ISSN 1479-3889 (print), 1479-3897 (online)
International Journal of Nonlinear Science
Vol.3(2007) No.2,pp.103-110

The Stability Analysis and Control of Nonminimum Phase Nonlinear Systems

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(Received 11 January 2007, accepted 6 March 2007)

Abstract: This paper studies nonlinear lower triangular systems with uncertainties, which zero dynamics may not be stable. We transform the systems into an canonical form which is minimum phase with respect to an new coordinate output, Through a special observer design and back stepping method, we can obtain the control input which can make the systems asymptoticly stable.

Key words:nonlinear system, asymptotic stability ,relative degree, minimum phase, zero-dynamics

1 Introduction

In last decades, state or output feedback of nonlinear systems have attracted many scholars' attention, many constructive design methods for state or output feedback controllers yielding asymptotic stability have been obtained [1]-[6],[14]-[17]. Generally, robustness of the systems having structured uncertainties is dealt with by seeking a feedback law that imposes some positive definite function to become a Lyapunov function for the closed-loop system.

Especially for lower triangular structure systems, we yield very powerful recursive design methods [7], and we also get the conclusion that only if a system is minimum phase is it possible to achieve an arbitrarily small level of attenuation between disturbance and output (see D.Karagiannis[8] and Shoulie Xie[10]). The common method in tackling with the stability of lower triangular systems with uncertain function is that the zero dynamics of the considered systems possess some stability property, ie. They are the input-to-state stable (ISS) or input-to-state-practical-stable (ISPS) (see sontag[10]), Is idori,[11], but D.Karaginnis [8] have obtained the global out-feedback stabilization for this lower triangular systems with uncertain functions, Whose zero dynamics are not necessarily stable. L.Parly[12] proves that a linear observer-based output feedback can globally regulate an equilibrium of nonlinear systems.

This paper studies nonlinear lower triangular systems with uncertainties, which zero dynamics may not be stable. Enlightened by Riccardo Marino [13], we transform the systems into an canonical form which is minimum phase with respect to an new coordinate output, Through a special observer design and backstepping method, we can obtain the control input which can make the systems global asymptotic stability.

2 System description and problem formulation

We consider a family of uncertain nonlinear systems of the form

$$\begin{cases} \dot{x}\left(t\right) = A_c x\left(t\right) + bu + F\left(y\left(t\right)\right) + \Phi\left(\bar{x}\left(t\right), \bar{x}\left(t-\tau\right), y\left(t\right), y\left(t-\tau\right)\right) \\ y = C_C x\left(t\right) \end{cases} \tag{1}$$

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IJNS.2007.04.15/067

where $F(x) = (F_1(y(t)), F_2(y(t)), \cdots, F_n(y(t)))\bar{x}_i(t) = (x_1(t), \cdots, x_i(t)), i = 1, 2, \cdots, n$

$$x(t) = (x_1(t), \cdots, x_n(t))$$

$$\Phi(x) = \begin{pmatrix} \Phi_1(\bar{x}_1(t), \bar{x}_1(t-\tau), y(t), y(t-\tau)) \\ \Phi_2(\bar{x}_2(t), \bar{x}_2(t-\tau), y(t), y(t-\tau)) \\ \vdots \Phi_n(\bar{x}_n(t), \bar{x}_n(t-\tau), y(t), y(t-\tau)) \end{pmatrix}, A_c = \begin{pmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ & & & \ddots & \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{pmatrix}$$

$$b = (0, \cdots, b_\rho, \cdots, b_n), C_C = (1, 0, \cdots, 0), b_\rho \neq 0$$

Suppose the relative degree of system (1) is ρ, Consider a new output $\mu = t_1 x = (t_{11}, \cdots, t_{1n})\, x$ such that the relative degree of system (1) with respect to μ is still equal to ρ. Though the similar coordinate change see Riccardo Marino [11], we perform the linear change of coordinates

$$\begin{pmatrix} \mu \\ \xi \end{pmatrix} = \begin{pmatrix} t_1 & t_2 & \cdots & t_n \end{pmatrix}^T x = \begin{pmatrix} t_1 & t_1 A_c & \cdots & t_1 A_c^{\rho-1} \; t & t_{\rho+1} & \cdots & t_n \end{pmatrix}^T x \triangleq Tx$$

with $t_1 b = 0, \cdots, t_1 A_c^{\rho-2} b = 0, t_1 A_c^{\rho-1} b \neq 0, t_{\rho+i} b = 0 1 \leq i \leq n-\rho$
and $t_1 A_c T^{-1} = (-k_1, 1, 0, \cdots, 0), t_1 A_c^2 T^{-1} = (-k_2, 0, 1, \cdots, 0), t_1 A_c^{\rho-1} T^{-1} = (-k_\rho, 0, \cdots, 0, c, 0 \cdots, 0)$, $t_{\rho+1}\bar{\Phi}(\bar{x}(t), \bar{x}(t-\tau), y(t), y(t-\tau)) = 0, \cdots, t_n \bar{\Phi}(\bar{x}(t), \bar{x}(t-\tau), y(t), y(t-\tau)) = 0$, with c being the $(\rho+1)\, th$ component of $t_1 A_c^\rho T^{-1}$. Such that in the new coordinates system (1) becomes

$$\begin{cases} \dot{\xi}_a = A_1 \xi_a + f_a(y(t)) + \bar{\Phi}_\rho^1(\bar{x}(t), \bar{x}(t-\tau), y(t), y(t-\tau)) + \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{pmatrix} (b_\rho u + c\xi_\rho) \\ \dot{\xi}_b = F\xi_b + d\mu + f_b(y(t)) \end{cases} \tag{2}$$

where $\xi_a = (\mu, \xi_1, \cdots, \xi_{\rho-1}), \xi_b = (\xi_\rho, \cdots, \xi_n)$

$$f_a(y(t)) = (f_1(y(t)), \cdots, f_\rho(y(t))), f_b(y(t)) = (f_{\rho+1}(y(t)), \cdots, f_n(y(t)))$$

$$A_1 = \begin{pmatrix} -k_1 & 1 & 0 & \cdots & 0 \\ -k_2 & 0 & 1 & \cdots & 0 \\ \vdots & & & \ddots & \\ -k_{\rho-1} & 0 & 0 & \cdots & 1 \\ -k_\rho & 0 & 0 & \cdots & 0 \end{pmatrix}, \quad \overline{\Phi}_\rho^1(x) = \begin{pmatrix} \overline{\Phi}_1(\bar{x}_1(t), \bar{x}_1(t-z), y(t), y(t-\tau)) \\ \overline{\Phi}_2(\bar{x}_2(t), \bar{x}_2(t-z), y(t), y(t-\tau)) \\ \vdots \\ \overline{\Phi}_\rho(\bar{x}_\rho(t), \bar{x}_\rho(t-z), y(t), y(t-\tau)) \end{pmatrix}.$$

Then $y = C_C T^{-1} \begin{pmatrix} \mu \\ \xi \end{pmatrix} = \alpha\mu + \theta_1^T \xi_a + \theta_2^T \xi_b$.

Through choosing appropriate transformation T such that $\theta_1 = 0$, then

$$y = \alpha\mu + \theta_2^T \xi_b. \tag{3}$$

The zero-dynamics of (2), viewing μ as output, are given by

$$\dot{\xi}_b = F\xi_b + f_b\left(\theta_2^T \xi_b\right). \tag{4}$$

Assumption 1 The zero-dynamics (4) is global asymptotic stability. With respect to μ, and there is a Lyapunov function $V(\eta)$ satisfying:

$$h_1 \|\eta\|^2 \le V(\eta) \le h_2 \|\eta\|^2 ,$$

$$\frac{\partial V(\eta)}{\partial \eta}\left(F\xi_b + f_b\left(\theta_2^T \xi_b\right)\right) \le -h_3 \|\eta\|^2 .$$

From (4), we distinguish two case 1).if $\theta_2 = 0$,.then the global asymptotic stability of zero-dynamics(4) with respect to μ is the same as that with respect to y. 2) if $\theta_2 \neq 0$, there exists a linear transformation of coordinates

$$\begin{cases} \mu = \mu, \\ z_i = \xi_i \qquad , 1 \le i \le \rho - 1, \\ z_i = \xi_i + \frac{\alpha\theta_{2i}}{\|\theta_2\|^2}\mu \qquad , \rho \le i \le n-1 \end{cases} \tag{5}$$

where $\theta_2 = (\theta_{2\rho}, \theta_{2\rho+1}, \cdots, \theta_{2n-1})$, which transforms systems (2) into the following form

$$\begin{cases} \dot{Z}_a = A_1 Z_a + f_a(y(t)) + \bar{\Phi}_\rho^1(\overline{x}(t), \overline{x}(t-\tau), y(t), y(t-\tau)) + \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{pmatrix}(b_\rho u + c z_\rho) - \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{pmatrix}\frac{\alpha\theta_{2\rho}}{\|\theta_2\|^2}\mu \\ \dot{Z}_b = F Z_b + d\mu + f_b(y(t)) + \frac{\alpha\theta_2}{\|\theta_2\|^2}(-k_1\mu + z_1 + f_1(y(t)) + \bar{\Phi}_1(\overline{x_1}(t), \overline{x_1}(t-\tau), y(t), y(t-\tau)) \\ -\alpha F \frac{\theta_2}{\|\theta_2\|^2}\mu \end{cases} \tag{6}$$

Correspondence $y = \theta_2^T Z_b$

Where $Z_a = (\mu, z_1, z_2, \cdots, z_{\rho-1})$, $Z_b = (z_\rho, z_{\rho+1}, \cdots z_{n-1})$,

In the new coordinates, (4) becomes the following form

$$\dot{Z}_b = F Z_b + f\left(\theta_2^T Z_b\right) \tag{7}$$

From assumption 1, (7) is global asymptotic stability and there exists positive definite function $V(Z_b)$, such that

$$h_1 \|Z_b\|^2 \le V(Z_b) \le h_2 \|Z_b\|^2$$

$$\frac{\partial V}{\partial Z_b}\left(F \cdot Z_b + f_b(\theta_2^T Z_b)\right) \le -h_3 \|Z_b\|^2 \tag{8}$$

3 Observer design

Relative to systems (6), we design the following observer

$$\begin{cases} \dot{\hat{x}} = A_c \hat{x}(t) + bu + F(y(t)) + (l_1^1 e^r, l_2^1 e^{2r}, \cdots, l_n^1 e^{nr})^T (y - \hat{x}_1) \\ \dot{\hat{Z}}_{a_1} = A_{11}\hat{Z}_a + f_{a_1}(y(t)) + \left(l_1^2 e^r, l_2^2 e^{2r}, \cdots, l_{\rho-1}^2 e^{(\rho-1)r}\right)^T \cdot (t_1 x - t_1 \hat{x}) \\ \hat{\mu} = t_1 \hat{x} \\ \dot{\hat{z}}_{\rho-1} = b_\rho u - k_\rho t_1 \hat{x} + f_\rho(y(t)) + c\hat{z}_\rho - \frac{\alpha\theta_{2\rho}}{\|\theta_2\|^2}\hat{\mu} + l_\rho^2 e^{\rho r}(t_1 x - t_1 \hat{x}) \end{cases} \tag{9}$$

Where, $\hat{Z}_{a_1} = (\hat{\mu}, \hat{z}_1, \hat{z}_2, \cdots, \hat{z}_{\rho-2})$, $f_{a_1}(y(t)) = (f_1(y(t)), f_2(y(t)), \cdots, f_{\rho-1}(y(t)))$
$b_1 = (0, \cdots, 0, b_0, 0, \cdots, 0)$, b_0 being the ρ th component of b_1

$$A_{11} = \begin{pmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & & & \ddots & \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{pmatrix}$$

From (1), (6) and (9), we have

$$\begin{cases} \dot{e}_i(t) = e_{i+1}(t) - l_i^1 e^{ir} e_1 + \Phi_i\left(\bar{x}_i(t), \bar{x}_i(t-\tau), y(t), y(t-\tau)\right) & 1 \le i \le n \\ \dot{\hat{Z}}_{a_1}(t) = A_{11}\hat{Z}_a(t) + f_{a_1}(y(t)) + \left(l_1^2 e^r, l_2^2 e^{2r}, \cdots, l_{\rho-1}^2 e^{(\rho-1)r}\right)^T (tx - t\hat{x}) \\ \dot{\hat{z}}_{\rho-1}(t) = b_\rho u - k_\rho t_1 \hat{x} + f_\rho(y(t)) + c\hat{z}_\rho - \frac{\alpha\theta_{2\rho}}{\|\theta_2\|^2}\mu + l_\rho^2 e^{\rho r}(tx - t\hat{x}) \end{cases} \tag{10}$$

Where $E(t) = x(t) - \hat{x}(t) = (e_1(t), e_2(t), \cdots, e_n(t))$, $\dot{r}(t) = -be^{r(t)} + \sigma\left(y, e^{r(t)}\right)$ and $r(t) \ge 0$ let $\varepsilon_i(t) = \frac{e_i(t)}{e^{ir}}$, then

$$\dot{\varepsilon}_i(t) = \left[-l_i^1 \varepsilon_1(t) + ib\varepsilon_i(t) + \varepsilon_{i+1}(t)\right] e^r - i\varepsilon_i \sigma(y \cdot e^r) + \frac{1}{e^{ir}} \Phi_i\left(\bar{x}_i(t), \bar{x}_i(t-\tau), y(t), y(t-\tau)\right)$$

$$1 \le i \le n$$

The matrix form is

$$\dot{\varepsilon} = A_2 \varepsilon \cdot e^r + G(t) \tag{11}$$

$$A_2 = \begin{pmatrix} -l_1^1 + b & 1 & 0 & \cdots & 0 \\ -l_2^1 & 2b & 1 & \cdots & 0 \\ \vdots & & \ddots & & \\ -l_{(n-1)}^1 & & & (n-1)b & 1 \\ -l_n^1 & & & & nb \end{pmatrix},$$

$$G(t) = -\begin{pmatrix} \varepsilon_1 \sigma(y, e^r) \\ 2\varepsilon_2 \sigma(y, e^r) \\ \vdots \\ n\varepsilon_n \sigma(y, e^r) \end{pmatrix} + \begin{pmatrix} \frac{1}{e^r}\Phi_1\left(x_1(t), x_1(t-\tau), y(t), y(t-\tau)\right) \\ \frac{1}{e^{2r}}\Phi_2\left(\bar{x}_2(t), \bar{x}_2(t-\tau), y(t), y(t-\tau)\right) \\ \vdots \\ \frac{1}{e^{nr}}\Phi_n\left(\bar{x}_n(t), \bar{x}_n(t-\tau), y(t), y(t-\tau)\right) \end{pmatrix} \tag{12}$$

Let $\hat{v}_i = \frac{\hat{z}_{i-1}}{e^{(i-1)r}} \quad 1 \le i \le \rho$ then

$$\begin{cases} \dot{\hat{v}}_i(t) = -k_i \frac{1}{e^{(i-1)r}} \hat{v}_1 + e^r \hat{v}_{i+1} - (i-1)\hat{v}_i \dot{r} + \frac{1}{e^{(i-1)r}} f_i(y(t)) + l_i^2 e^r t_1 E(t), & 1 \le i \le \rho \\ \dot{\hat{v}}_\rho(t) = \frac{1}{e^{(\rho-1)r}}\left(b_\rho u - k_\rho t_1 \hat{x} + f_\rho(y(t)) + c\hat{z}_\rho - \frac{\alpha\theta_{2\rho}}{\|\theta_2\|^2}\mu\right) - (\rho-1)\dot{r}\hat{v}_\rho + l_\rho^2 e^r t_1 E(t) \end{cases} \tag{13}$$

$$1 \le i \le \rho - 1$$

Let $\tilde{v}_i(t) = \hat{v}_i(t) - v_i^*\left(\bar{\hat{x}}_{i-1}(t)\right)$, where $\bar{\hat{x}}_{i-1}(t) = (\hat{x}_1(t), \hat{x}_2(t), \cdots, \hat{x}_{i-1}(t))$

Then $\dot{\tilde{v}}_i(t) = \dot{\hat{v}}_i(t) - \sum\limits_{j=1}^{i-1} \frac{\partial v_i^*}{\partial \hat{x}_j(t)} \cdot \dot{\hat{x}}_j(t)$

Let $V_1 = \varepsilon^T P \varepsilon$, $V_2 = \sum\limits_{i=1}^{\rho} \tilde{v}_i^2$,$V_3 = \lambda_{\max}(Q)\int_{t-\tau}^t \varphi_2(\bar{x}_{\rho-1}(u))\,du + \lambda_{\max}(Q)\int_{t-\tau}^t \varphi_4(y(u))\,du$ and

$$V = \gamma V(Z_b) + \delta V_1 + \delta^2 V_2 + \delta V_3 \tag{14}$$

Then

$$\dot{V}_1 = 2\varepsilon^T P \dot{\varepsilon} = 2\varepsilon^T P \left(A_2 \varepsilon e^r + G(t) \right) = e^r \varepsilon^T \left(P A_2 + A_2^T P \right) \varepsilon + 2\varepsilon^T P G(t) \tag{15}$$

Assumption 2 There exists positive real number α_{ij} $(i, j = 1, 2, \cdots, n)$ such that the following inequalities hold

$$\left| F_i(\bar{x}_i(t)) - F_i\left(\bar{\hat{x}}_i(t)\right) \right| \le \sum_{j=1}^{i} \alpha_{ij} \left| x_j(t) - \hat{x}_j(t) \right|, \quad i = 1, 2, \cdots, n \tag{16}$$

Assumption 3 There exist K_∞ functions $\varphi_1(\cdot)$,$\varphi_2(\cdot)$, $\varphi_3(\cdot)$ and $\varphi_4(\cdot)$ such that the following inequality holds

$$\|\Phi(t)\|^2 \le \left(\varphi_1(\bar{x}_{\rho-1}(t)) + \varphi_2(\bar{x}_{\rho-1}(t-\tau)) + \varphi_3(y(t)) + \varphi_4(y(t-\tau)) \right) \tag{17}$$

From (12) the last term of (15) is

$$\begin{array}{l} 2\varepsilon^T P G(t) \le \varepsilon^T P Q^{-1} P \varepsilon + \\ + \left(\frac{1}{e^r} \Phi_1(x_1(t), x_1(t-\tau), y(t), y(t-\tau)), \frac{1}{e^{2r}} \Phi_2(\bar{x}_2(t), \bar{x}_2(t-\tau), y(t), y(t-\tau)), \cdots, \right. \\ \left. \frac{1}{e^{nr}} \Phi_n(\bar{x}_n(t), \bar{x}_n(t-\tau), y(t), y(t-\tau)) \right) \cdot \\ Q \left(\begin{array}{l} \frac{1}{e^r} \Phi_1(x_1(t), x_1(t-\tau), y(t), y(t-\tau)) \\ \frac{1}{e^{2r}} \Phi_2(\bar{x}_2(t), \bar{x}_2(t-\tau), y(t), y(t-\tau)) \\ \vdots \\ \frac{1}{e^{nr}} \Phi_n(\bar{x}_n(t), \bar{x}_n(t-\tau), y(t), y(t-\tau)) \end{array} \right) - 2\varepsilon^T P \left(\begin{array}{l} \varepsilon_1 \sigma(y, e^r) \\ 2\varepsilon_2 \sigma(y, e^r) \\ \vdots \\ n\varepsilon_n \sigma(y, e^r) \end{array} \right) \end{array} \tag{18}$$

From (18), (15) and (17) ,we can obtain the following inequalities

$$2\varepsilon^T P G(t) \le \|P\|_2^2 \lambda_{\max}\left(Q^{-1}\right) \varepsilon^T \varepsilon + \lambda_{\max}(Q) \frac{1}{e^{2r}} \left(\varphi_1(\bar{x}_{\rho-1}(t)) + \varphi_2(\bar{x}_{\rho-1}(t-\tau)) \right.$$

$$\left. + \varphi_3(y(t)) + \varphi_4(y(t-\tau)) \right) + \lambda_{\max}(P) \varepsilon^T \varepsilon + n^2 \lambda_{\max}(P) \sigma^2(y, e^r) \varepsilon^T \varepsilon \tag{19}$$

Assumption 4 There exist positive definite matrix P and positive real number β, such that the following inequality holds

$$P A_2 + A_2^T P \le -\beta I \tag{20}$$

From (15), (19) and (20), we have

$$\begin{array}{l} \dot{V}_1 \le \left[-\beta e^r + \|P\|_2^2 \lambda_{\max}\left(Q^{-1}\right) + \lambda_{\max}(P) + n^2 \lambda_{\max}(P) \sigma^2(y, e^r) \right] \varepsilon^T \varepsilon \\ + \lambda_{\max}(Q) \frac{1}{e^{2r}} \left(\varphi_1(\bar{x}_{\rho-1}(t)) + \varphi_2(\bar{x}_{\rho-1}(t-\tau)) + \varphi_3(y(t)) + \varphi_4(y(t-\tau)) \right) \end{array} \tag{21}$$

From (14) let's consider

$$\left(\dot{\tilde{v}}_i\right)^2 = 2\tilde{v}_i \cdot \dot{\tilde{v}}_i = 2\tilde{v}_i \left(\dot{\hat{v}}_i - \sum_{j=1}^{i-1} \frac{\partial v_i^*}{\partial \hat{x}_j(t)} \dot{\hat{x}}_j(t) \right)$$

$$= 2\tilde{v}_i \cdot \left[\begin{array}{l} -k_i \frac{1}{e^{(i-1)r}} \hat{v}_1 + e^r \tilde{v}_{i+1} + e^r v_{i+1}^* - (i-1) \tilde{v}_i \dot{r} - (i-1) v_i^* \dot{r} \\ + \frac{1}{e^{(i-1)r}} f_i(y(t)) + l_i^2 e^r t_1 E(t) - \sum\limits_{j=1}^{i-1} \frac{\partial v_i^*}{\partial \hat{x}_j(t)} \dot{\hat{x}}_j(t) \end{array} \right] \tag{22}$$

We get the virtual controller

$$v_{i+1}^* = -\frac{1}{2} e^{-r} \lambda_i \tilde{v}_i - \tilde{v}_{i-1} + k_i \frac{1}{e^{ir}} \hat{v}_1 + \frac{1}{e^r} \left((i-1) \tilde{v}_i \dot{r} - (i-1) v_i^* \dot{r} \right)$$

$$-\frac{1}{e^{ir}}f_i(y(t)) - l_i^2 t_1 E(t) + \frac{1}{e^r}\sum_{j=1}^{i-1}\frac{\partial v_i^*}{\partial \hat{x}_j(t)}\dot{\hat{x}}_j(t), \quad 1 \le i \le \rho - 1 \tag{23}$$

Also, we have

$$\begin{aligned} \left(\dot{\tilde{v}}_\rho\right)^2 &= 2\tilde{v}_\rho \cdot \dot{\tilde{v}}_\rho = 2\tilde{v}_\rho \cdot \left[\dot{\hat{v}}_\rho(t) - \dot{v}_\rho^*\right] \\ &= 2\tilde{v}_\rho \cdot \left[\begin{array}{l} -(\rho-1)\tilde{v}_\rho \dot{r} - (\rho-1)v_\rho^*\dot{r} + l_\rho^2 e^r t_1 E(t) - \sum_{j=1}^{\rho-1}\frac{\partial v_\rho^*}{\partial \hat{x}_j(t)}\dot{\hat{x}}_j(t) \\ + \frac{1}{e^{(\rho-1)r}}\left[b_\rho u - k_\rho t_1 \hat{x} + f_\rho(y(t)) + c\hat{z}_\rho - \frac{\alpha\theta_{2\rho}}{\|\theta_2\|^2}\mu\right] \end{array}\right] \end{aligned} \tag{24}$$

We get the controller

$$\begin{aligned} u = &-\frac{e^{\rho r}}{b_\rho}\tilde{v}_{\rho-1} + \frac{k_\rho t_1 \hat{x}}{b_\rho} - \frac{1}{b_\rho}(f_\rho(y(t)) + c\hat{z}_\rho - \frac{\alpha\theta_{2\rho}}{\|\theta_2\|^2}\mu) \\ &+ \frac{e^{(\rho-1)r}}{b_\rho}\left[-\frac{1}{2}\lambda_\rho\tilde{v}_\rho + (\rho-1)\tilde{v}_\rho\dot{r} + (\rho-1)v_\rho^*\dot{r} - l_\rho^2 e^r t_1 E(t) + \sum_{j=1}^{\rho-1}\frac{\partial v_\rho^*}{\partial \hat{x}_j(t)}\dot{\hat{x}}_j(t)\right] \end{aligned} \tag{25}$$

From (22)-(25), we have $\dot{V}_2 \le -\sum_{i=1}^{\rho}\lambda_i\tilde{v}_i^2$, Where $\lambda_i > 0 \quad i = 1, 2, \cdots, \rho$

From (14), we have

$$\dot{V}_3 = \lambda_{\max}(Q)\left(\varphi_2(\bar{x}_{\rho-1}(t)) - \varphi_2(\bar{x}_{\rho-1}(t-\tau)) + \varphi_4(y(t)) - \varphi_4(y(t-\tau))\right) \tag{26}$$

From (6), we have

$$\begin{aligned} &\frac{\partial V(Z_b)}{\partial Z_b}\cdot\dot{Z}_b \\ =&\frac{\partial V(Z_b)}{\partial Z_b} \\ &\left[FZ_b + d\mu + f_b(y(t)) + \frac{\alpha\theta_2}{\|\theta_2\|^2}(-k_1\mu + z_1 + f_1(y(t)) + \bar{\Phi}_1(\overline{x_1}(t), \overline{x_1}(t-\tau), y(t), y(t-\tau)) - \alpha F\frac{\theta_2}{\|\theta_2\|^2}\mu\right] \\ &\le -h_3\|Z_b\|^2 + \frac{\partial V(Z_b)}{\partial Z_b}[d\mu + \frac{\alpha\theta_2}{\|\theta_2\|^2}(-k_1\mu + z_1 + f_1(y(t)) + \bar{\Phi}_1(\overline{x_1}(t), \overline{x_1}(t-\tau), y(t), y(t-\tau)) \\ &-\alpha F\frac{\theta_2}{\|\theta_2\|^2}\mu] \end{aligned} \tag{27}$$

We have $\dot{V} = \gamma\frac{\partial V(Z_b)}{\partial Z_b}\cdot\dot{Z}_b + \delta\dot{V}_1 + \delta^2\dot{V}_2 + \delta\dot{V}_3$

$$\begin{aligned} &\le -\gamma h_3\|Z_b\|^2 + \delta\left[\begin{array}{l} -\beta\cdot e^r + \|P\|_2^2\cdot\lambda_{\max}\left(Q^{-1}\right) \\ +\lambda_{\max}(P) + n^2\lambda_{\max}(P)\sigma^2(y, e^r) \end{array}\right]\varepsilon^T\varepsilon - \delta^2\sum_{i=1}^{\rho}\lambda_i v_i^2 \\ &+\delta\lambda_{\max}(Q)\left(\bar{\varphi}_1(\bar{x}_{\rho-1}(t)) + \bar{\varphi}_3(y(t))\right) + \gamma\frac{\partial V(Z_b)}{\partial Z_b} \\ &[d\mu + \frac{\alpha\theta_2}{\|\theta_2\|^2}(-k_1\mu + z_1 + f_1(y(t)) + \bar{\Phi}_1(\overline{x_1}(t), \overline{x_1}(t-\tau), y(t), y(t-\tau)) - \alpha F\frac{\theta_2}{\|\theta_2\|^2}\mu] \end{aligned} \tag{28}$$

where $\bar{\varphi}_1(\bar{x}_{\rho-1}(t)) = \varphi_1(\bar{x}_{\rho-1}(t)) + \varphi_2(\bar{x}_{\rho-1}(t))$, $\bar{\varphi}_3(\bar{x}_{\rho-1}(t)) = \varphi_3(y(t)) + \varphi_4(y(t))$

Let $\frac{\beta}{2} - 2\|P\|_2^2\lambda_{\max}\left(Q^{-1}\right) - {\alpha^*}^2 n^2\lambda_{\max}(Q) - \lambda_{\max}(P) > 0$

Since $L(r(t)) = \frac{\beta}{2}e^r - 2\|P\|_2^2\lambda_{\max}\left(Q^{-1}\right) - {\alpha^*}^2 n^2\lambda_{\max}(Q) - \lambda_{\max}(P)$ is a continuous function, there exists $t_0 > 0$, when $t \in [t_0, +\infty)$, $L(r(t)) > 0$

Let

$$\sigma^2\left(y, e^r\right) \leq \frac{e^{2r}}{n-1}\left[\frac{\beta}{2}e^r - 2\left\|P\right\|_2^2\lambda_{\max}\left(Q^{-1}\right) - {\alpha^*}^2 n^2\lambda_{\max}\left(Q\right) - \lambda_{\max}\left(P\right)\right] \tag{29}$$

From (28), (29), we have

$$\dot{V} \leq -h_3\left\|Z_b\right\|^2 - \frac{\beta}{2}e^r\varepsilon^T\varepsilon - \sum_{i=1}^{\rho}\lambda_i V_i^2 \quad , \quad t \in [t_0, +\infty) \tag{30}$$

From (30), we get the conclusion that system (1) is global asymptotic stability.

4 Example

Consider the following nonminimum phase systems in the form (1) with $n = 2$ and $\rho = 1$

$$\begin{cases} \dot{x}_1 = x_2 + b_1 u + x_1\left(t\right) + \left(\sin t\right)\cdot x_2\left(t\right) + \Phi_1\left(x_1\left(t\right), x_1\left(t-\tau\right)\right) \\ \dot{x}_2 = b_2 u + x_1\left(t\right) + 5x_2\left(t\right) + \left(\sin 3t\right)x_2\left(t\right) + \Phi_2\left(\bar{x}_2\left(t\right), \bar{x}_2\left(t-\tau\right), y\left(t\right), y\left(t-\tau\right)\right) \\ y = x_1 \qquad\qquad b_1 \neq 0, b_2 > 0 \end{cases} \tag{31}$$

in which $\phi_i\left(0\right) = 0$,$i = 1, 2$. The zero-dynamics of (31) are given by $\dot{\eta} = b_2\eta$, which makes the following linear change of coordinates

$$\begin{cases} \mu = t_1 x_1 + t_2 x_2 \\ \eta = -b_2 x_1 + b_1 x_2 \end{cases},$$

let $t_1 b_1 + t_2 b_2 \neq 0$,and system(31) has the relative degree with respect to μ. The zero-dynamics of system (31) viewing μ as an output are given by

$$\begin{array}{l} \dot{\eta} = \frac{-b_2+\eta}{b_2t_2+t_1b_1} + b_1\left[x_1\left(t\right) + \left(\sin t\right)x_2\left(t\right) + \Phi_1\left(x_1\left(t\right), x_1\left(t-\tau\right)\right)\right] \\ \quad - b_2\left[x_1\left(t\right) + 5x_2\left(t\right) + \left(\sin 3t\right)x_2\left(t\right) + \Phi_2\left(\bar{x}_2\left(t\right), \bar{x}_2\left(t-\tau\right), y\left(t\right), y\left(t-\tau\right)\right)\right] \end{array}.$$

Let $b_2t_2 + b_1t_1 > 0$, $h_1 = \frac{1}{2}$, $h_2 = 2$, $h_3 = \frac{b_2t_1}{b_2t_2+b_1t_1}$, and Assumption 1 is satisfied
Let $\alpha_{11} = \alpha_{12} = 1$, $\alpha_{21} = 1$, $\alpha_{22} = 6$, and Assumption 2 is satisfied.
Let $\left\|\Phi\left(t\right)\right\|^2 \leq \varphi_3\left(y\left(t\right)\right) + \varphi_4\left(y\left(t-\tau\right)\right)$, and Assumption 3 is satisfied, where
$\Phi\left(t\right) = \left(\Phi_1\left(x_1\left(t\right), x_1\left(t-\tau\right)\right), \Phi\left(\bar{x}_2\left(t\right), \bar{x}_2\left(t-\tau\right), y\left(t\right), y\left(t-\tau\right)\right)\right)^T$.
Let $P_1 = P_2 = 1$, $l_1 = 1$, $l_2 = -1$, $\beta < 2$, and Assumption 4 is satisfied,where

$$P = \begin{pmatrix} P_1 & 0 \\ 0 & P_2 \end{pmatrix}, A_2 = \begin{pmatrix} -1 & 1 \\ 1 & 1 \end{pmatrix}$$

Let $Q = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$,

$$\begin{array}{l} u = -\frac{e^a}{2b_1}\lambda_1\tilde{V}_1 + \frac{k_1t_1}{b_1}\hat{x} - \frac{1}{b_1}\left(\hat{x}_1\left(t\right) + \left(\sin t\right)\hat{x}_2\left(t\right)\right) - \frac{l_1e^rt_1}{b_1}\left(x\left(t\right) - \hat{x}\left(t\right)\right) - e^a\left(\varphi_3\left(y\left(t\right)\right)\right) \\ + \frac{\alpha V(\eta)}{\alpha\eta}\left[F\cdot\frac{\alpha\theta_2}{\left\|\theta_2\right\|}\mu + \frac{\alpha\theta_2}{\left\|\theta_2\right\|^2}\left(g_2\left(\mu\right) + \bar{\Phi}\left(y\left(t\right)\right)\mu - g_1\left(\mu\right)\right)\right] \end{array},$$

where

$$g_1\left(\mu\right) = \frac{t_1b_1^2 - b_1b_2 - 5b_2^2 - b_2^2\sin 3t}{t_1b_1 + t_2b_2}\mu + b_1\sin t\frac{b_2\mu}{t_1b_1 + t_2b_2},$$

$$g_2\left(\mu\right) = \frac{-t_2b_1^2 + t_2b_2 - 5t_1b_2 - t_1b_2^2\sin\left(3t\right)}{t_1b_1 + t_2b_2}\mu + b_1\sin t\frac{t_1\mu}{t_1b_1 + t_2b_2}.$$

Let $\dot{r} = -e^r + \frac{e^r}{2}$, and we have $r\left(t\right) = \ln\frac{2}{t+c}$, then system (31) are global asymptotic stability,where c is arbitrary real number such that $t + c > 0$.

Acknowledgements

This work was supported by the Natural Science Foundation of China(No. 60574036), and the Specialized Research Fund for the Doctoral Program of Higher Education of China (No. 20050055013).

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