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Backstepping Control for a Class of Uncertain Nonlinear Systems with Neural Network

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Abstract: In this paper, an adaptive L_2 -gain control scheme is presented for a class of uncertain nonlinear system with backstepping method and neural network. Based on the output of neural network, the given controller can not only guarantee robust stability of the closed loop system, but also make it has L_2-gain performance index which less than or equal to $\gamma > 0$.

Key words: Uncertain nonlinear systems; RBF Neural Network; Backstepping control; L_2-gain control

1 Introduction

In the last several years, considerable research has been taken on nonlinear control[1−4]. Integrator backstepping control is an effective control method in the nonlinear system control area due to the robust performance with respect to known nonlinearities not satisfying the matching conditions [5]. Recently, the neural network has been successfully used in the control of uncertain nonlinear systems and there are a lot of research results about them. Combining backstepping control with neural network to control nonlinear systems has received increasing attention because of its advantage [6−7]. Integrator backstepping requires cancellation of nonlinearities. Sometimes we my not have precise knowledge about the system nonlinearities, or the nonlinearities may be changing with time. In these cases, we can use neural network to approximate the nonlinearities. A direct adaptive backstepping neural-network control scheme has been proposed for a class of affine nonlinear systems with unknown uncertainties in [6]. Ref.[7] has considered the problem of controlling uncertain nonlinear systems by combining backstepping design with neural networks. But these papers did not consider parameter uncertainty of the nonlinear system.

In modern control area, demanding the nonlinear closed loop system can not only guarantee robust stability, but also has some performance index. In this paper, a robust adaptive L_2-gain controller is proposed with neural network for a class of uncertain nonlinear system. The structure of the paper is as follows. Section 2 gives the robust adaptive L_2-gain control problem formulation of a class of uncertain nonlinear system. Section 3 describes the robust adaptive L_2-gain control scheme based on neural network. Some concluding remarks are given in Section 4.

2 Problem formulation

Consider the uncertain nonlinear system in the form of

$$\begin{aligned} &\dot{x}_i = x_{i+1} + \gamma_i^T(x_1,\cdots,x_i)\theta + \psi_i(\mathbf{x}), \quad 1 \le i \le n-1 \\ &\dot{x}_n = f(\mathbf{x}) + \gamma_n^T(\mathbf{x})\theta + (g_0(\mathbf{x}) + g^T(\mathbf{x})\theta)u + \psi_n(\mathbf{x}) \\ &y = x_1 \end{aligned} \tag{2.1}$$

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where $\mathbf{x} = [x_1, \cdots, x_n]^T \in R^n$ is the state vectors of uncertain nonlinear system (2.1) . $u \in R$ is the control input. $y \in R$ is the output of the nonlinear system.$f(.)$ is a smooth nonlinear function terms which can be unknown. $\gamma_i(\mathbf{x}), g_0(\mathbf{x}), g(\mathbf{x}), 1 \leq i \leq n$ are known smooth nonlinear function term with corresponding dimensions. $\psi_i(\mathbf{x}), 1 \leq i \leq n$ are the system uncertainties with unknown upper bound of the nonlinear system. θ is the parameter uncertainty of the nonlinear system.To proceed with the design of robust adaptive L_2-gain controller, the following assumption and definition are required.

Assumption 1: To random$\theta \in L_2(0, \infty)$, there has $g_0 + g^T(x)\hat{\theta} \neq 0$. where $\hat{\theta}$ is the estimate value of θ.

Definition: Given a scalar $\gamma > 0$, the system (2.1) possesses L_2-gain $\leq \gamma$ for $T \geq 0$ and $\theta \in L_2(0, \infty)$if and only if

$$\int_0^T \|y(t)\|^2 dt \leq \gamma^2 \int_0^T \|\theta\|^2 dt + N \tag{2.2}$$

where $N > 0$.

In this paper, the neural networks are chosen as RBF neural networks. Therefore, the approximations of compound uncertainty $d_i(Z_i), 1 \leq i \leq n$ of i-th subsystem can be expressed as

$$\hat{d}_i(Z_i, t) = W_i^T \phi_i(Z_i) \tag{2.3}$$

where $d_i(Z_i)$and Z_i will be defined in following section. $\phi_i(\mathbf{x})$,$1 \leq i \leq n$ are base functions of corresponding RBF neural networks.

The optimization weight value of RBF neural network is defined as

$$W_i^* = \arg \min_{W_i \in \Omega_{d_i}} \left[\sup_{Z_i \in s_{Z_i}} |d_i(Z_i/W_i) - d_i(Z_i)| \right] \tag{2.4}$$

where $\Omega_{d_i} = \{W_{d_i} : \|W_{d_i}\| \leq M_{d_i}\}$ is the valid field of the parameter. $M_{d_i}$is the designed parameter. $s_{Z_i} \subset R^{n_i}$is the variable space of the state vector.

Under optimization weight value, the unknown uncertainty can be expressed as

$$d_i = W_i^{*T} \phi_i(Z_i) + \varepsilon_i \tag{2.5}$$

where ε_i is the smallest approximation error of RBF neural networks. Suppose that

$$|\varepsilon_i| \leq \varepsilon_i^* \tag{2.6}$$

where $\varepsilon_i^* > 0$ is upper bound of the approximation error of $d_i(Z_i)$ using RBF neural network.

3 Controller Design for Uncertain Nonlinear Systems with Backstepping method and RBF Neural Network

The task of this section is to design controller with neural network and backstepping method.

Step 1: Let $e_1 = y_1 - y_{1d}$ and suppose that $y_{1d} = 0$, so $e_1 = x_1$. Its derivative

$$\dot{e}_1 = x_2 + \gamma_1^T(x_1)\theta + \psi_1(\mathbf{x}) \tag{3.1}$$

Let $V_{z1} = e_1^2/2$, then its derivative is

$$\dot{V}_{z1} = e_1\dot{e}_1 = e_1(x_2 + \gamma_1^T(x_1)\theta + \psi_1(\mathbf{x})) \tag{3.2}$$

Because the uncertainty $d_1 = \psi_1(\mathbf{x})$is unknown, the RBF neural network is used to approximate it. Eq.(3.2) can be rewritten as

$$\dot{V}_{z1} = e_1\dot{e}_1 = e_1(x_2 + \gamma_1^T(x_1)\theta + W_1^{*T}\phi_1(Z_1) + \varepsilon_1) \tag{3.3}$$

where $Z_1 = [x_1, \cdots, x_n, e_1]^T$.

By viewing x_2 as a virtual control input, let us choose virtual controller x_2^* as follows:

$$x_2^* = -c_1 e_1 - \frac{1}{4}\tau e_1(1+\gamma_1^T(x_1)\gamma_1(x_1)) - \hat{W}_1^T\phi_1(Z_1) \tag{3.4}$$

where $c_1 > 0, \tau > 0$.

Defining $e_2 = x_2 - x_2^*$, there yields

$$\dot{V}_{z1} = e_1(e_2 - c_1 e_1 + \gamma_1^T(x_1)\theta - \frac{1}{4}\tau e_1(1+\gamma_1^T(x_1)\gamma_1(x_1)) - \tilde{W}_1^T\phi_1(Z_1) + \varepsilon_1) \tag{3.5}$$

where $\tilde{W}_1 = \hat{W}_1 - W_1^*$.

Consider the following Lyapunov candidate:

$$V_1 = V_{z1} + \frac{1}{2}\tilde{W}_1^T\Gamma_1^{-1}\tilde{W}_1 = \frac{1}{2}e_1^2 + \frac{1}{2}\tilde{W}_1^T\Gamma_1^{-1}\tilde{W}_1 \tag{3.6}$$

where $\Gamma_1 = \Gamma_1^{-1} > 0$.

Considering (3.5), the derivative of V_1 is

$$\dot{V}_1 = e_1(e_2 - c_1 e_1 + \gamma_1^T(x_1)\theta - \frac{1}{4}\tau e_1(1+\gamma_1^T(x_1)\gamma_1(x_1)) - \tilde{W}_1^T\phi_1(Z_1) + \varepsilon_1) + \tilde{W}_1^T\Gamma_1^{-1}\dot{\tilde{W}}_1 \tag{3.7}$$

Choose the following adaptive law

$$\dot{\tilde{W}}_1 = \dot{\hat{W}}_1 = \Gamma_1[\phi_1(Z_1)e_1 - \sigma_1\hat{W}_1] \tag{3.8}$$

where$\sigma_1 > 0$. Substituting (3.8) into (3.7) yields

$$\dot{V}_1 = e_1(e_2 - c_1 e_1 + \gamma_1^T(x_1)\theta - \frac{1}{4}\tau e_1(1+\gamma_1^T(x_1)\gamma_1(x_1))) + e_1\varepsilon_1 - \sigma_1\tilde{W}_1^T\hat{W}_1 \tag{3.9}$$

There is

$$-\frac{1}{4}\tau e_1^2(1+\gamma_1^T(x_1)\gamma_1(x_1) + e_1\gamma_1^T(x_1)\theta \le \frac{(\gamma_1^T(x_1)\theta)^2}{\tau(1+\gamma_1^T(x_1)\gamma_1(x_1))} \le \frac{1}{\tau}\left\|\theta\right\|^2 \tag{3.10}$$

Let $c_1 = c_{10} + c_{11}$, with $c_{10} > 0, c_{11} > 0$. Similarly by completion of squares, we have

$$-\sigma_1\tilde{W}_1^T\hat{W}_1 \;=\; -\sigma_1\tilde{W}_1^T(\tilde{W}_1 + W_1^*) \le -\frac{\sigma_1\left\|\tilde{W}_1\right\|^2}{2} + \frac{\sigma_1\left\|W_1^*\right\|^2}{2} \tag{3.11}$$

$$-c_{11}e_1^2 + e_1\varepsilon_1 \le -c_{11}e_1^2 + |e_1|\,|\varepsilon_1| \le \frac{\varepsilon_1^2}{4c_{11}} \le \frac{\varepsilon_1^{*2}}{4c_{11}} \tag{3.12}$$

Substituting (3.10) – (3.12) into (3.9) yield

$$\dot{V}_1 \le e_1e_2 - c_{10}e_1^2 + \frac{1}{\tau}\left\|\theta\right\|^2 + \frac{1}{4c_{11}}\varepsilon_1^{*2} - \frac{\sigma_1\left\|\tilde{W}_1\right\|^2}{2} + \frac{\sigma_1\left\|W_1^*\right\|^2}{2} \tag{3.13}$$

where the coupling term $e_1e_2$will be canceled in the following step.

Step i$(2 \le i \le n-1)$: This step is to make the error between x_i and x_i^* as small as possible. Defining $e_i = x_i - x_i^*$ and differentiating it yield

$$\dot{e}_i = \dot{x}_i - \dot{x}_i^* = x_{i+1} + \gamma_i^T(x_1,\cdots,x_i)\theta + \psi_i(\mathbf{x}) - \dot{x}_i^* \tag{3.14}$$

Let $d_i = \psi_i(\mathbf{x}) - \dot{x}_i^*$. Because the compound uncertainty d_i is unknown, the RBF neural network is used to approximate it. Let $V_{zi} = V_{i-1} + e_i^2/2$, then its derivative is

$$\begin{aligned}\dot{V}_{zi} \;\le\; & e_{i-1}e_i - \sum_{k=1}^{i-1}c_{k0}e_k^2 + \frac{i-1}{\tau}\left\|\theta\right\|^2 + \sum_{k=1}^{i-1}\left(\frac{1}{4c_{k1}}\right)\varepsilon_k^{*2} - \sum_{k=1}^{i-1}\frac{\sigma_k\left\|\tilde{W}_k\right\|^2}{2} \\ +\; & \sum_{k=1}^{i-1}\frac{\sigma_k\left\|W_k^*\right\|^2}{2} + e_i(x_{i+1} + \gamma_i^T(x_1,\cdots,x_i)\theta + W_i^{*T}\phi_i(Z_i) + \varepsilon_i)\end{aligned} \tag{3.15}$$

where $Z_i = [x_1, \cdots, x_n, e_1, \cdots, e_i, \varsigma_i]^T$.

By viewing x_{i+1} as a virtual control input, let us choose virtual controller x_{i+1}^* as follows:

$$x_{i+1}^* = -e_{i-1} - c_i e_i - \frac{1}{4}\tau e_i(1 + \gamma_i^T(x_1, \cdots, x_i)\gamma_i(x_1, \cdots, x_i)) - \hat{W}_i^T \phi_i(Z_i) \tag{3.16}$$

Defining $e_{i+1} = x_{i+1} - x_{i+1}^*$ and considering (3.16), there yields

$$\begin{aligned} \dot{V}_{zi} \leq\ & e_i e_{i+1} - \sum_{k=1}^{i-1} c_{k0} e_k^2 + \frac{i-1}{\tau}\|\theta\|^2 + \sum_{k=1}^{i-1}\left(\frac{1}{4c_{k1}}\right)\varepsilon_k^{*2} - \sum_{k=1}^{i-1}\frac{\sigma_k \left\|\tilde{W}_k\right\|^2}{2} + \sum_{k=1}^{i-1}\frac{\sigma_k \|W_k^*\|^2}{2} \\ -\ & c_i e_i^2 + e_i(-\tilde{W}_i^T \phi_i(Z_i) - \frac{1}{4}\tau e_i(1 + \gamma_i^T(x_1, \cdots, x_i)\gamma_i(x_1, \cdots, x_i)) + \gamma_i^T(x_1, \cdots, x_i)\theta + \varepsilon_i) \end{aligned} \tag{3.17}$$

where $\tilde{W}_i = \hat{W}_i - W_i^*$.

Consider the following Lyapunov candidate:

$$V_i = V_{zi} + \frac{1}{2}\tilde{W}_i^T \Gamma_i^{-1} \tilde{W}_i \tag{3.18}$$

where $\Gamma_i = \Gamma_i^{-1} > 0$.

Considering (3.17), the derivative of V_i is

$$\begin{aligned} \dot{V}_i \leq\ & e_i e_{i+1} - \sum_{k=1}^{i-1} c_{k0} e_k^2 + \frac{i-1}{\tau}\|\theta\|^2 + \sum_{k=1}^{i-1}\left(\frac{1}{4c_{k1}}\right)\varepsilon_k^{*2} - \sum_{k=1}^{i-1}\frac{\sigma_k \left\|\tilde{W}_k\right\|^2}{2} + \sum_{k=1}^{i-1}\frac{\sigma_k \|W_k^*\|^2}{2} - c_i e_i^2 \\ & + e_i(-\tilde{W}_i^T \phi_i(Z_i) - \frac{1}{4}\tau e_i(1 + \gamma_i^T(x_1, \cdots, x_i)\gamma_i(x_1, \cdots, x_i)) + \gamma_i^T(x_1, \cdots, x_i)\theta + \varepsilon_i) + \tilde{W}_i^T \Gamma_i^{-1} \dot{\tilde{W}}_i \end{aligned} \tag{3.19}$$

Choose the following adaptive law

$$\dot{\hat{W}}_i = \dot{\tilde{W}}_i = \Gamma_i[\phi_i(Z_i)e_i - \sigma_i \hat{W}_i] \tag{3.20}$$

where $\sigma_i > 0$.Then (3.19) becomes

$$\begin{aligned} \dot{V}_i \leq\ & e_i e_{i+1} - c_i e_i^2 - \sum_{k=1}^{i-1} c_{k0} e_k^2 + \frac{i-1}{\tau}\|\theta\|^2 + \sum_{k=1}^{i-1}\left(\frac{1}{4c_{k1}}\right)\varepsilon_k^{*2} - \sum_{k=1}^{i-1}\frac{\sigma_k \left\|\tilde{W}_k\right\|^2}{2} + \sum_{k=1}^{i-1}\frac{\sigma_k \|W_k^*\|^2}{2} \\ & + e_i(-\frac{1}{4}\tau e_i(1 + \gamma_i^T(x_1, \cdots, x_i)\gamma_i(x_1, \cdots, x_i)) + \gamma_i^T(x_1, \cdots, x_i)\theta + \varepsilon_i) - \sigma_i \tilde{W}_i^T \hat{W}_i \end{aligned} \tag{3.21}$$

Let $c_i = c_{i0} + c_{i1}$, with $c_{i0} > 0, c_{i1} > 0$. Similarly by completion of squares, the derivative of V$_i$ becomes

$$\dot{V}_i \leq e_i e_{i+1} - \sum_{k=1}^{i} c_{k0} e_k^2 + \frac{i}{\tau}\|\theta\|^2 + \sum_{k=1}^{i}\left(\frac{1}{4c_{k1}}\right)\varepsilon_k^{*2} - \sum_{k=1}^{i}\frac{\sigma_k \left\|\tilde{W}_k\right\|^2}{2} + \sum_{k=1}^{i}\frac{\sigma_k \|W_k^*\|^2}{2} \tag{3.22}$$

Step n: This step is to design the control input which make the uncertain nonlinear system has L_2-gain performance index less than or equal to γ. Defining $e_n = x_n - x_n^*$ and differentiating it yield

$$\dot{e}_n = \dot{x}_n - \dot{x}_n^* = f(\mathbf{x}) + \gamma_n^T(\mathbf{x})\theta + (g_0(\mathbf{x}) + g^T(\mathbf{x})\theta)u + \psi_n(\mathbf{x}) - \dot{x}_n^* \tag{3.23}$$

Let $d_n = f(\mathbf{x}) + \psi_n(\mathbf{x}) - \dot{x}_n^*$. Because the compound uncertainty d_n is unknown, the RBF neural network is used to approximate it. Let $V_{zn} = V_{n-1} + e_n^2/2$, then its derivative is

$$\begin{aligned} \dot{V}_{zn} \leq\ & e_{n-1}e_n - \sum_{k=1}^{n-1} c_{k0} e_k^2 + \frac{n-1}{\tau}\|\theta\|^2 + \sum_{k=1}^{n-1}\left(\frac{1}{4c_{k1}}\right)\varepsilon_k^{*2} - \sum_{k=1}^{n-1}\frac{\sigma_k \left\|\tilde{W}_k\right\|^2}{2} \\ +\ & \sum_{k=1}^{n-1}\frac{\sigma_k \|W_k^*\|^2}{2} + e_n(\gamma_n^T(\mathbf{x})\theta + (g_0(\mathbf{x}) + g^T(\mathbf{x})\theta)u + W_n^{*T}\phi_n(Z_n) + \varepsilon_n) \end{aligned} \tag{3.24}$$

where $Z_n = [x_1, \cdots, x_n, e_1, \cdots, e_n]^T$.

Choose the control law of the uncertain system as:

$$u = \frac{1}{g_0 + g^T\hat{\theta}}(-e_{n-1} - c_n e_n - \frac{1}{4}\tau e_n(1 + \gamma_n^T(\mathbf{x})\gamma_n(\mathbf{x})) - \hat{W}_n^T\phi_n(Z_n)) \tag{3.25}$$

Let

$$\rho = \frac{(-e_{n-1} - c_n e_n - \frac{1}{4}\tau e_n(1 + \gamma_n^T(\mathbf{x})\gamma_n(\mathbf{x})) - \hat{W}_n^T\phi_n(Z_n))g^T}{g_0 + g^T\hat{\theta}} \tag{3.26}$$

and choose

$$\dot{\hat{\theta}} = \rho e_n \tag{3.27}$$

then substituting (3.25) into (3.24) yields

$$\dot{V}_{zn} \le -\sum_{k=1}^{n-1} c_{k0}e_k^2 + \frac{n-1}{\tau}\|\theta\|^2 + \sum_{k=1}^{n-1}\left(\frac{1}{4c_{k1}}\right)\varepsilon_k^{*2} - \sum_{k=1}^{n-1}\frac{\sigma_k\left\|\tilde{W}_k\right\|^2}{2} + \sum_{k=1}^{n-1}\frac{\sigma_k\|W_k^*\|^2}{2} - c_n e_n^2$$
$$+e_n(-\frac{1}{4}\tau e_n(1 + \gamma_n^T(\mathbf{x})\gamma_n(\mathbf{x})) + \gamma_n^T(\mathbf{x})\theta + g^T(\mathbf{x})(\theta - \hat{\theta})u - \tilde{W}_n^T\phi_n(Z_n) + \varepsilon_n) \tag{3.28}$$

where $\tilde{W}_n = \hat{W}_n - W_n^*$.

Consider the following Lyapunov candidate:

$$V_n = V_{zn} + \frac{1}{2}\tilde{W}_n^T\Gamma_n^{-1}\tilde{W}_n + \frac{1}{2}(\theta - \hat{\theta})^T(\theta - \hat{\theta}) \tag{3.29}$$

where $\Gamma_n = \Gamma_n^{-1} > 0$.

Considering (3.28), the derivative of V_n is

$$\begin{aligned}\dot{V}_n \;\le\; & -c_n e_n^2 - \sum_{k=1}^{n-1} c_{k0}e_k^2 + \frac{n-1}{\tau}\|\theta\|^2 + \sum_{k=1}^{n-1}\left(\frac{1}{4c_{k1}}\right)\varepsilon_k^{*2} - \sum_{k=1}^{n-1}\frac{\sigma_k\left\|\tilde{W}_k\right\|^2}{2}\\ + & \sum_{k=1}^{n-1}\frac{\sigma_k\|W_k^*\|^2}{2} + e_n(-\frac{1}{4}\tau e_n(1 + \gamma_n^T(\mathbf{x})\gamma_n(\mathbf{x})) + \gamma_n^T(\mathbf{x})\theta + g^T(\mathbf{x})(\theta - \hat{\theta})u\\ - & \tilde{W}_n^T\phi_n(Z_n) + \varepsilon_n) + \tilde{W}_n^T\Gamma_n^{-1}\dot{\hat{W}}_n - (\theta - \hat{\theta})^T\dot{\hat{\theta}}\end{aligned} \tag{3.30}$$

Choose the following adaptive law

$$\dot{\tilde{W}}_n = \dot{\hat{W}}_n = \Gamma_n[\phi_n(Z_n)e_n - \sigma_n\hat{W}_n] \tag{3.31}$$

where $\sigma_n > 0$.Then (3.30) becomes

$$\dot{V}_n \le -c_n e_n^2 - \sum_{k=1}^{n-1} c_{k0}e_k^2 + \frac{n-1}{\tau}\|\theta\|^2 + \sum_{k=1}^{n-1}\left(\frac{1}{4c_{k1}}\right)\varepsilon_k^{*2} - \sum_{k=1}^{n-1}\frac{\sigma_k\left\|\tilde{W}_k\right\|^2}{2} + \sum_{k=1}^{n-1}\frac{\sigma_k\|W_k^*\|^2}{2}$$
$$+e_n(-\frac{1}{4}\tau e_n(1 + \gamma_n^T(\mathbf{x})\gamma_n(\mathbf{x})) + \gamma_n^T(\mathbf{x})\theta + g^T(\mathbf{x})(\theta - \hat{\theta})u + \varepsilon_n) - \sigma_n\tilde{W}_n^T\hat{W}_n - (\theta - \hat{\theta})^T\dot{\hat{\theta}} \tag{3.32}$$

Let $c_n = c_{n0} + c_{n1}$, with $c_{n0} > 0, c_{n1} > 0$. Similarly by completion of squares, the derivative of V_n becomes

$$\dot{V}_n \le -\sum_{k=1}^{n} c_{k0}e_k^2 + \frac{n}{\tau}\|\theta\|^2 + \sum_{k=1}^{n}\left(\frac{1}{4c_{k1}}\right)\varepsilon_k^{*2} - \sum_{k=1}^{n}\frac{\sigma_k\left\|\tilde{W}_k\right\|^2}{2} + \sum_{k=1}^{n}\frac{\sigma_k\|W_k^*\|^2}{2} + e_n g^T(\mathbf{x})(\theta - \hat{\theta})u - (\theta - \hat{\theta})^T\dot{\hat{\theta}} \tag{3.33}$$

Substituting (3.25) and (3.26) into (3.33) yields

$$\dot{V}_n \leq -\sum_{k=1}^{n} c_{k0}e_k^2+\frac{n}{\tau}\left\|\theta\right\|^2+\sum_{k=1}^{n}\left(\frac{1}{4c_{k1}}\right)\varepsilon_k^{*2}-\sum_{k=1}^{n}\frac{\sigma_k\left\|\tilde{W}_k\right\|^2}{2}+\sum_{k=1}^{n}\frac{\sigma_k\left\|W_k^*\right\|^2}{2}+e_n\rho^T(\theta-\hat{\theta})-(\theta-\hat{\theta})^T\dot{\hat{\theta}} \tag{3.34}$$

Substituting (3.27) into (3.34) yields

$$\dot{V}_n \leq -\sum_{k=1}^{n} c_{k0}e_k^2+\frac{n}{\tau}\left\|\theta\right\|^2+\sum_{k=1}^{n}\left(\frac{1}{4c_{k1}}\right)\varepsilon_k^{*2}-\sum_{k=1}^{n}\frac{\sigma_k\left\|\tilde{W}_k\right\|^2}{2}+\sum_{k=1}^{n}\frac{\sigma_k\left\|W_k^*\right\|^2}{2} \tag{3.35}$$

The above design procedure and the property of the adaptive controller can be summarized in the following Theorem.

Theorem: Considering the closed-loop system consisting of the uncertain nonlinear system (2.1), the controller can be designed as (3.25), and the RBF NN weight updating laws can be chosen as (3.8), (3.20), and (3.31), the unknown parameter adaptive law can be chosen as (3.27), suppose that

$$\gamma^2=\frac{n}{\tau}, \quad N=V_n(0)>0 \tag{3.36}$$

and when

$$\sum_{k=1}^{n}\left(\frac{1}{4c_{k1}}\right)\varepsilon_k^{*2}+\sum_{k=1}^{n}\frac{\sigma_k\left\|W_k^*\right\|^2}{2}<\sum_{k=1}^{n}\frac{\sigma_k\left\|\tilde{W}_k\right\|^2}{2} \tag{3.37}$$

then the nonlinear system (2.1) has L_2-gain performance index which less than or equal toγ. Namely

$$\int_0^T \left\|y(t)\right\|^2 dt \leq \gamma^2\int_0^T \left\|\theta\right\|^2 dt+N$$

Proof: Considering Eq.(3.37), Eq.(3.35) can be written as

$$\dot{V}_n < -\sum_{k=1}^{n} c_{k0}e_k^2+\frac{n}{\tau}\left\|\theta\right\|^2 \tag{3.38}$$

Let $c_{10}\geq 1$ $(1\leq i\leq n)$, Eq.(3.38) becomes

$$\dot{V}_n \leq -e_1^2+\frac{n}{\tau}\left\|\theta\right\|^2 \tag{3.39}$$

Considering (3.36), $y=e_1$ and $V(T)>0$, Taking integral on equation (3.39), we yield

$$\int_0^T \left\|y(t)\right\|^2 dt \leq \gamma^2\int_0^T \left\|\theta\right\|^2 dt+N$$

Thus the nonlinear system (2.1) has L_2-gain performance index which less than or equal to γ under controller (3.25).

Apparently, choose appropriate design parameter $c_k(c_{k0},c_{k1}),\sigma_k,\tau,(1\leq k\leq n)$ can make $\dot{V}_n<0$ from (3.37) and (3.38) due to $\theta\in L_2(0,\infty)$. So the closed loop uncertain system is uniformly ultimately bounded.

4 Conclusion

This paper apply RBF neural network to approximate system uncertainty and the output of the neural network is used to design robust L_2-gain controller for a class of uncertain nonlinear systems. Under the given parameter's update law, the approximate error of the compound uncertainty is bounded. The proposed robust adaptive controller can make the uncertain closed loop system has L_2 performance index.

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