

Game Theory Analysis of Price Decision in Real Estate Industry

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Abstract: Suppose the real estate system is composed of land developer and real estate developer. A model of government, land developer and real estate developer is established. Through using game theory the price decision of land and housing is analyzed. The equilibrium solutions about cooperative game and non-cooperative game are also discussed. Efficiencies of each price decisions are further compared when government profit as well as housing prices is taken into consideration. Research result indicates that cooperation is the optimal strategy and regulating the tax rate in a moderate range is an efficiency way for government to increase its profit while decreasing housing prices so as to maintain social stability.

Key words: game, cooperative, price decision, real estate

1 Introduction

In China, the land has been developed by the government in accordance with urban planning, functional positioning and economic development. It includes land compensation, removal and resettlement, land formation, land use planning and design etc. Recently, a joint effort of developing the land has come into existence. This is a useful attempt in the process of land development marketization. In this way, the government reduces the burden and return to the position of management and service, while developers no longer do such work as infrastructure construction that is unrelated to the real estate construction. Though the government select the suitable developers through public bidding, it is difficult for them to evaluate the profit exactly. Moreover whether the land repurchase price is reasonable or not is something the government must consider. The imperfect pricing mechanisms may lead to the loss of state assets.

Price decision in real estate industry includes land and housing price decision. How to make reasonable price decision is an important problem for the government and real estate companies. In recent years, with the development and application of game theory [1], the pricing of products has new solutions. More and more experts and scholars began to explore product pricing strategy from the perspective of game theory. Ercetin Ozgur and Leandros Tassioulas applied duopoly game to investigate the performance of the pricing scheme and showed that under certain conditions the competition results in peering of the surrogates [2]. Zhong Lei-gang, Lin Lin and Ma Qin-hai discussed the repetitive game results of the two partners under different conditions and illustrated the feasibility of the long term cooperation in a supply chain to realize the "two-win" objective [3]. Qin Xue-zhi established signal game model of firm's manager and investors, and analyzed the asset pricing problem. He also gave the equilibrium of firm's market values under three conditions [4]. Levinson David using game theory studied a two-player game and then extended it to the case of the three-player game. It showed that the emergence of congestion depends on the players relative valuations of early arrival, late arrival, and journey delay [5]. Game theory can be widely used in the field of product pricing, however, it is rarely applied in the study of real estate pricing in the literature. Ding Bao-hui in his paper summarized the common pricing methods such as market to market pricing, psychological

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pricing, market-skimming pricing etc [6]. Traditional real estate products pricing is mainly derived from the perspective of individual developers to determine the ultimate price of products with little regard to the impact of other participants' price determination [7]. In fact these influences should not be ignored. Pricing based on game theory can fully take these factors into account, thus enables price decision more accurate and reasonable.

Since there is a lack of the reference to real estates pricing, this paper is aiming to use game theory to analyze the real estate products pricing with full consideration of the influence of other participants' price decision. The game model of the government land developer and real estate developer is established. We also give the system equilibriums respectively under the cooperative and non-cooperative conditions and compare the efficiency of two pricing strategies, so as to propose reasonable pricing strategy.

2 The establishment of game model

The main body of the real estate market is composed of the government, the land developer, the real estate developer and consumers. The relationship among them can be seen in Fig.1. The government provides the land to land developer by way of bidding, auction and license and the latter is responsible for developing the land. At present, the profits that land developers gain are consulted by the government. Real estate developers obtain the land from land transaction market and eventually sell it to consumers in the form of buildings so as to achieve benefits. This paper mainly focuses on the profit relation between government, land developers and real estate developers, and suppose their main goals are to maximize their own profits (government targets are diverse, (one of which may be) the objective to maximize profits. To simplify the computation, we assume there is only one group of land developer with common benefits mode and cost structure. Real estate developers are in the same situation. $R_i (i = 0, 1, 2)$ is the profit of land developer, government and real estate developer respectively. $p_i (i = 0, 1, 2)$ denote the price of net land, land transfer, and housing respectively. The net land is the raw land after preliminary development and sorting work. The relation between net land price and land transfer price is $p_1 = (1 + r)p_0, r (r > 0)$ is marginal profit rate. $c_i (i = 0, 1, 2)$ are the costs of three main bodies in real estate market and $c_i < p_i (i = 1, 2, 3)$. The government net revenue from the land transfer fund accounts for β_1 percent. The profit margin of land developer is β_0 . Rate of advalorem duty that real estate developer pay to government is β_2 and $\beta_i < 1 (i = 0, 1, 2)$.

Suppose demand function $Q(p_i)$ is a monotonically decreasing function of price p_i . That is, $Q(p_i) = a_i - b_i p_i, a_i$ is market-clearing price—the price of a piece of commodity or service at which quantity supplied is equal to quantity demanded, and $b_i (b_i > 0)$ is price demand elasticity, $i = 0, 1, 2$. After preliminary development and sorting work has been done by the land developer, the government callback the land so it's easy to see that $Q(p_0) = Q(p_1)$.

In this paper, government total revenue comprises the tax paid by real estate developer and land transfer fund deducting the supporting urban infrastructure charges and land compensation. The profit of each main body can be denoted in the following forms:

$$R_1 = \beta_1 p_1 Q(p_1) + \beta_2 p_2 Q(p_2) \quad (1)$$

$$R_0 = \beta_0 p_0 Q(p_0) \quad (2)$$

$$R_2 = (p_2 - c_2 - f p_1 - \beta_2 p_2) Q(p_2) \quad (3)$$

where f is reciprocal of floor- area ratio.

Gross profit of developer is:

$$R = \beta_0 p_0 Q(p_0) + (p_2 - c_2 - \beta_2 p_2 - f p_1) Q(p_2) \quad (4)$$

Because $\frac{\partial^2 R_0}{\partial p_0^2} = \frac{\partial^2 R}{\partial p_0^2} = -2b_0\beta_0 < 0, \frac{\partial^2 R_2}{\partial p_2^2} = \frac{\partial^2 R}{\partial p_2^2} = -2b_2(1 - \beta_2) < 0, R_0$ is the concave function of $p_0; R$ is the concave function of $p_0, p_2; R_2$ is the concave function of p_2 .

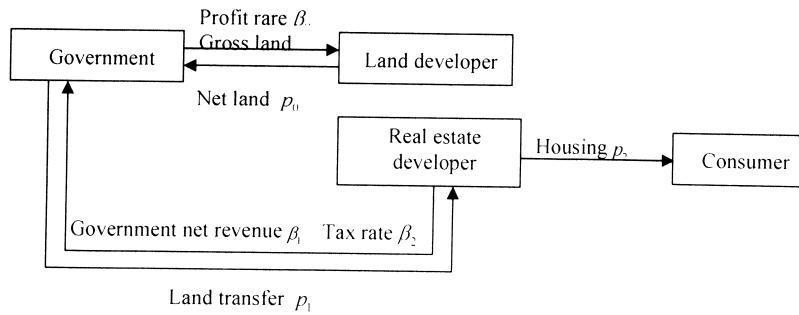


Figure 1: The relation between government, land developer and real estate developer

3 Non-cooperative game model

Here we establish a sequential non-cooperative game model according to the relation among the government, the land developer and the real estate developer. At the first stage, land developer bring forward the net land price p_0 to government, which will maximize its profit. At the second stage, government give the base price p_1 for land transfer according to the planning, the programming and land use conditions provided by land departments and land use charges etc.. At the third stage real estate developer make price decision of p_2 based on the known value of p_0, p_1 .

Since R_0 is the concave function of p_0 , the max profit of land developer is determined by the first-order derivative of R_0 : $\frac{\partial R_0}{\partial p_0} = a_0\beta_0 - 2b_0\beta_0p_0 = 0$ then $p_0 = \frac{a_0}{2b_0}$, so $p_1 = (1+r)\frac{a_0}{2b_0}$ can be obtained. In this model, for a given price p_0 and p_1 the best solution for real estate developer to maximize its profit is to let p_2 satisfy the formula (5):

$$\frac{\partial R_2}{\partial p_2} = (1 - \beta_2)(a_2 - b_2p_2) - b_2 [(1 - \beta_2)p_2 - c_2 - fp_1] = 0 \tag{5}$$

That is,

$$(1 - \beta_2)(a_2 - b_2p_2) - b_2 \left[(1 - \beta_2)p_2 - c_2 - f(1+r)\frac{a_0}{2b_0} \right] = 0 \tag{6}$$

So,

$$p_2 = \frac{a_2(1 - \beta_2) + b_2(c_2 + fp_1)}{2b_2(1 - \beta_2)}$$

Thus the equilibriums of non-cooperative game are:

$$(p_0, p_1, p_2) = \left(\frac{a_0}{2b_0}, (1+r)\frac{a_0}{2b_0}, \frac{a_2(1 - \beta_2) + b_2(c_2 + fp_1)}{2b_2(1 - \beta_2)} \right) \tag{7}$$

Formula (7) is the optimal price strategy for the main bodies under the sequential non-cooperative game condition. Their profits can be gained as follows:

$$R_0 = \frac{a_0^2\beta_0}{4b_0} \tag{8}$$

$$R_1 = \frac{a_0^2\beta_1(1+r)}{4b_0} + \frac{a_2^2\beta_2}{4b_2} - \frac{b_2(c_2 + f(1+r)\frac{a_0}{2b_0})^2}{4(1 - \beta_2)^2} \tag{9}$$

$$R_2 = \frac{[a_2(1 - \beta_2) - b_2(c_2 + \frac{a_1f}{2b_1} + (1+r)f\frac{a_0}{2b_0})]^2}{4b_2(1 - \beta_2)} \tag{10}$$

$$R = \frac{a_0^2\beta_0}{4b_0} + \frac{[a_2(1 - \beta_2) - b_2(c_2 + \frac{a_1f}{2b_1} + (1+r)f\frac{a_0}{2b_0})]^2}{4b_2(1 - \beta_2)} \tag{11}$$

4 Game model under cooperative condition

When land developers and real estate developers unite, their goal is maximize the union profit denoted by R' , R'_1 denotes government revenue, $Q(p_0)'$, $Q(p_2)'$ is the sale quantity of land developer and real estate developer, p'_0 , p'_2 is the optimal price of corresponding product. Because R is the concave function of p_0, p_2 , according to simultaneous first order condition $\begin{cases} \partial R / \partial p_0 = 0 \\ \partial R / \partial p_2 = 0 \end{cases}$, the value of p'_0, p'_2 that maximize R' can get:

$$\begin{cases} \frac{\partial R}{\partial p_0} = a_0\beta_0 - 2b_0\beta_0p_0 - f(1+r)(a_2 - b_2p_2) = 0, \\ \frac{\partial R}{\partial p_2} = (1 - \beta_2)(a_2 - b_2p_2) - b_2[(1 - \beta_2)p_2 - c_2 - f(1+r)p_0] = 0. \end{cases} \quad (12)$$

Let $T = f(1+r)(a_2 - b_2p_2)$, as demand function and floor- area ratio are positive, so $T > 0$, and

$$p'_0 = \frac{a_0\beta_0 - T}{2b_0\beta_0}, p'_2 = \frac{a_2(1 - \beta_2) + b_2(c_2 + fp_1) - b_2fT \frac{1+r}{2b_0\beta_0}}{2b_2(1 - \beta_2)}. p'_0 < p_0, p'_1 < p_1, p'_2 < p_2.$$

$$\begin{cases} p'_0 = \frac{a_0}{2b_0} - \Delta p_0 \\ p'_2 = \frac{a_2(1 - \beta_2) + b_2(c_2 + fp_1)}{2b_2(1 - \beta_2)} - \Delta p_2 \end{cases} \quad (13)$$

where

$$\begin{cases} \Delta p_0 = p_0 - p'_0 = \frac{2b_0f(1+r)[a_2(1 - \beta_2) - b_2c_2] - a_0b_2f^2(1+r)^2}{2b_0[4b_0\beta_0(1 - \beta_2) - b_2f^2(1+r)^2]} \\ \Delta p_2 = p_2 - p'_2 = \frac{f(1+r)\Delta p_0}{2(1 - \beta_2)} \end{cases} \quad (14)$$

Thus the equilibriums of the cooperative game are:

$$(p'_0, p'_1, p'_2) = \left(\frac{a_0}{2b_0} - \Delta p_0, \left(\frac{a_0}{2b_0} - \Delta p_0 \right) (1+r), \frac{a_2(1 - \beta_2) + b_2(c_2 + fp_1)}{2b_2(1 - \beta_2)} - \Delta p_2 \right)$$

The sale quantity of land developers and real estate developers are as follows:

$$Q(p_0)' = Q(p_0) + b_0\Delta p_0,$$

so

$$Q(p_0)' > Q(p_0); Q(p_2)' = Q(p_2) + b_2\Delta p_2,$$

so

$$Q(p_2)' > Q(p_2).$$

The union profit R' is:

$$R' = R + \Delta p_2[(1 - \beta_2)(2b_2p_2 - a_2 - b_2\Delta p_2) - b_0c_2] + f(1+r)(2b_2\Delta p_2\Delta p_0 - a_2\Delta p_0 - b_2\Delta p_2p_0).$$

The necessary and sufficient condition of $R' > R$ is:

$$\beta_2 < \frac{\Delta p_2(2b_2p_2 - a_2 - b_2\Delta p_2) + f(1+r)(2b_2\Delta p_2\Delta p_0 - a_2\Delta p_0 - b_2\Delta p_2p_0)}{\Delta p_2(2b_2p_2 - a_2 - b_2\Delta p_2)} \quad (15)$$

Government revenue R'_1 is:

$$R'_0 = R_0 - \beta_1(1+r)b_0\Delta p_0^2 + \beta_2\Delta p_2(2b_2p_2 - a_2 - b_2\Delta p_2)$$

The necessary and sufficient condition of $R'_0 > R_0$ is:

$$\beta_2 > \frac{\beta_1(1+r)b_0\Delta p_0^2}{\Delta p_2(2b_2p_2 - a_2 - b_2\Delta p_2)} \quad (16)$$

5 Efficiency comparison between cooperative & non- cooperative game

Through the above analysis we can compare the efficiency between cooperative and non- cooperative games. The result is shown in table1. We can see that in the cooperative game, despite the net land price and housing price decline, the increase of demand can lead to the rise in system profits. This partially explains the phenomenon that at present many real estate developers are enthusiastic about the land development though the profit margin is only 8%. In fact, during the process of land development, real estate developers who participate in it can have a thorough understanding of development cost and project particulars. So when the real estate developers construct buildings they can control both cost and pricing of their products with greater certainty.

Table 1: The comparison of efficiency of cooperative & non- cooperative game

	non- cooperative game	relation	cooperative game	condition
Net land price	p_1	$>$	p'_1	
Housing price	p_2	$>$	p'_2	
Demand of net land	$Q(p_1)$	$<$	$Q(p_1)'$	
Demand of housing	$Q(p_2)$	$<$	$Q(p_2)'$	
Total profit of union	R	$<$	R'	(15)
government revenue	R_0	$<$	R'_0	(16)

In addition, in the non-cooperative game, land developers' profits are only related to the net land price, therefore they tend to artificially raise the quoted price, thus will increase the cost of real estate development and eventually lead to the rise in building prices. In the cooperative game, the profit of union including real estate and land developers relate with not only net land price but also the housing price so the offer would be more reasonable. As long as tax rate is low enough, developers can maximize their own profits through cooperative pricing strategy.

When condition (16) is met government revenue in cooperative game will be higher than that of non-cooperative game. That's to say, as tax bearing rate that developers have to pay is high enough, cooperative pricing strategy will enhance the government revenue. However, the upper limit of tax bearing rate is that developers can cooperate, otherwise, the government revenue increase but the real estate prices also increase. It is obvious that regulating the tax rate in a moderate rang is an efficient way for government to increase its profit and decrease housing prices so as to maintain social stability.

6 Conclusions

This paper investigates the pricing strategy about land and real estate with the application of game theory. The non-cooperative game and cooperative game between developers are discussed. The corresponding pricing strategies are given. Research shows that when land and real estate developers make price decision independently, the optimal price is higher than that of cooperative game.

In short, land development marketization alleviates the government's burden and enhances its ability to regulate land supply and maximize the value of land resources. Developers with abundant experience can obtain relatively stable proceeds through land development. Other real estate developers will no longer do the works unrelated to real estate construction. Those who have rich economic strength can participate in land and real estate development simultaneously which not only solve the problem of land shortage but also gain better profit margins.

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