

An Exact Linearization Feedback Control of CHEN Equation

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Abstract: In this paper, by applying the theory of Li-qun and the tool of Li-differential coefficient we change the general ODE into the according linear standard former, and with the classical control method of chaos controlling—feedback control, we carry out the exact linearization controlling of CHEN equation. From the numerical simulation, we can compare the dynamic phase figures before controlling and after controlling, and find: the exact linearization feedback control can control the chaotic state into steady state effectively.

Key words: CHEN equation; exact linearization; feedback control; Lorenz equation

1 Introduction

Since Ott, Grebogi and Li-York posed the method of OGY[1], chaos control and chaos synchronization have been the important part of nonlinear science theory and application, and have been the hot topics and important project. There is much work about them ([2],[3],[4],[5], etc.). There exist many effective controlling methods, such as parameter perturbation, feedback, delaying and etc. Among these methods, the feedback method has very wide application and good controlling effect. In this paper, we introduce the exact linearization feedback controlling method[6], which has gained good theory and many good mathematical results[7]. But applying this method to control chaos in more systems is not deployed. CHEN equation is evolved from Lorenz equation, which is a chaotic system having different typical dynamic characters and behaviors[8]. Studying its chaos controlling can reflect the dynamic conformation and extend the application of the exact linearization feedback controlling method in the chaos controlling. In this paper, we apply this method to control chaos in CHEN equation, and control chaotic state into steady state effectively. This work prepares more for the study of this method's universality and validity.

2 Theory

Theorem : To the system

$$\begin{cases} \dot{x} = f(x) + ug(x) \\ y = h(x) \end{cases} \quad (2.1)$$

where $x \in R^n$, $u \in R$, $y \in R$, $f(x)$ and $g(x)$ are both vector fields, $h(x)$ is scalar quantity. If: (1) The Lie derivation relative to vector field $g(x)$ for the k ($0 \leq k \leq r - 2$) rank Lie derivations of the output function $h(x)$ relative to vector field $f(x)$ are all equal to 0 in the neighborhood of x_0 , i.e. $:L_g L_f^k h(x) = 0 (\forall x \in$

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$\cup(x_0)$.(2) The Lie derivation relative to vector field $g(x)$ for the $r - 1$ rank Lie derivation of the output function $h(x)$ relative to vector field $f(x)$ is not equal to 0 in the neighborhood of x_0 , i.e. $:L_g L_f^{r-1} h(x) \neq 0$.

Then the system (2.1) can be Li-linearized in the neighborhood of x_0 , and if the expected output $y_R(t)$ is determined, the controller u has such expression:

$$u = \frac{1}{l_g l_f^{r-1} h(x)} (-l_f^r h(x) + y_R^{(r)} - \sum_{i=1}^r c_{i-1} (l_f^{i-1} h(x) - y_R^{(i-1)})) \tag{2.2}$$

The proof is passed over (see article [9]).

3 The exact linearization controlling of CHEN equation

3.1 The dynamic character and behavior of CHEN equation

We consider CHEN equation:

$$\begin{cases} \frac{dx}{dt} = a(y - x) \\ \frac{dy}{dt} = (c - a)x - xz + cy \\ \frac{dz}{dt} = xy - bz \end{cases} \tag{3.1}$$

We can discuss some mathematic qualities, equilibrium's bifurcation character, period solutions' character and chaotic attractor's dynamic structure. It has some characteritic in nature different from other typical models(such as: its original system—Lorenz system). In the above equation(3.1), we choose $a = 35, b = \frac{8}{3}, c = 28$, and apply the software of Mathematica in numerical simulation. We gain the dynamic phase figures as Fig.1 and Fig.2.

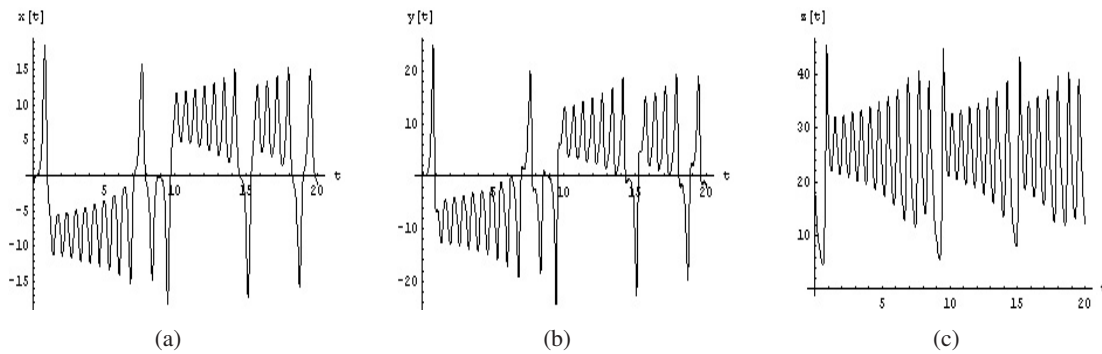


Figure 1: The solutions $x(t), y(t), z(t)$ of CHEN equation

3.2 The exact linearization control of CHEN equation

In CHEN equation(3.1), we choose a, b, c as above, and order $x_1(t) = x(t), x_2(t) = y(t), x_3(t) = z(t)$, then CHEN equation changes into:

$$\begin{cases} \dot{x}_1 = 35x_2 - 35x_1 \\ \dot{x}_2 = -7x_1 - x_1x_3 + 28x_2 \\ \dot{x}_3 = x_1x_2 - \frac{8}{3}x_3 \end{cases} \tag{3.2}$$

In the following, we apply the exact linearization feedback controlling to control chaos in CHEN equation(3.2).

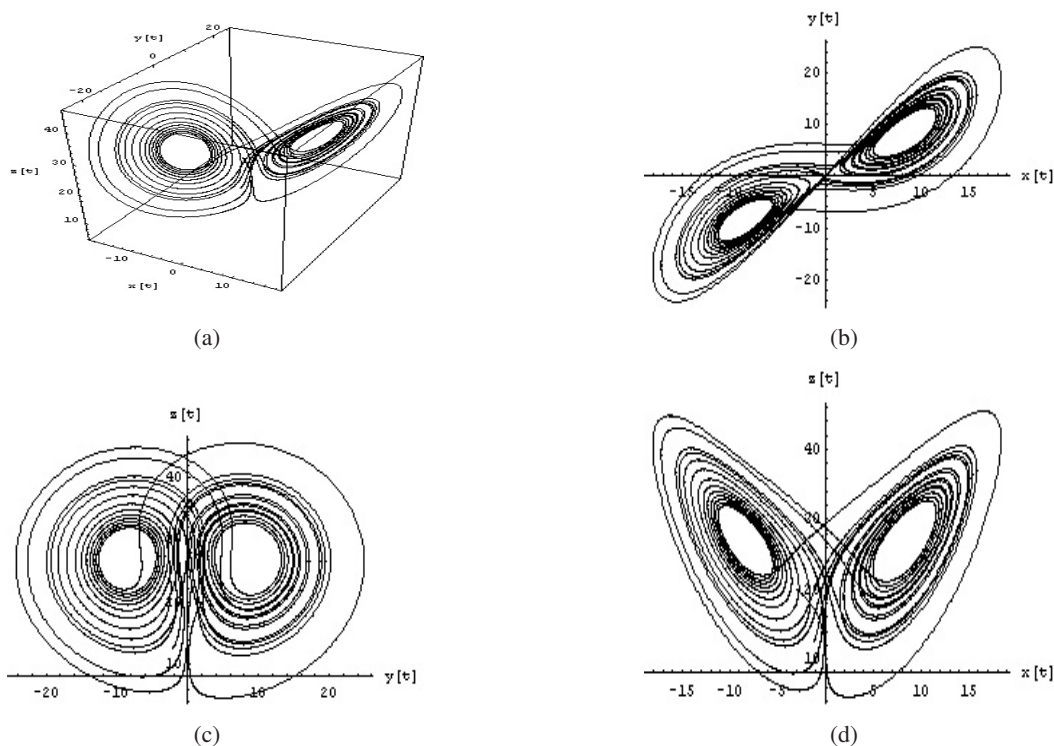


Figure 2: The relations of the solutions $x(t), y(t), z(t)$ and two of them of CHEN equation

3.2.1 Case 1

Choosing $h(x) = x_1, g(x) = \begin{pmatrix} 0 \\ 0 \\ x_3 \end{pmatrix}$, we get $r = 3$ according to the above theorem. With putting $r = 3$ into the expression and choosing $y_R = 8.5$, we can get the controller u as following:

$$u = \frac{1}{-35x_1x_3} (34300x_1 - 34300x_2 - 1225x_1x_3 + 1225x_2x_3 - 1715x_1 - 245x_1x_3 + 6860x_2) + 35x_1^2x_2 - \frac{280}{3}x_1x_3 - 125x_1 + 183(-35x_1 + 35x_2) + 450(-245x_2 + 980x_1 - 35x_1x_3)$$

We put u into the right side of CHEN equation, and get the results of controlling as Fig.3.

Comparing the numerical results of CHEN equation before controlling and after controlling (see Fig.1, fig.2 and Fig.3), we can find: the controller gained by exact linearization can effectively control chaotic state into steady state.

3.2.2 Case 2

Giving $h(x) = x_2, g(x) = \begin{pmatrix} x_1 \\ 0 \\ -7 - x_3 \end{pmatrix}$, we get $r = 3$ according to the above theorem. With putting $r = 3$ into the expression and choosing $y_R = 27$ we can get the controller u as following:

$$u = \frac{1}{-\frac{56}{3}x_1 + 245x_2 + 35x_2x_3 - 2x_1^2x_2} (-(49 + c)x_1^2x_2 - x_1^3(7 + x_3)) + \frac{1}{3}x_2(-6160 + 3a + 84b + 1617c - 35(-76 + 3c)x_3) - \frac{1}{9}x_1(21(-880 + 3b - 21c) - 945x_2^2 + (1973 + 9b - 87c)x_3 + 315x_3^2)$$

We put u into the right side of CHEN equation, and get the results of controlling as Fig.4.

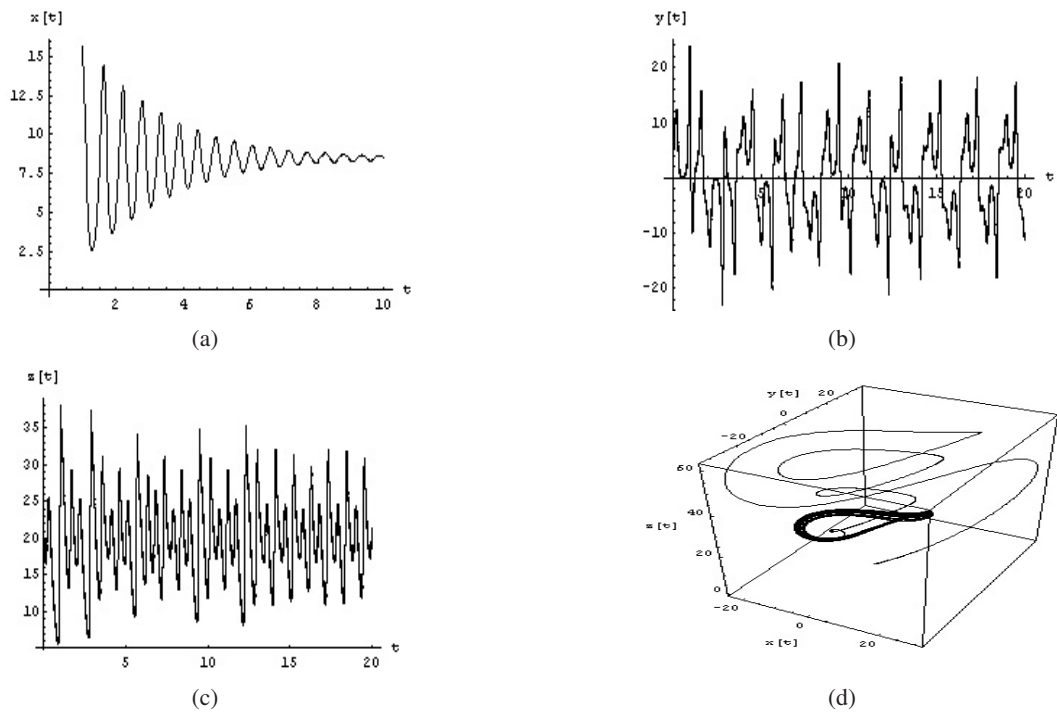


Figure 3: The solutions $x(t), y(t), z(t)$ and the relation of them of CHEN equation under controlling

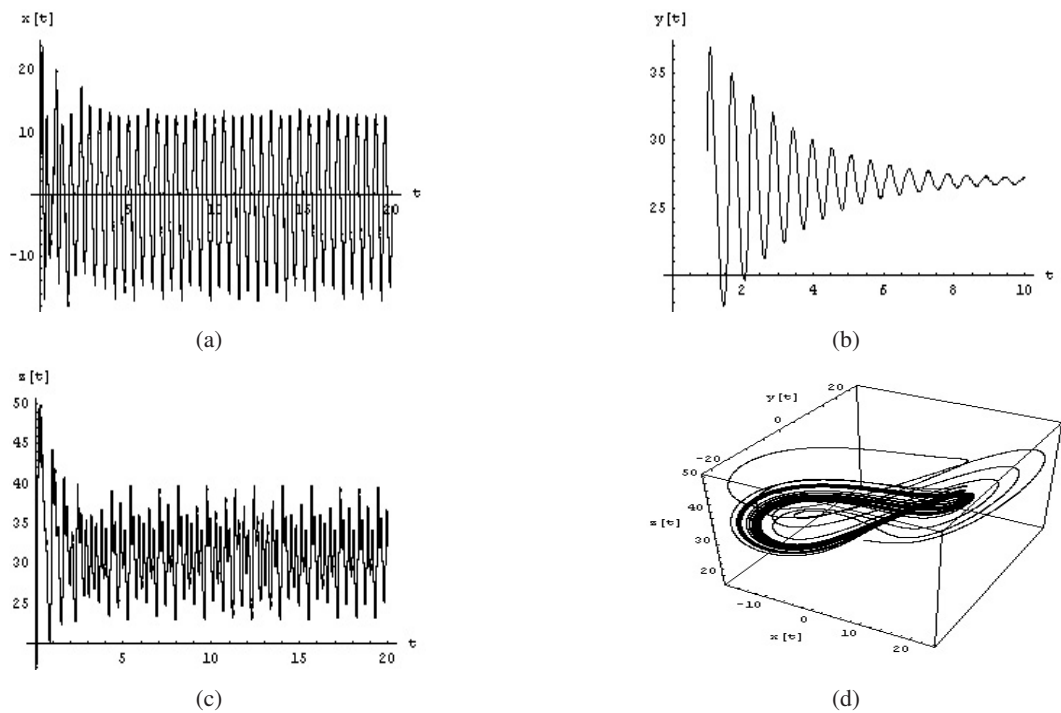


Figure 4: The solutions $x(t), y(t), z(t)$ and the relation of them of CHEN equation under controlling

As the same, comparing the numerical results of CHEN equation before controlling and after controlling(see Fig.1, fig.2 and fig.4),we can also find: the controller gained by exact linearization can effectively control chaotic state into steady state.

4 Results and expectation

As a new control method, the exact linearization feedback control method have gained very good results in some familiar systems, such as: Lorenz system, the oscillator of van der pol,etc.Here we apply this method in controlling chaos in CHEN equation. By the numerical simulation, we can find: the exact linearization feedback control can control chaotic state into steady state, and has good controlling effect. Of course we have such question: can this method control chaotic state or steady state into periodic state and steady state into chaotic state, i.e. we can consider applying the exact linearization feedback control in reverse-control. We will try to study this method in the above applications step by step in the future, and expect to get deep results about theory and application.

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References

- [1] Edward Ott, Celso Grebogi and James A.Yorke: Controlling Chaos. *Physical Review Letter.* 64(11), 1196-1199 (1990)
- [2] Zhenyuan Xu: Synchronization of two discrete Ginzburg-Landau equations using local coupling. *International Journal of Nonlinear Science.* 1(1),19-32(2006)
- [3] Xuedi Wang, Lixin Tian, Liqin Yu: Linear feedback controlling and synchronization of the Chen's chaotic system. *International Journal of Nonlinear Science.*2(1),43-49(2006)
- [4] Guoliang Cai, Juanjuan Huang, Lixin Tian, Qingchao Wang: Adaptive control and slow manifold analysis of a new chaotic system. *International Journal of Nonlinear Science.*2(1),50-60(2006)
- [5] Lixin Tian, Shulai Zhang, Guangjuan Yang: Some results of a class of systems and their application in chaotic synchronization. *International Journal of Nonlinear Science.* 2(3),159-165(2006)
- [6] Weiguo Zhang: The introduction of advanced control theory and method.*Shanxi: Northwestern Polytechnical University Press.*4(2000)
- [7] Li-qun Chen, Yan-zhu Liu: A modified exact linearization control for chaotic oscillators.*Nonlinear Dynamics.* 20(1999)
- [8] Guanrong Chen: Bifurcation analysis of CHEN's equation.*International Journal of Bifurcation Chaos.* 10(8)(2000)
- [9] Xiaofeng Shi, Lixin Tian: An exact linearization control for AIM of KS equation.*Journal of Jiangsu University.* 23(6),11-14(2002)
- [10] Changpin Li, Guanrong Chen: Bifurcation of one-dimensional reaction-diffusion equation.*International Journal of Bifurcation Chaos.* 11(5),1295-1306(2001)
- [11] Gang Hu, Jinghua Xiao, Zhigang Zheng: Chaos control.*Shanghai: Shanghai Science and Technology Educational Publishing House.* 12(2002)