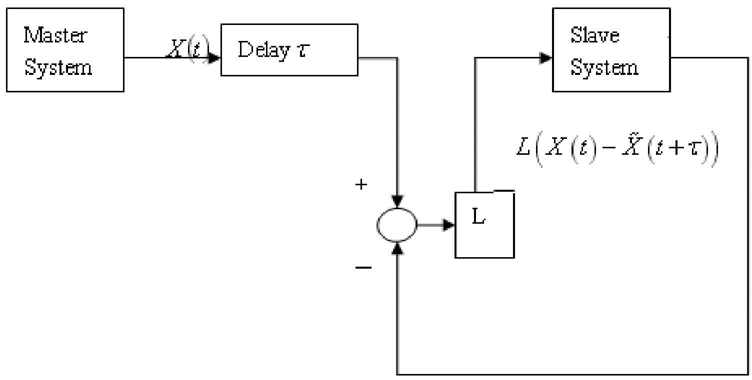

Figure 1: Synchronization with time-delayThe attractor of HCSA system

In Fig.1 and formula (3), the error $X(t) - \tilde{X}(t+\tau)$ is used for control, where $X(t)$ is sent from the master system through the channel and hence it has a time delay as compared to the signal that arrives at the slave system. The value of time-delay τ is not required to be known for executing the control action.

Define the error signal by

$$e(t) = X(t) - \tilde{X}(t+\tau) \tag{4}$$

which is the state error between the master system at time t and the slave system at time $t+\tau$ Synchronization requires that $\|e(t)\| \to 0$ as $t \to \infty$. Then, from (3) and (4), the following error system equation is obtained:

$$\begin{aligned}\dot{e}(t) &= AX(t) + g(X(t)) - A\tilde{X}(t+\tau) - g\left(\tilde{X}(t+\tau)\right) - L\left(X(t) - \tilde{X}(t+\tau)\right)\\ &= Ae(t) + M_{X,\tilde{X}}e(t) - Le(t) = \left(A + M_{X,\tilde{X}} - L\right)e(t)\end{aligned}$$

Using Lyapunov function method, the synchronization of system (1) and (3) can realized.

3 Synchronization for time-delay of HCSA system

HCSA system is a physical system, which is described in paper [10]. It can be described by

$$\begin{cases}\dot{x} = -ax - 4y - 4z - y^2\\ \dot{y} = -ay - 4z - 4x - z^2\\ \dot{z} = -az - 4x - 4y - x^2\end{cases} \tag{5}$$

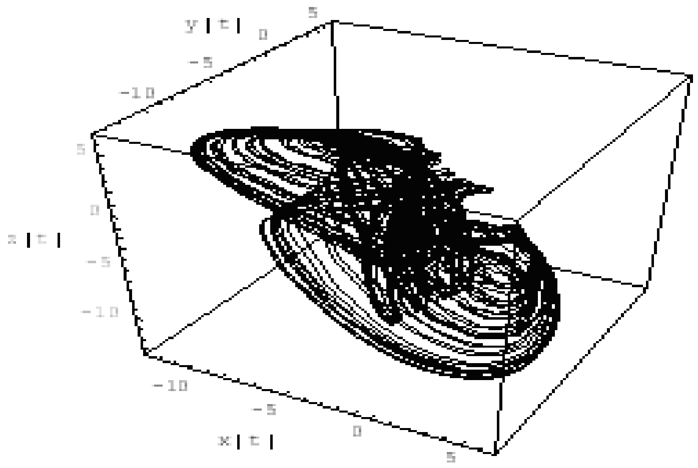

Figure 2: The attractor of HCSA system

From [10] we learn: when $a = 1.27$, the Lyapunov exponents are $(0.7899, 0, -4.5999)$ then system (5) is chaotic .(see Fig.2). The following slave system is constructed by using (3) for (5)

$$\begin{cases} \dot{\tilde{x}}(t+\tau) = -a\tilde{x}(t+\tau) - 4\tilde{y}(t+\tau) - 4\tilde{z}(t+\tau) - \tilde{y}^2(t+\tau) + l_1(x(t) - \tilde{x}(t+\tau)) \\ \dot{\tilde{y}}(t+\tau) = -a\tilde{y}(t+\tau) - 4\tilde{z}(t+\tau) - 4\tilde{x}(t+\tau) - \tilde{z}^2(t+\tau) + l_2(y(t) - \tilde{y}(t+\tau)) \\ \dot{\tilde{z}}(t+\tau) = -a\tilde{z}(t+\tau) - 4\tilde{x}(t+\tau) - 4\tilde{y}(t+\tau) - \tilde{x}^2(t+\tau) + l_3(z(t) - \tilde{z}(t+\tau)) \end{cases} \tag{6}$$

Subtracting (6) from (5), we obtain

$$\begin{cases} \dot{e}_x(t) = -ae_x(t) - 4e_y(t) - 4e_z(t) - \left[y^2(t) - \tilde{y}^2(t+\tau)\right] - l_1 e_x(t) \\ \dot{e}_y(t) = -ae_y(t) - 4e_z(t) - 4e_x(t) - \left[z^2(t) - \tilde{z}^2(t+\tau)\right] - l_2 e_y(t) \\ \dot{e}_z(t) = -ae_z(t) - 4e_x(t) - 4e_y(t) - \left[x^2(t) - \tilde{x}^2(t+\tau)\right] - l_3 e_z(t) \end{cases}$$

where

$$e_x(t) = x(t) - \tilde{x}(t+\tau),$$

$$e_y(t) = y(t) - \tilde{y}(t+\tau),$$

$$e_z(t) = z(t) - \tilde{z}(t+\tau),$$

which can be rewritten as

$$\dot{e}(t) = Ae(t) + g(X(t)) - g\left(\tilde{X}(t+\tau)\right) - Le(t) \tag{7}$$

where

$$A = \begin{bmatrix} -a & -4 & -4 \\ -4 & -a & -4 \\ -4 & -4 & -a \end{bmatrix},$$

$$L = \begin{bmatrix} l_1 & 0 & 0 \\ 0 & l_2 & 0 \\ 0 & 0 & l_3 \end{bmatrix},$$

$$e(t) = \begin{bmatrix} x(t) - \tilde{x}(t+\tau) \\ y(t) - \tilde{y}(t+\tau) \\ z(t) - \tilde{z}(t+\tau) \end{bmatrix},$$

$$g(X) = \begin{bmatrix} y^2 \\ z^2 \\ x^2 \end{bmatrix}$$

Then

$$g\left(X\left(t\right)\right)-g\left(\tilde{X}\left(t+\tau\right)\right)=\left[\begin{array}{c} y^2\left(t\right)-\tilde{y}^2\left(t+\tau\right) \\ z^2\left(t\right)-\tilde{z}^2\left(t+\tau\right) \\ x^2\left(t\right)-\tilde{x}^2\left(t+\tau\right) \end{array}\right]$$
$$=\left[\begin{array}{ccc} 0 & y\left(t\right)+\tilde{y}\left(t+\tau\right) & 0 \\ 0 & 0 & z\left(t\right)+\tilde{z}\left(t+\tau\right) \\ x\left(t\right)+\tilde{x}\left(t+\tau\right) & 0 & 0 \end{array}\right]\left[\begin{array}{c} x\left(t\right)-\tilde{x}\left(t+\tau\right) \\ y\left(t\right)-\tilde{y}\left(t+\tau\right) \\ z\left(t\right)-\tilde{z}\left(t+\tau\right) \end{array}\right] \quad (8)$$
$$=M_{X,\tilde{X}}e\left(t\right)$$

where

$$M_{X,\tilde{X}}=\left[\begin{array}{ccc} 0 & y\left(t\right)+\tilde{y}\left(t+\tau\right) & 0 \\ 0 & 0 & z\left(t\right)+\tilde{z}\left(t+\tau\right) \\ x\left(t\right)+\tilde{x}\left(t+\tau\right) & 0 & 0 \end{array}\right]$$

Theorem 1 *If there exists a positive definite symmetric consta nt matrix such that*

$$\left(A-L+M_{X,\tilde{X}}\right)^T P+P\left(A-L+M_{X,\tilde{X}}\right)\leq \mu I<0 \quad (9)$$

uniformly for all in the phase space, where is a negative constant and is the identity matrix, then the error system (7) *is globally exponentially stable about zero, implying that the two systems* (5) *and* (6) *are globally asymptotically synchronized.*

Proof. Choose aLyapunov function of the form

$$V=e\left(t\right)^T Pe\left(t\right)$$

where P is a positive definite symmetric constant matrix. Its derivative is

$$\begin{aligned} \dot{V}&=\dot{e}\left(t\right)Pe\left(t\right)+e\left(t\right)^T P\dot{e}\left(t\right) \\ &=[(A-L+M_{X,\tilde{X}})e\left(t\right)]^T Pe\left(t\right)+e\left(t\right)^T P[(A-L+M_{X,\tilde{X}})e\left(t\right)] \\ &=e\left(t\right)^T[(A-L+M_{X,\tilde{X}})^T P+P(A-L+M_{X,\tilde{X}})]e\left(t\right)\leq \mu\left\|e\left(t\right)\right\|^2<0 \end{aligned}$$

where $\|.\|$ denotes the Euclidean norm.
Based on the Lyapunov stability theory, system (7) is globally exponentially stable about zero. Hence the two systems (5) and (6) are globally asymptotically synchronized. □

Theorem 1 proves the effectiveness of the method, next work is to determine L .
Let $Q=\left(A-L+M_{X,\tilde{X}}\right)^T P+P\left(A-L+M_{X,\tilde{X}}\right)$, $\lambda_i, i=1,2,3$ are the eigenvalves of Q. If inequality (9) holds, then

$$\lambda_i\leq \mu<0, \qquad i=1,2,3 \quad (10)$$

So we choose L , so as to satisfy (10) .

Theorem 2 *Choose* $P=diag\left(p_1,p_2,p_3\right)$ *, and let*

$$P\left(A+M_{X,\tilde{X}}\right)+\left(A+M_{X,\tilde{X}}\right)^T P=[\bar{a}_{ij}]$$

If a suitable L is chosen such that

$$l_i\geq \frac{1}{2p_i}\left(\bar{a}_{ii}+R_i-\mu\right), \qquad i=1,2,3 \qquad and \qquad R_i=\sum_{j=1,j\neq i}^{3}\left|\bar{a}_{ij}\right|$$

then the two coupled chaotic systems (7) *and* (8) *are globally synchronized.*

Proof:

$$\begin{aligned} Q &= \left(A - L + M_{X,\tilde{X}}\right)^T P + P\left(A - L + M_{X,\tilde{X}}\right) \\ &= P\left(A + M_{X,\tilde{X}}\right) + \left(A + M_{X,\tilde{X}}\right)^T P - LP - PL \\ &= P\left(A + M_{X,\tilde{X}}\right) + \left(A + M_{X,\tilde{X}}\right)^T P - 2LP \end{aligned}$$

From Gerschgorin's theorem in matrix theory we know, every eigenvalve is in its Gerschgorin, i.e.$\lambda_i - \bar{a}_{ii} - 2l_ip_i \le R_i$, while $\mu \ge \lambda_i$, it means μ is out of all Gerschgoeins, so $\mu - \bar{a}_{ii} - 2l_ip_i \ge R_i$, then we get following inequalities,

$$l_i \ge \frac{1}{2p_i}\left(\bar{a}_{ii} + R_i - \mu\right)$$

In particular, we choose $P = I$, we obtain

$$\left(A + M_{X,\tilde{X}}\right) + \left(A + M_{X,\tilde{X}}\right)^T$$

$$= \begin{bmatrix} -a & -4+y+\tilde{y} & -4 \\ -4 & -a & -4+z+\tilde{z} \\ -4+x+\tilde{x} & -4 & -a \end{bmatrix} \begin{bmatrix} -a & -4 & -4+x+\tilde{x} \\ -4+y+\tilde{y} & -a & -4 \\ -4 & -4+z+\tilde{z} & -a \end{bmatrix}$$

$$= \begin{bmatrix} -2a & -8+y+\tilde{y} & -8+x+\tilde{x} \\ -8+y+\tilde{y} & -2a & -8+z+\tilde{z} \\ -8+x+\tilde{x} & -8+z+\tilde{z} & -2a \end{bmatrix}$$

We can choose

$$\begin{aligned} l_1 &\ge 1/2\left(-2a + |-8+y+\tilde{y}| + |-8+x+\tilde{x}| - u\right) \\ l_2 &\ge 1/2\left(-2a + |-8+y+\tilde{y}| + |-8+z+\tilde{z}| - u\right) \\ l_3 &\ge 1/2\left(-2a + |-8+x+\tilde{x}| + |-8+z+\tilde{z}| - u\right) \end{aligned}$$

From Fig.1, we can get$-15 < x < 5, -15 < y < 5, -15 < z < 5$, so we choose $x = -5, y = 0, z = 0, \tilde{x} = -4, \tilde{y} = 1, \tilde{z} = 0.5$ and choose $\mu = -1$, we can obtain $l_1 = 13, l_2 = 21, l_3 = 18$, the inequality (3) holds. According to Theorem 1, the two chaotic systems (5) and (6) are globally asymptotically synchronized (see Fig. 3-5). □

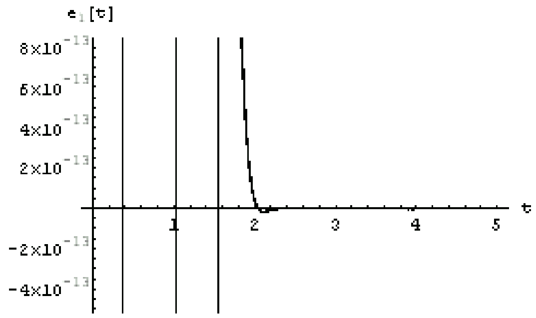

Figure 3: Time evolution of valuable $e_x(t)$

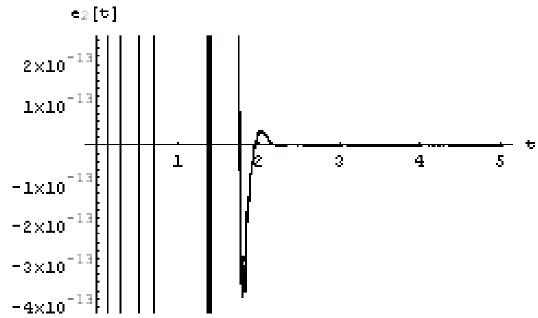

Figure 4: Time evolution of valuable $e_y(t)$

Theorem 3 *The two chaotic systems* (1) *and* (3) *are globally asymptotically synchronized, if one condition is satisfied at least as follows:*

$$(1)\ max_j\{\tilde{a}_{jj} + \sup\sum_{i=1,i\neq j}^{3}|\tilde{a}_{ij}|\} < 0$$

$$(2)\ max_j\{\tilde{a}_{ii} + \sup\sum_{j=1,j\neq i}^{3}|\tilde{a}_{ij}|\} < 0$$

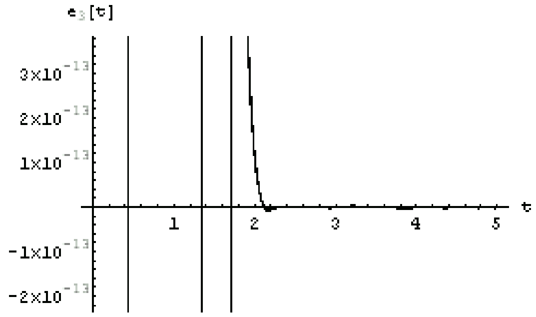

Figure 5: Time evolution of valuable $e_z(t)$

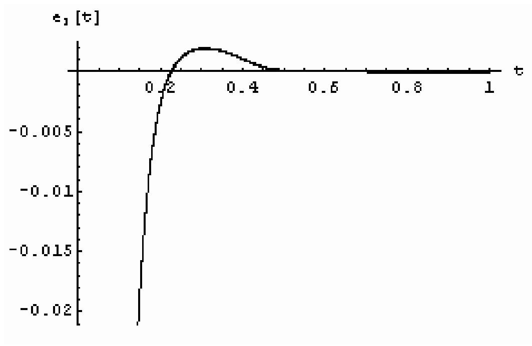

Figure 6: Time evolution of valuable $e_x(t)$

where $A - L + M_{X,\tilde{X}} = (\tilde{a}_{ij})_{3\times3}$.

In this paper, we have

$$A - L + M_{X,\tilde{X}} = \begin{bmatrix} -a-l_1 & -4+y+\tilde{y} & -4 \\ -4 & -a-l_2 & -4+z+\tilde{z} \\ -4+x+\tilde{x} & -4 & -a-l_3 \end{bmatrix}$$

We choose $l_1,\ l_2,\ l_3$ to satisfy

$$max\{-a-l_1+4+|-4+x+\tilde{x}|,\ -a-l_2+4+|-4+y+\tilde{y}|,\ -a-l_3+4+|-4+z+\tilde{z}|\}<0$$

or

$$max\{-a-l_1+4+|-4+y+\tilde{y}|,\ -a-l_2+4+|-4+z+\tilde{z}|,\ -a-l_3+4+|-4+x+\tilde{x}|\}<0$$

Then, We can determine the bound of $l_1,\ l_2,\ l_3$ and realize the chaos synchronization by choosing the proper parameters(see Fig.6-8)

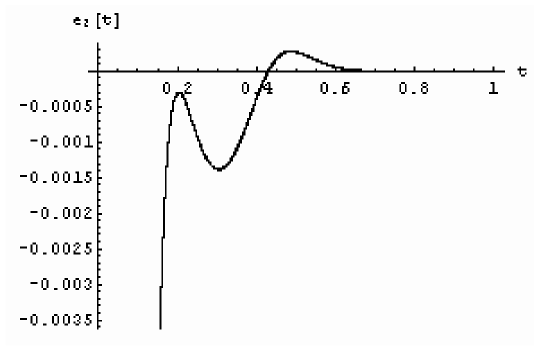

Figure 7: Time evolution of valuable $e_y(t)$

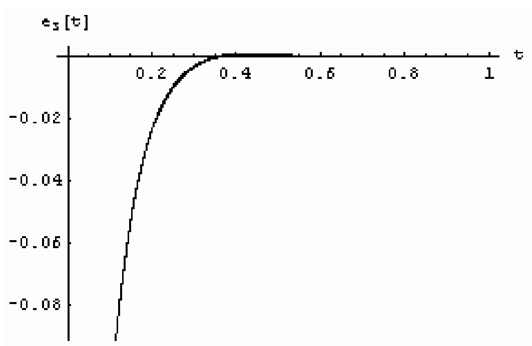

Figure 8: Time evolution of valuable $e_z(t)$

In practical appliances, systems get interfere inevitably from environment. Disturb to the error system , assuming that $z = (1.0 + 0.002 \sin t)z$. And the Mathematic software is used to simulate, the simulation result show the effectiveness and anti-interference ability of this method(see Fig.9-11).

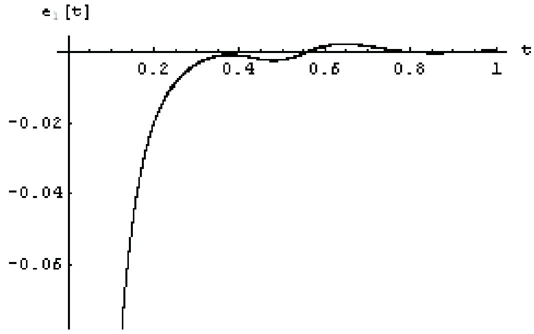

Figure 9: Time evolution of valuable $e_x(t)$

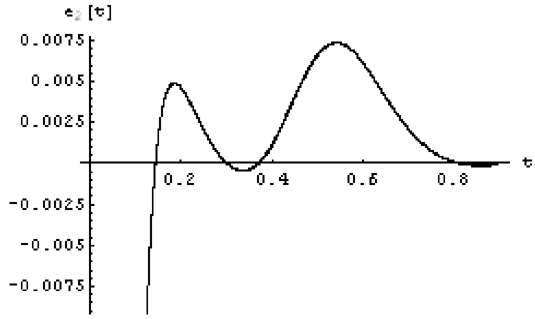

Figure 10: Time evolution of valuable $e_y(t)$

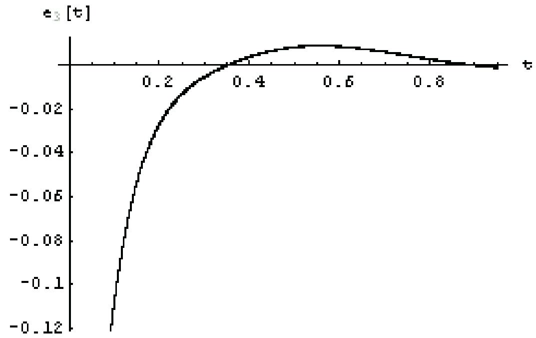

Figure 11: Time evolution of valuable $e_z(t)$

4 Conclusion

The chaos synchronization problem, with time-delay in the channel, has been studied from the typical approach of unidirectional linear error feedback using the received time-delay signal directly without requiring any knowledge of the constant unknown time-delay. This paper address a practical issue in chaos synchronization of HCSA system.From the attractor of the system, we can get the bound of the parameters.Then we can choose the proper coupling parameters, which can make the systems globally synchronize. How to choose the better coupling parameter is the further research.

References

[1] Chang Lin, Xiu-lian Zhang, Hai-Chang Mo: Tracing control of chaos by driving the implanted parameter. Chaos Solitons and Fractals.19, 673-677(2004)

[2] Chen G, Dong X. From chaos to order-methodologies, perspectivesand applications. Singapore: World Scientific;1998

[3] Mei Sun, Lixin Tian, Jian Yin: Hopf bifurcation analysis of the energy resource chaotic System. International Journa of Nonlinear Science.1(1), 49-53(2006)

[4] Sevdzhan Hakkaev, Kiril Kirchev: On the well-posedness and stability of Peakons for a generalized Camassa-Holm equation. International Joural of Nonlinear Science. 3(1), 139-148(2006)

[5] Xinghua Fan, Lixin Tian, Lihong Ren: New compactons in nonlinear atomic chain equations with firstand- second-neighbour interactions.International Journal of Nonlinear Science.1(2), 105-110(2006)

[6] Jitao Sun. Global synchronization criteria with channel time-delay for chaotic time-delay system. Chaos, Solitons and Fractals 2004;21: 967-975.

[7] Liao T-L, Tsai S-H. Adaptive synchronization of chaotic systems and its application to secure communications. Chaos, Solitons and Fractals 2000;11(9):1387-96.

[8] Guo-Ping Jiang, Wei Xing Zheng, Guanrong Chen. Global chaos synchronization with channel time-delay [J]. Chaos, Solitons and Fractals 20(2004)267-275.

[9] Jitao Sun. Global synchronization criteria with channel time-delay for chaotic time-delay system. Chaos, Solitons and Fractals 2004;21:967-975.

[10] Julien Clinton Sprott. Chaos and Time-series Analysis. Oxford University Press. 2003.