The Stability Analysis of Duopoly Investment Model with Bounded Rationality Based on China’s Entry into the WTO

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Abstract: Under the background of China’s entry into the WTO and the reforms of Chinese state-owned enterprises, this paper brings the duopoly game model with bounded rationality into investment field, and sets up a dynamic investment model. It uses the non-linear theory to analyze the stability of the model; furthermore, it expounds influential factors in this investment. The research finds that the change of each factor may influence the stability of investment. Finally, it indicates that investors’ expansion in investment for profits may cause the phenomena of chaos.

Key words: Entry into the WTO; investment; dynamic model; Nash equilibrium; Chaos

1 Introduction

With the process of globalization speeding up, the international trade and transnational investment play more important roles in the global economy. China’s entry into the WTO could facilitate Chinese globalization. Moreover, Chinese state-owned enterprises are facing the third reform. Presently government has allowed to close out the state-owned enterprises’ shares to foreigner, and foreign businessmen have been increasing the investment to Chinese state-owned enterprises (see [1]). Simultaneously Chinese state-owned enterprises have been the focus of investment for Chinese financial institutions. Up on that, Chinese State-owned enterprises become the hot spot of investment at home and abroad. Foreign merchants and Chinese financial organizations will launch the competition to invest into Chinese state-owned enterprises, which may affect the stability of process of investment and have a bad impact on the profits of both sides of investment, and then influence steady development of national economy (see [2-4]). So the research of this question has great significance.

In recent years, oligarch game with bounded rationality has been the hot spot of research. The linear cost double-monopoly game model based on the bounded rational expectation has been studied by Bischi and Naimzada [5]. Ahmed, Agiza and Hassan gain the double monopoly game model with bounded rationality by modification of Puu’s dynamical duopoly (see [6]). And then Agiza et al. study the linear cost double monopoly game model with bounded rationality (see [7]). [8] applies the double monopoly game model with bounded rationality to advertisement investment. When the participators change the reaction speed to the market these studies all point out that the complex phenomena like bifurcation and chaos will be observed. In the present paper, we endeavor to bring the duopoly game model of bounded rationality into financial field, and set up a non-linear investment model to describe the above act of investing. And the other objective of this paper is to study the reasons as to why and when the investors increase investment and discuss the influence on the system stability caused by this behavior of investment. The remainder of this paper is organized as follows. Section 2 presents an investment model with bounded rationality through

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modifying the duopoly game model of bounded rationality. In section 3, we analyze and discuss the stability of singular points in the model by using non-linear theory, and in section 4, we discover that those investors, whose rates of adjusting investment are lower, gain lower profit than that of others through simulation of the system with computer. So investors prefer to leapfrog the rate of adjusting investment, but this may result in instability in the system and may cause the phenomenon of chaos.

2 The model

Suppose both a foreign merchant and a Chinese financing institution (or called financing institution) invest in a Chinese state-owned enterprise (or called state-owned enterprise), and then let this model meet the requirements as follows:

1) Investors often pour money into state-owned enterprise by stages in order to avoid risks, so this model is regarded as a discrete dynamics model. Here, \( x(n) \) represents the investment of foreign businessmen in China at \( nth \) stage, \( y(n) \) represents the investment of financial institution at \( nth \) stage.

2) The fund held by this state-owned enterprise is limited in a stage, because of the extent of state-owned shares evacuation and development of the state-owned enterprise. Suppose the maximum investment allowance of the state-owned enterprise is \( K \) in some certain stage. If overinvestment, the fund would be idle or waste; rather, investors would not realize the largest profit. Moreover the service efficiency of fund is different, which is influenced by whether the fund is forthcoming and the level of management in the operation of the fund, \( \beta_1, \beta_2 \) represent foreign merchants’ and the financial organizations’ service efficiency of fund respectively. At last, we get the profit function of state-owned enterprise:

\[
\pi = a - b[K - (\beta_1 x + \beta_2 y)]^2
\]

(2.1)
a represents the biggest profit which the enterprise obtains, \( b \) represents the loss coefficient because of the shortage or overflow of the capital.

3) The marginal profit of foreign investor is \( \frac{\partial \pi}{\partial x} = 2b\beta_1[K - (\beta_1 x + \beta_2 y)] \), which means the change of profit is brought by the slight alteration about \( x \). We suppose that the foreigner’s yield not only relates with investment directly, but also with the profit of the state-owned enterprise brought by this investment. In other words, the state-owned enterprise will make responses to the investor according to the profit, with the form of capital bounds or bounds stock, and the rate of return on investment is \( \gamma_1 \). Moreover, the investment is \( x(n) \) in some \( nth \) stage. We get the foreigner’s income: \( 2b\beta_1 \gamma_1[K - (\beta_1 x + \beta_2 y)] \); in the same way, we can also get the marginal profit of financial institution: \( \frac{\partial \pi}{\partial y} = 2b\beta_2[K - (\beta_1 x + \beta_2 y)] \), and its income function:

\[
2b\beta_2 \gamma_2[K - (\beta_1 x + \beta_2 y)].
\]

4) All the funds have the cost, such as the value of time, opportunity cost and so on. Here, we still use the production cost function \( c(x) = c + d(x) \) to indicate the cost of funds. Here, \( c \) is used to show the fixed expenses, and \( d \) is the expenses of using the unit of fund, namely variable cost. Suppose \( c_1 \) stands for the fixed cost of foreign investment, \( c_2 \) stands for that of Chinese financial institutions, and variable costs of theirs are respectively: \( d_1, d_2 \). In the end, we gain the foreign cost function is \( c_1 + d_1 x(n) \), and that of China is \( c_2 + d_2 y(n) \). Through the hypothesis upwards, we obtain the profit function of the foreign merchant and financial institution separately in the \( nth \) stage:

\[
\begin{align*}
\pi_n(x) &= 2b\beta_1 \gamma_1[K - (\beta_1 x(n) + \beta_2 y(n))] - (c_1 + d_1 x(n)) \\
\pi_n(y) &= 2b\beta_2 \gamma_2[K - (\beta_1 x(n) + \beta_2 y(n))] - (c_2 + d_2 y(n))
\end{align*}
\]

(2.2)

Their marginal profits in the \( nth \) stage separately are:

\[
\begin{align*}
\frac{\partial \pi_n(x)}{\partial x} &= 2b\beta_1 \gamma_1[K - (\beta_1 x(n) + \beta_2 y(n))] - 2b\beta_1^2 \gamma_1 x(n) - d_1 \\
\frac{\partial \pi_n(y)}{\partial y} &= 2b\beta_2 \gamma_2[K - (\beta_1 x(n) + \beta_2 y(n))] - 2b\beta_2^2 \gamma_2 y(n) - d_2
\end{align*}
\]

In practice, both foreign merchant and Chinese financial institution are not able to collected complete information and make decisions on the basis of complete rationality, so they are not rational enough. Besides,
Because the investment has just started and the financing capacity can be raised. And enterprises also may stimulate investment with superior rate of return on investment. improve the management mechanism. In this case, cost can be reduced and the service efficiency of the funds safe and stable environment for investors. Meanwhile the state-owned enterprises are also doing their best to have a bad influence on the development of enterprises. So Chinese governments tries to create a healthy, this situation doesn’t expect to be seen by Chinese governments and the enterprises. The shortage of funds 

\[
\begin{align*}
\alpha_1, \alpha_2 & \text{ respectively stand for the rates of adjusting the investment of foreign investors and financial institutions, which means they transform the unit income into investment of the next time.}
\end{align*}
\]

3 Model analysis

In order to study the qualitative behavior of the nonlinear algebraic system (2.3), we figure out the nonnegative fixed points of this system. We see the graph of system (2.3). And four singular points are found: 

\[
E_0 = (0, 0), E_1 = (0, \frac{2bK\beta_2\gamma_2 - d_2}{4bK\beta_2\gamma_2}), E_2 = (\frac{2bK\beta_1\gamma_1 - d_1}{4bK\beta_1\gamma_1}, 0),
\]

\[
E_3 = (\frac{2bK\beta_1\beta_2\gamma_1\gamma_2 - 2bK\beta_2\gamma_2 + d_2\beta_1\gamma_1}{6b^2\beta_1^2\beta_2\gamma_1\gamma_2}, \frac{2bK\beta_1\beta_2\gamma_1\gamma_2 - 2d_2\beta_1\gamma_1 + d_1\beta_2\gamma_1}{6b^2\beta_1^2\beta_2\gamma_1\gamma_2}).
\]

We define the points \( E_0, E_1, E_2 \) as bounded equilibrium points. And \( E_3 \) is regarded as the Nash equilibrium point, because both foreign investor’s and financial institution’s marginal profits are zero at this point.

3.1 Stability analysis of the point \( E_0 \)

The Jacobin matrix of \( E_0 \) is:

\[
J = \begin{pmatrix}
1 + \alpha_1(2bK\beta_1\gamma_1 - d_1) & 0 \\
0 & 1 + \alpha_2(2bK\beta_2\gamma_2 - d_2)
\end{pmatrix}
\]

And its two eigenvalues are separately: \( \lambda_1 = 1 + \alpha_1(2bK\beta_1\gamma_1 - d_1) \), \( 1 + \alpha_2(2bK\beta_2\gamma_2 - d_2) \). Make \( E_0 \) stable, both \( \lambda_1 < 1 \) and \( \lambda_2 < 1 \) must be satisfied. Because rates of adjusting the investment \( \alpha_1, \alpha_2 \) are greater than zero, we need both \( 2bK\beta_1\gamma_1 - d_1 < 0 \) and \( 2bK\beta_2\gamma_2 - d_2 < 0 \) to be satisfied.

In the realistic sense, \( E_0 \) represents that there is not investment from foreign merchants and financing institutions. At this time, investors will firstly forecast whether they can reap a very fair profit through this investment. And then investment will be made, if they can. Otherwise, investment willingness can be very low. We suppose the cost function is \( c(x) = c + d(x) \) as above. In general, fixed cost \( c \) is stable at stated periods, but the variable cost \( d \) is varying in response to the circumstance of investment. So we refer chiefly to variable cost here. When the variable cost of investment is too high, and the service efficiency of funds \( \beta_1, \beta_2 \) as well as the rates of return on investment \( \gamma_1, \gamma_2 \) are low, it is possible to appear that \( 2bK\beta_1\gamma_1 - d_1 < 0, 2bK\beta_2\gamma_2 - d_2 < 0, \) namely \( \lambda_1 < 1, \lambda_2 < 1 \). And this point is stable crunode. However, this situation doesn’t expect to be seen by Chinese governments and the enterprises. The shortage of funds is a main barrier which stands in the way of economic development. If funds were not enough, it would have a bad influence on the development of enterprises. So Chinese governments tries to create a healthy, safe and stable environment for investors. Meanwhile the state-owned enterprises are also doing their best to improve the management mechanism. In this case, cost can be reduced and the service efficiency of the funds can be raised. And enterprises also may stimulate investment with superior rate of return on investment. Because the investment has just started and the financing capacity \( K \) is full bottomed, \( 2bK\beta_1\gamma_1 - d_1 > 0 \) or \( 2bK\beta_2\gamma_2 - d_2 > 0 \) is soon satisfied, namely \( \lambda_1 > 1 \) or \( \lambda_2 > 1 \). It is impossible to come forth the stable situation where both \( \lambda_1 < 1 \) and \( \lambda_2 < 1 \) lie. Therefore, \( E_0 \) is not stable.

3.2 Stability analysis of the point \( E_1 \)

The Jacobin matrix of \( E_1 = (0, \frac{2bK\beta_2\gamma_2 - d_2}{4bK\beta_2\gamma_2}) \) is:

\[
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\[ J = \begin{pmatrix} 1 + \alpha_1(bK\beta_1\gamma_1 + \frac{d_2b\beta_2\gamma_2}{\beta_2\gamma_2} - d_1) & 0 \\ \alpha_2(-bK\beta_1\gamma_1 + \frac{d_2\gamma_1}{\beta_2\gamma_2}) & 1 - \alpha_2(2bK\beta_2\gamma_2 - d_2) \end{pmatrix}. \]

And its two eigenvalues are separately: \( \lambda_1 = 1 + \alpha_1\beta_1\gamma_1(bK + \frac{d_2}{\beta_2\gamma_2} - \frac{d_1}{\beta_1\gamma_1}) \), \( \lambda_2 = 1 - \alpha_2\beta_2\gamma_2(2bK - \frac{d_2}{\beta_2\gamma_2}). \)

Foreign businessmen are entitled to national treatment after China’s entry into the World Trade Organization. Citizenship are the same for foreign merchants’ and Chinese financial institutions’. Moreover, the variable cost, service efficiency of funds and rates of return on investment of foreign merchants’ are all similar to that of Chinese financial institutions’. However, \( K \), the maximum fund allowance of Chinese state-owned enterprise, outrasses the variable cost. So it is easy to satisfy \( bK + \frac{d_2}{\beta_2\gamma_2} - \frac{d_1}{\beta_1\gamma_1} > 0 \), \( 2bK - \frac{d_2}{\beta_2\gamma_2} > 0 \), namely \( \lambda_1 > 1 \lambda_2 < 1 \). Therefore, \( E_1 \) is an unstable saddle point.

\( E_1 \) represents that there is no investment from foreign merchant, but only that of the financial institute. China formerly enacted and implemented many policies and laws not only limiting the investment of foreign merchant, but also making some ramparts. So the investment cost of foreign merchant \( d_1 \) was too high, and foreign merchant would not like to invest. It means this point is stable, namely \( \lambda_1 = 1 + \alpha_1\beta_1\gamma_1(bK + \frac{d_2}{\beta_2\gamma_2} - \frac{d_1}{\beta_1\gamma_1}) < 1 \), \( \lambda_2 = 1 - \alpha_2\beta_2\gamma_2(2bK - \frac{d_2}{\beta_2\gamma_2}) < 1 \). And it also indicates that foreign businessmen would not invest, if \( d_1 > bK\beta_1\gamma_1 + \frac{d_2\beta_1\gamma_1}{\beta_2\gamma_2} \). As far as China was concerned, the rampart, used to limit the investment of foreign merchant, can be made in this case. However, After China joins the World Trade Organization, she will abide the promises that these ramparts are gradually repealed, and reduce the cost \( d_1 \), simultaneously increase the rate of return on investment, in order to attract the foreign capital. Moreover, foreign investors usually regard Chain as a very big market, so they would not hesitate to invest, when \( d_1 \) were only diminished to some certain extent. Here, \( y = \frac{2bK\beta_2\gamma_2 - d_2}{4bK\beta_2\gamma_2} < K \), it does accord with the actual state of China that the fund offered by Chinese financial institutions can not meet the needs of domestic state-owned enterprises, while foreigners still have room for investment. The cost is reduced, and then foreigners begin to invest. Therefore, this point is also not stable, and here \( \lambda_1 > 1 \), \( \lambda_2 < 1 \).

We will not discuss the stability of point \( E_2 \), for it has symmetrical structure with \( E_1 \), and has the similar situation.

### 3.3 Stability analysis of the point \( E_3 \)

**Theorem:** \( E_3 \) is stable, if \( \frac{1}{12}\alpha_1\alpha_2\beta_1\beta_2\gamma_1\gamma_2(2bK - \frac{2d_1}{\beta_1\gamma_1} + \frac{d_2}{\beta_2\gamma_2})(2bK - \frac{2d_1}{\beta_1\gamma_1} + \frac{d_2}{\beta_2\gamma_2}) < \frac{1}{4}\alpha_1\beta_1\gamma_1(2bK - \frac{2d_1}{\beta_1\gamma_1} + \frac{d_2}{\beta_2\gamma_2}) + \frac{1}{12}\alpha_2\beta_2\gamma_2(2bK - \frac{2d_1}{\beta_1\gamma_1} + \frac{d_2}{\beta_2\gamma_2}) < \frac{1}{4}\alpha_1\beta_1\gamma_1(2bK - \frac{2d_1}{\beta_1\gamma_1} + \frac{d_2}{\beta_2\gamma_2}) < 1. \)

**Proof.** \( E_3 = \left( \frac{2bK\beta_1\gamma_1\gamma_2d_2}{12\beta_1\gamma_1\gamma_2} - \frac{2d_1\beta_1\gamma_1}{\beta_1\gamma_1} + \frac{d_2\beta_1\gamma_1}{\beta_1\gamma_1}, \frac{2bK\beta_1\gamma_1\gamma_2d_2}{12\beta_1\gamma_1\gamma_2} - \frac{2d_1\beta_1\gamma_1}{\beta_1\gamma_1} + \frac{d_2\beta_1\gamma_1}{\beta_1\gamma_1} \right) \) is the Nash equilibrium point of system (2.3), and its Jacobian matrix is:

\[
\begin{pmatrix}
-\frac{2}{3}\alpha_1\beta_1\gamma_1(2bK - \frac{2d_1}{\beta_1\gamma_1} + \frac{d_2}{\beta_2\gamma_2}) & -\frac{2}{3}\alpha_2\beta_2\gamma_2(2bK - \frac{2d_1}{\beta_1\gamma_1} + \frac{d_2}{\beta_2\gamma_2}) \\
-\frac{1}{3}\alpha_1\beta_1\gamma_1(2bK - \frac{2d_1}{\beta_1\gamma_1} + \frac{d_2}{\beta_2\gamma_2}) & 1 - \frac{2}{3}\alpha_2\beta_2\gamma_2(2bK - \frac{2d_1}{\beta_1\gamma_1} + \frac{d_2}{\beta_2\gamma_2})
\end{pmatrix},
\]

its characteristic equation is: \( \rho(\lambda) = \lambda^2 - trJ\lambda + detJ \). \( trJ \) denotes the trace of matrix, and \( detJ \) denotes determinant of the matrix.

\[
trJ = 2 - \frac{2}{3}\alpha_1\beta_1\gamma_1(2bK - \frac{2d_1}{\beta_1\gamma_1} + \frac{d_2}{\beta_2\gamma_2}) - \frac{2}{3}\alpha_2\beta_2\gamma_2(2bK - \frac{2d_1}{\beta_1\gamma_1} + \frac{d_2}{\beta_2\gamma_2})
\]

\[
detJ = 1 - \frac{2}{3}\alpha_1\beta_1\gamma_1(2bK - \frac{2d_1}{\beta_1\gamma_1} + \frac{d_2}{\beta_2\gamma_2}) - \frac{2}{3}\alpha_2\beta_2\gamma_2(2bK - \frac{2d_1}{\beta_1\gamma_1} + \frac{d_2}{\beta_2\gamma_2})
\]

\[
trJ^2 - 4detJ = \left[ \frac{2}{3}\alpha_1\beta_1\gamma_1(2bK - \frac{2d_1}{\beta_1\gamma_1} + \frac{d_2}{\beta_2\gamma_2}) - \frac{2}{3}\alpha_2\beta_2\gamma_2(2bK - \frac{2d_1}{\beta_1\gamma_1} + \frac{d_2}{\beta_2\gamma_2}) \right]^2
\]

\[
+ \frac{4}{9}\alpha_1\alpha_2\beta_1\beta_2\gamma_1\gamma_2(2bK - \frac{2d_1}{\beta_1\gamma_1} + \frac{d_2}{\beta_2\gamma_2})(2bK - \frac{2d_1}{\beta_1\gamma_1} + \frac{d_2}{\beta_2\gamma_2})
\]

IINS homepage: http://www.nonlinearscience.org.uk/
In general, $K$, which is the fund capacity of enterprise, is very big, but the variable cost of fund is not big, so it is very easy that $(2bK - \frac{2d_1}{\beta_1 \gamma_1} + \frac{d_2}{\beta_2 \gamma_2}) > 0$, $(2bK - \frac{2d_2}{\beta_2 \gamma_2} + \frac{d_1}{\beta_1 \gamma_1}) > 0$. Basing on this preconditional, we get $tr.J^2 - 4det.J > 0$, namely the eigenvalue of the Jacobin matrix is true. According to July-qualification, the necessary and sufficient conditions to stabilize the Nash equilibrium point are listed as follows:

1. $1 + tr.J + det.J > 0$

The first condition becomes

$$4 - \frac{4}{3} \alpha_1 \beta_1 \gamma_1 (2bK - \frac{2d_1}{\beta_1 \gamma_1} + \frac{d_2}{\beta_2 \gamma_2}) - \frac{4}{3} \alpha_2 \beta_2 \gamma_2 (2bK - \frac{2d_2}{\beta_2 \gamma_2} + \frac{d_1}{\beta_1 \gamma_1})$$

$$+ \frac{1}{2} \alpha_1 \alpha_2 \beta_1 \beta_2 \gamma_1 \gamma_2 (2bK - \frac{2d_1}{\beta_1 \gamma_1} + \frac{d_2}{\beta_2 \gamma_2}) (2bK - \frac{2d_2}{\beta_2 \gamma_2} + \frac{d_1}{\beta_1 \gamma_1}) > 0 \quad (3.1)$$

2. $1 - tr.J + det.J > 0$

The second condition becomes

$$\frac{1}{3} \alpha_1 \alpha_2 \beta_1 \beta_2 \gamma_1 \gamma_2 (2bK - \frac{2d_1}{\beta_1 \gamma_1} + \frac{d_2}{\beta_2 \gamma_2}) (2bK - \frac{2d_2}{\beta_2 \gamma_2} + \frac{d_1}{\beta_1 \gamma_1}) > 0 \quad (3.2)$$

3. $|det.J| > 0$

The third condition becomes

$$|1 - \frac{2}{3} \alpha_1 \beta_1 \gamma_1 (2bK - \frac{2d_1}{\beta_1 \gamma_1} + \frac{d_2}{\beta_2 \gamma_2}) - \frac{2}{3} \alpha_2 \beta_2 \gamma_2 (2bK - \frac{2d_2}{\beta_2 \gamma_2} + \frac{d_1}{\beta_1 \gamma_1})$$

$$+ \frac{1}{3} \alpha_1 \alpha_2 \beta_1 \beta_2 \gamma_1 \gamma_2 (2bK - \frac{2d_1}{\beta_1 \gamma_1} + \frac{d_2}{\beta_2 \gamma_2}) (2bK - \frac{2d_2}{\beta_2 \gamma_2} + \frac{d_1}{\beta_1 \gamma_1})| < 1 \quad (3.3)$$

Incorporate formulas (3.1) (3.2) (3.3), and then get:

$$\frac{1}{12} \alpha_1 \alpha_2 \beta_1 \beta_2 \gamma_1 \gamma_2 (2bK - \frac{2d_1}{\beta_1 \gamma_1} + \frac{d_2}{\beta_2 \gamma_2}) (2bK - \frac{2d_2}{\beta_2 \gamma_2} + \frac{d_1}{\beta_1 \gamma_1})$$

$$< \frac{1}{3} \alpha_1 \beta_1 \gamma_1 (2bK - \frac{2d_1}{\beta_1 \gamma_1} + \frac{d_2}{\beta_2 \gamma_2}) + \frac{1}{3} \alpha_2 \beta_2 \gamma_2 (2bK - \frac{2d_2}{\beta_2 \gamma_2} + \frac{d_1}{\beta_1 \gamma_1})$$

$$- \frac{1}{12} \alpha_1 \alpha_2 \beta_1 \beta_2 \gamma_1 \gamma_2 (2bK - \frac{2d_1}{\beta_1 \gamma_1} + \frac{d_2}{\beta_2 \gamma_2}) (2bK - \frac{2d_2}{\beta_2 \gamma_2} + \frac{d_1}{\beta_1 \gamma_1}) < 1 \quad (3.4)$$

Thus, theorem is proved.

When Eq.(3.4) is satisfied, the foreign investors and the financial institutions will go to the Nash equilibrium point $E_3$. At this point, both sides reach a stable phase where they get along well with each other, and their gains are separately: $\pi(x) = \frac{(2bK \beta_1 \beta_2 \gamma_1 \gamma_2 - 2d_1 \beta_1 \gamma_1 + d_2 \beta_2 \gamma_2)^2}{18bK^2 \beta_1^2 \beta_2^2 \gamma_1^2 \gamma_2^2} - c_1$,

$\pi(y) = \frac{(2bK \beta_1 \beta_2 \gamma_1 \gamma_2 - 2d_1 \beta_1 \gamma_1 + d_2 \beta_2 \gamma_2)^2}{18bK^2 \beta_1^2 \beta_2^2 \gamma_1^2 \gamma_2^2} - c_2$.

It is note-worthy that, the foreign merchant and the financial institution are passing through a process of dynamic gambling before they arrive at $E_3$, which is the stable point. The Chinese financial institutions are carrying out self-improvement gradually, and the foreign merchants are also shifting investment according to the profit. When the parameters influencing the stability are beyond the stabilizing region, the system will be not stable. Seen from the Eq. (3.4), foreign investor’s and financial institution’s rates of adjusting the investment $\alpha_1$, $\alpha_2$ have impact on the stability of system, but it could not affect the investment in the equilibrium point. It only influences the profit of investors in the process of going to equilibrium point. Moreover, the rates of return on investment $\gamma_1$, $\gamma_2$ (or the service efficiency of funds $\beta_1$, $\beta_2$) not only affect the stability of equilibrium point, but the investment in the equilibrium point. When $\gamma_1$ or $\gamma_2$ ($\beta_1$ or $\beta_2$) come to a useful quantity, especially $\gamma_1$, $\gamma_2$ ($\beta_1$, $\beta_2$)are both big, there will be a more fierce competition between the two investors. And then the stability may be caused to fall. On the other hand, the stable region would expand. This case will be shown from simulation in the following part. As far as variable costs $d_1$, $d_2$ are concerned, they not only affect the amount of investment and profit of their own in the equilibrium point, but
that of the other side in this investment as well. The lower the variable cost \(d_1(\text{or} d_2)\) is, the more money is invested in the equilibrium point for foreign merchant (or Chinese financial institution), and the more profit is taken by foreign merchant; but the less money is invested in the equilibrium point for Chinese financial institution (or foreign merchant), the less profit is taken by Chinese financial institution. The maximum fund capacity \(K\) in Chinese state-owned enterprise can have impact on the process of investment. The investment of foreign merchant and Chinese financial institution is increasing with the growing of \(K\), when the other parameters have no change, so does the profit. It shows that the bigger fund capacity of Chinese state-owned enterprise is, the more money can be absorbed, i.e. Chinese state-owned enterprise could attract more capital, if it has enough room for development.

4 Numerical simulations

To begin with, we research the stability of the system. We get the parameters: \(b = 4, k = 6, \beta_1 = \beta_2 = 1, \gamma_1 = \gamma_2 = 0.4, d_1 = 0.9, d_2 = 1.1\), and then we simulate the stable region in the equilibrium point, as Fig. 1(a). When \(\gamma_1\) rises from 0.4 to 0.6 by number and the others do not change, as we can see from Fig. 1(b), the stable area will be reduced in the direction of \(\alpha_1\). If we get \(\gamma_1 = \gamma_2 = 0.6\) and the others do not change, there will be the reduction of stable region further, as Fig.1(c). If the changes manifested in \(\beta_1, \beta_2\), it is similar to the situation above as that of \(\gamma_1, \gamma_2\). Lake of space forbid further treatment on \(\beta_1, \beta_2\) and simulation figures. In summation, the more \(\gamma_1, \gamma_2 (\beta_1, \beta_2)\) are, the less the stability of the system is. However, both sides of investment are always making great efforts to increase the service efficiency of funds throughout the investment, which can cause the reduction of stable zone in this case. So the level-headed investment circumstance can be built by Chinese government with controlling the rates of return on investment for Chinese state-owned enterprise on a proper occasion.

![Figure 1: Stable region in the equilibrium point when\(\gamma_1 = \gamma_2 = 0.4\)](image)

Because the focal point to which the investors pay the most attention is profit, we study the parameters how to influence it secondly. When we get the parameters: \(b = 4, k = 6, \beta_1 = \beta_2 = 1, \gamma_1 = \gamma_2 = 0.4, \alpha_1 = \alpha_2 = 0.1, d_1 = 0.9, d_2 = 1.1\); \(c_1 = 1.0\); \(c_2 = 0.8\), and initial value \(x(0) = 1.5\); \(y(0) = 1.2\), we get the investments of foreign merchants’ and Chinese financial institutions’ through system (2.3), and then put this value into system (2.2), in the end, their profit are gained respectively. Through the computer simulation, the changing situations of their investment and profit are separately demonstrated, as Fig. 2(a).

While \(\gamma_2\) reduces from 0.4 to 0.3 and other parameters are invariable, as Fig. 2(b) shows, the profit of Chinese financial institution will fall, for the rate of return on investment has a direct impact on profit. Seen from Fig. 2(c), the profit will be falling with the reducing of fund service efficiency, while \(\beta_2\) reduces from 1.0 to 0.8 and other parameters are invariable. So this is the essential reason why both sides of investment try to leapfrog the service efficiency of fund. When \(\alpha_2\) is reduced to 0.06 and others are unchanged, we get Fig. 2(d). Although the amount of investment in equilibrium point is irrelevant with parameters \(\alpha_1, \alpha_2\), the reduction of it not only brings on the decline of one’s profit, but can prosper the other. So both sides of investment make great efforts to improve rates of adjusting the investment. However, if \(\alpha_1, \alpha_2\) are too high, chaos will arise in the system.
Figure 2: The profit of foreign enterprises and Chinese financial institutions

In order to know the system better, we evaluate the parameters to simulate the evolvement of system. When we get the parameters: $b = 4, k = 6, \beta_1 = \beta_2 = 1, \gamma_1 = \gamma_2 = 0.4, \alpha_1 = 0.1, d_1 = 0.9, d_2 = 1.1, c_1 = 1.0; c_2 = 0.8$ and initial value $x(0) = 1.5; y(0) = 1.2$, the profit and investments of foreign merchant and Chinese financial institution are gotten, as Fig. 3(a) and Fig. 3(b) show. If the rate of adjusting the investment gradually increases, and reaches a certain degree, this system will pass through doubles periodicity to the chaos. However, the investors would not like to see the phenomenon of chaos, because not only the investors could not predict the exact amount of investment, but also the chaos of profit would give the investors bad impact, which may be as seriously as trapping the investors’ operation into crisis, even bankruptcy. The state-owned enterprises at present need a large amount of capital to back themselves developing steadily during this reformative process, when state-owned properties are withdrawn gradually. On the other hand, the chaotic phenomena, which may bring about excessive or insufficient investments, will inevitably hinder the steady and healthy development of Chinese state-owned enterprises. If this kind of phenomenon take place in China in a large scale, the national economy, after China’s entry into the World Trade Organization, would be impacted badly, even end in financial crisis. Therefore, Chinese government and enterprises exert their efforts to limit the system into the stable zone. Lack of space, we will discuss how to control the chaos in another paper.

5 Conclusions

China will carry out what she had promised to do after entering into the WTO, and the state will gradually give up its holding shares in some industries. In this case, it will stimulate the drastic competition among the investors both home and abroad, while the State-owned assets are withdrawing gradually. However, information about the investors and Chinese state-owned enterprise in mostly incomplete, and the investment decision-making could not be completely rational but bounded rationality only. On this basis, we set up a dynamics model with bounded rationality to describe a situation that both foreign enterprises and domestic financial institutions are investing in domestic enterprises and the duopoly game with bounded rationality is brought into financial field. The research uses the nonlinear theory to analyze the stability of the singular
point in this model, figure out its stable region. What’s more, it points out the change of each factor in
the system may influence the stability of investment, and chaos would be caused if the rate of adjusting
investment are too high. In the course of discussion on this paper, this model can better explain each
situation faced by China after WTO, so we try to offer academic reference for this act of investing in reality
and avoid financial crisis through analyzing and evolving these cases in this system.

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