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Effects of Thermal Radiation and Magnetic Field on Unsteady Mixed Convection Flow and Heat Transfer Over a Porous Stretching Surface

E. M.A.Elbashbeshy[1] *, D.A.Aldawody[2]
[1]Department of Mathematics, Faculty of Science, Ain-Shams University, Cairo, Egypt
[2]Department of Mathematics, Faculty of Science, Port Said, Suez Canal University, Egypt
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Abstract: The aim of this paper is to investigate the effects of thermal radiation and magnetic field on unsteady boundary layer mixed convection flow and heat transfer problem from a vertical porous stretching surface. The fluid is assumed to be viscous and incompressible. The governing partial differential equations are transformed into a system of ordinary differential equations. Numerical solutions of these equations are obtained with the help of shooting method. The solution is found to be dependent on the governing parameters including the Prandtl number, unsteadiness parameter, mixed convection parameter, magnetic parameter, suction parameter and thermal radiation parameter. Comparison of numerical results is made with previously published results under the special cases, and found to be in good agreement.

Keywords: unsteady flow; boundary layer flow; stretching porous surface; mixed convection; magnetic field; thermal radiation

1 Introduction

The problem of flow and heat transfer over a continuously moving surface find numerous and wide-ranging applications in industrial manufacturing processes, such as wire drawing, glass fiber production, and many others. Since the pioneering work of Sakiadis [1], various aspects of the problem have been investigated by many authors. There has been a great deal of the work done on Newtonian fluid flow and heat transfer over a stretching surface, but only a few recent studies are cited here. Elbashbeshy [2] examined the effects of injection and suction on the heat transfer from stretching surface with variable surface heat flux. Chen [3] carried out an analysis to study mixed convection flow over a stretching surface. Nazar et al. [4] presented the development of the two-dimensional boundary layer flow in the region of stagnation point on a stretching surface. All the above mention studies consider the steady-state problem. But in certain practical problems, the motion of the stretched surface may start from the rest. In these problems, the transient or unsteady aspects become more interesting .The unsteady heat transfer problems over a stretching surface, which is started impulsively from rest or is stretched with a velocity that depends on time, are considered. Elbashbeshy and Bazid [5], presented an exact similarity solution for unsteady momentum and heat transfer flow whose motion is caused solely by the linear stretching of a horizontal stretching surface . Andersson et al. [6] presented a new similarity solution for the temperature field is devised, which transforms the time-dependent thermal energy equation to an ordinary differential equation.

In recent years, numerous investigation have been conducted on the magneto-hydrodynamic flows and heat transfer because of its important applications in metallurgical industry. In the presence of a transverse magnetic field ,the flow and heat transfer over a stretching surface have been investigated by [7-11]. All the above mention studies consider the steady state problem.

The aim of the present work is to study the effects of thermal radiation and magnetic field on unsteady mixed convection flow and heat transfer over a vertical stretching surface in the presence of wall suction.

*Corresponding author. *E-mail address*: elbashbeshy10@hotmail.com

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2 Equations of motion

Consider an unsteady two-dimensional mixed convection boundary layer flow of an incompressible viscous liquid over a vertical porous stretching surface and moving with velocity $U_w(x,t) = \frac{cx}{1-\alpha t}$ where c and $\alpha$are constants and with temperature distribution $T_w = T_\infty + \frac{T_0 cx}{2\nu(1-\alpha t)^2}$ (Andersson et al. [6]) where T_0 is a reference temperature such that $0 \le T_0 \le T_w$. Here the stretching surface is subjected to such amount of tension which does not alter the structure of the porous material. The x-axis is taken along the stretching surface in the direction of motion. The y-axis is perpendicular to it. A uniform magnetic field of strength B_0 is applied normal to the stretching surface which produces magnetic effect in the x-axis .The fluid is assumed to be gray, emitting and absorbing radiation, but non-scattering medium the Rosseland approximation is used to describe the radiative heat flux in the energy equation. The radiative heat flux in the x-direction is considered negligible in comparison to the y-direction. The continuity, momentum and energy equations governing such type of flow are written as

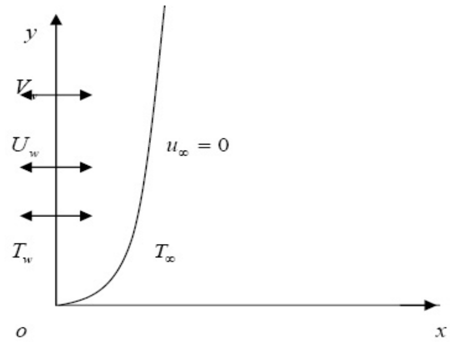

Figure 1: Physical model and coordinate system.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty) - \frac{\sigma B_0^2}{\rho}u, \tag{2}$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho c_p}\frac{\partial q_r}{\partial y}. \tag{3}$$

along with the boundary conditions

$$\begin{aligned} &u = U_w, v = V_w, T = T_w \quad at \;\; y = 0, \\ &u \to 0, T \to T_\infty \quad at \;\; y \to \infty, \end{aligned} \tag{4}$$

where u and v are the velocity components along x and y axes, respectively, t is the time, $V_w = -\sqrt{\frac{c\nu}{1-\alpha t}}$ is the velocity of suction ($V_w > 0$), ν is the kinematics viscosity and β is the volumetric coefficient of thermal expansion. g is the gravity field, B_0 is the uniform magnetic field, σ is the electrical conductivity, c_p is the specific heat at constant pressure, ρ is the density, T is the temperature, T_∞ is the temperature for away from the stretching surface, k is the coefficient of thermal conductivity of the fluid *and* q_r the radiation heat flux. By using Rosseland diffusion approximation for radiation $q_r = -\frac{4\sigma_s}{3k^*}\frac{\partial T^4}{\partial y}$ where σ_s is the Stefan-Boltzman constant and k^* is the absorption coefficient. We assume that the temperature difference within the flow is such that T^4 may be expanded in a Taylor's series. Expanding T^4 about T_∞ and neglecting higher orders we get $T^4 = 4T_\infty^3 T - 3T_\infty^4$.
Now Eq. (3) becomes

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = (\frac{k}{\rho c_p} + \frac{16\sigma_s T_\infty^3}{3\rho c_p k^*})\frac{\partial^2 T}{\partial y^2}. \tag{5}$$

The equation of continuity is satisfied if we choose a stream function $\psi(x,y)$ such that

$$u = \frac{\partial \psi}{\partial y}, v = -\frac{\partial \psi}{\partial x} \tag{6}$$

We now introduce the similarity variable η and the dimensionless variables f and θ as follows:

$$\begin{array}{l}\sqrt{\frac{c}{\nu(1-\alpha t)}}y,\\ \psi(x,y)=\sqrt{\frac{c\nu}{1-\alpha t}}xf(\eta), and \quad \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty}.\\ T=T_\infty+T_0[\frac{cx}{2\nu(1-\alpha t)^2}]\theta(\eta).\end{array} \tag{7}$$

With the help of the above relations, the governing equations finally reduce to

$$f'''+ff''-f'^2-A(f'+\tfrac{1}{2}\eta f'')-\lambda\theta-Mf'=0, \tag{8}$$

$$\theta''+\tfrac{\mathrm{Pr}}{(1+R)}[f\theta'-f'\theta-\tfrac{A}{2}(4\theta+\eta\theta')]=0, \tag{9}$$

where the parameter that measures the unsteadiness is $A=\frac{\alpha}{c}$, $\lambda=\frac{g\beta T_0}{2\nu c}$ is the mixed convection parameter, and $M=\frac{c\sigma B_0^2}{\rho}(1-\alpha t)$ is the magnetic field parameter, $\mathrm{Pr}=\frac{\mu c_p}{k}$ is the Prandtl number (μ is the viscosity), and $R=\frac{16\sigma T_\infty^3}{3kk^*}$ is the thermal radiation parameter.

The boundary conditions are transformed to:

$$\begin{array}{ll}\eta=0: & f(0)=f_0 \quad , \quad f'(0)=1 \quad and \quad \theta=1,\\ \eta\to\infty: & f'\to 0 \quad , \quad \theta\to 0 \quad ,\end{array} \tag{10}$$

where $f_0=-V_w\sqrt{\frac{1-\alpha t}{c\nu}}$, $f_0>0$ corresponding to suction.

The physical quantity of interest in this problem is the skin friction coefficient C_f and the local Nusselt number Nu_x, which are defined as

$$\begin{array}{ll}C_f=\frac{2\mu(\frac{\partial u}{\partial y})_{y=0}}{[\rho U_w^2]} \quad , & Nu_x=-\frac{x(\frac{\partial T}{\partial y})_{y=0}}{T_w-T_\infty}\\ \frac{1}{2}C_f\sqrt{Re_x}=f''(0) \quad , & Nu_x/\sqrt{Re_x}=-\theta'(0)\end{array} \tag{11}$$

where $Re_x=\frac{u_w x}{\nu}$ is the local Reynold's number.

3 Results and discussions

Computation through employed numerical scheme has been carried out for various values of the parameters such as unsteadiness parameter A, radiation parameter R, mixed convection parameter λ, magnetic parameter M, suction parameter f_0 and Prandtl number Pr. For illustrations of the results, numerical values are plotted in the figures. In order to check the accuracy of the numerical solution a comparison of heat transfer characteristics at the surface for $A=0$ (steady-state flow), $\lambda=0$, $M=0$ and $R=0$ at different values of Prandtl number are made with that of Elbashbeshy [2] and Ishak et al. [12]. From Table 1, we note that there is a close agreement with these approaches and thus verifies the accuracy of the method used. From Table 2-3 and Fig. 2-3, we note that the skin friction coefficient $F''(0)$ increases with the increase of unsteadiness parameter A and magnetic field parameter M and decreases with mixed convection parameter λ and thermal radiation parameter R. The local Nusselt number $-\theta'(0)$ increases with the increase of unsteadiness parameter A and mixed convection parameter λ and decreases with the increase of magnetic field parameter M and thermal radiation parameter R. In Fig. 4, it is seen that with increasing suction parameter ($f_0>0$), fluid velocity is found to decrease, i.e. suction causes to decrease the velocity of the fluid in the boundary layer region. This effect acts to decrease the wall shear stress. Increase in suction causes progressive thinning of the boundary layer. The physical explanation for such a behaviour is as follows, in case of suction the heated fluid is pushed towards the wall where the buoyancy forces can act to retard the fluid due to high influence of the viscosity. this effect acts to decrease the wall shear stress. Fig.5, the temperature in the boundary layer is also decreases with increasing suction parameter ($f_0>0$) . The thermal boundary thickness decreases with suction parameter ($f_0>0$), which causes an increase in the rate of heat transfer. The physical explanation for such a behaviour is that the fluid is brought closer to the surface and reduces the thermal boundary layer thickness in case of suction. Fig. 6, present horizontal velocity profile for various values of A when $\mathrm{Pr}=0.72, R=5, M=\lambda=0.1$ and $f_0=1.5$. We observe from Fig. 6 that the velocity decreases with the increase of the unsteadiness parameter A .It is interesting to note that the thickness of the boundary decreases with increasing values of A. We observe from Fig. 7 that the temperature is found to decrease with the increase of η. Significant change in the rate of decrease of the temperature for increasing values of A is noticed .Temperature at a point of the surface decreases significantly with the increase of A i.e. rate of heat transfer increases with increasing unsteadiness parameter A. The effect of the thermal radiation parameter

R on velocity and the temperature is illustrated in Fig. 8 and Fig. 9, for the same governing parameters as specified in Fig. 4. It is found that velocity increases as the thermal radiation parameter R increases. Also, the temperature increases as thermal radiation parameter R increases. This is agreement with the physical fact that the thermal boundary layer thickness increases with increasing R. Figs. 10 and 11 present velocity profile and temperature profile for various values of mixed convection parameter λ when $\mathrm{Pr} = 0.72, R = 5, M = 0.1, A = 0.1$ and $f_0 = 1.5$. The velocity increases with increasing values of mixed convection parameter λ, while the temperature decreases with values of mixed convection parameter λ, at $\lambda = 0$ gives the result of forced convection case. The effect of the magnetic parameter M on the velocity profile is illustrated in Fig. 12 for $\mathrm{Pr} = 0.72, R = 5, \lambda = 0.1, A = 0.1$ and $f_0 = 1.5$. It is seen that the presence of magnetic field causes higher restriction to the fluid, which reduced the fluid velocity. The effect of the magnetic parameter M on the temperature profile is illustrated in Fig. 13. It is obvious that the temperature gradient at the surface decreases and accordingly the temperature and thermal boundary layer thickness increases.

Table 1: Comparison of local Nusselt number at $\lambda = M = R = 0, A = 0$ for different values of f_0 and Pr with previously published data

A	f_0	Pr	Elbashbeshy [2]	Ishak et al[12]	Present results
0	-1.5	0.72		0.4570	0.4570
0	-1.5	1		0.5000	0.5000
0	-1.5	10		0.6452	0.6452
0	0	0.72	0.8161	0.8086	0.8086
0	0	1	1.0000	1.0000	1.0000
0	0	10	3.7202	3.7202	3.7204
0	1.5	0.72		1.4944	1.4944
0	1.5	1		2.0000	2.0000
0	1.5	10		16.0842	16.0635

Table 2: Results of skin friction coefficient $F''(0)$ and the local Nusselt number $-\theta'(0)$ for different values of unsteadiness parameter A and magnetic parameter M at $f_0 = 1.5$, $\mathrm{Pr} = 0.72$, mixed convection parameter λ=0.1 and radiation parameter R=5

A	0		0.1		0.3	
M	$F''(0)$	$-\theta'(0)$	$F''(0)$	$-\theta'(0)$	$F''(0)$	$-\theta'(0)$
0	1.8041	0.3269	1.8748	0.3769	1.9808	0.4558
0.2	1.9093	0.3177	1.9698	0.3713	2.0637	0.4528
0.4	2.0029	0.3103	2.0559	0.3667	2.1408	0.4501

Table 3: Results of skin friction coefficient $F''(0)$ and the local Nusselt number $-\theta'(0)$ for different values of radiation parameter R and mixed convection parameter λ at $f_0 = 1.5, Pr = 0.72, M = 0.1$ and $A = 0.1$.

λ	0.1		0.2		0.3	
R	$F''(0)$	$-\theta'(0)$	$F''(0)$	$-\theta'(0)$	$F''(0)$	$-\theta'(0)$
1	1.9944	0.8827	1.9187	0.8919	1.8458	0.9002
5	1.9236	0.3739	1.7953	0.3881	1.6792	0.3994
10	1.8819	0.2432	1.729	0.257	1.594	0.2676

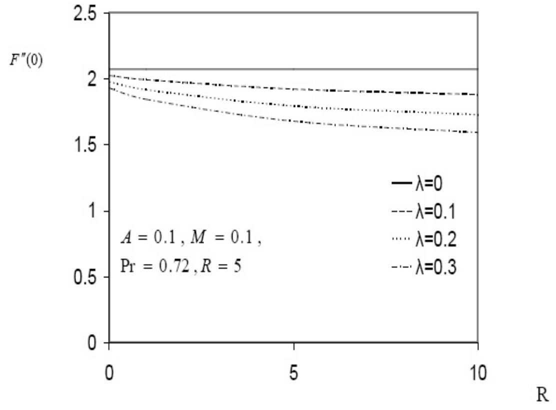

Figure 2: Variation of the skin friction coefficient as a function of radiation parameter R for various values of λ at $f_0 = 1.5$.

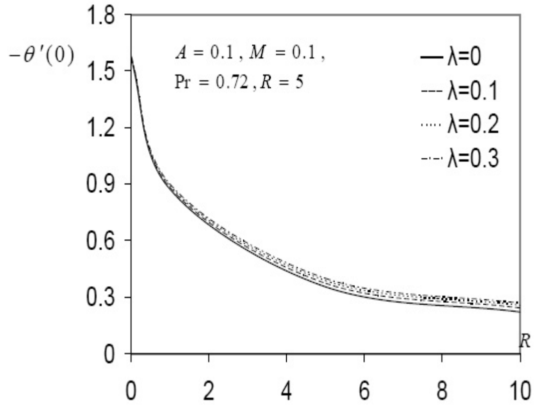

Figure 3: Variation of the local Nusselt number as a function of radiation parameter R for various values of λ at $f_0 = 1.5$.

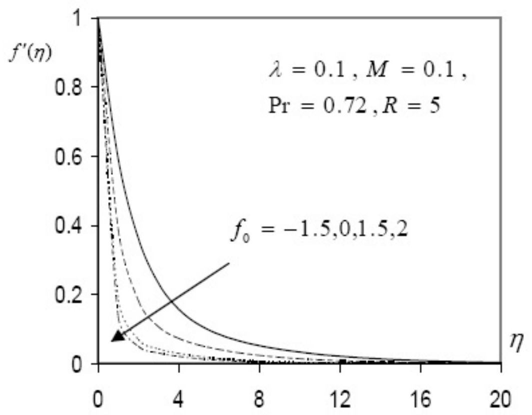

Figure 4: Velocity profiles for different values of suction/injection parameter at $A = 0.1$.

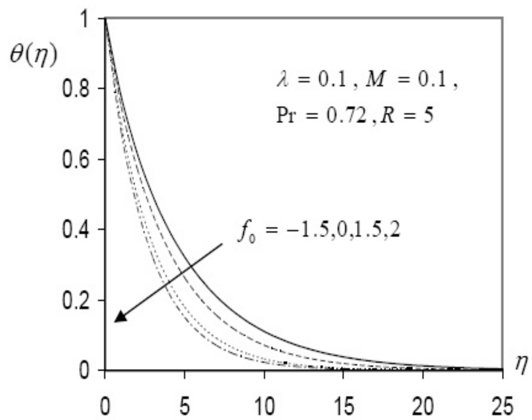

Figure 5: Temperature profiles for different values of suction/injection parameter $A = 0.1$.

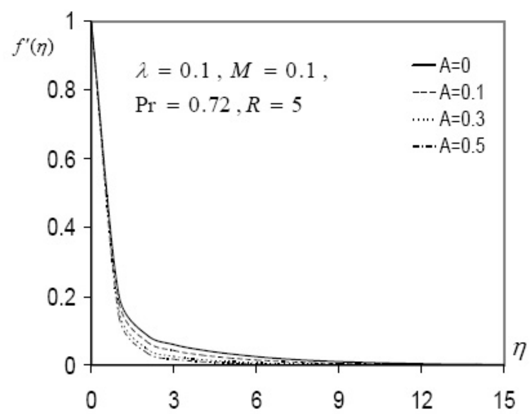

Figure 6: Velocity profiles for different values of unsteadiness parameter A at $f_0 = 1.5$.

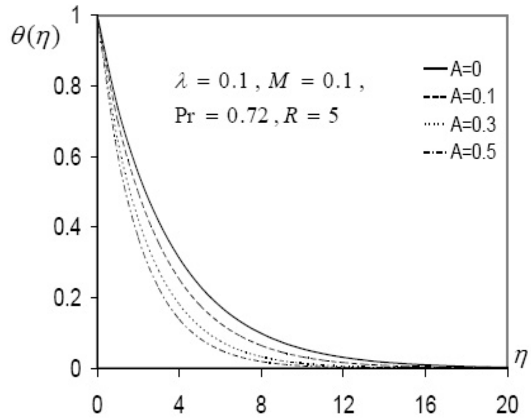

Figure 7: Temperature profiles for different values of unsteadiness parameter A at $f_0 = 1.5$.

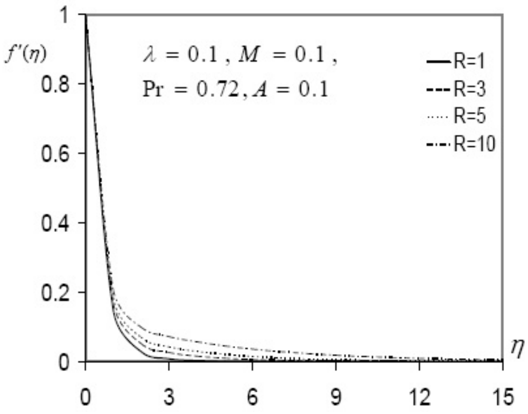

Figure 8: Velocity profiles for different values of radiation parameter R at $f_0 = 1.5$.

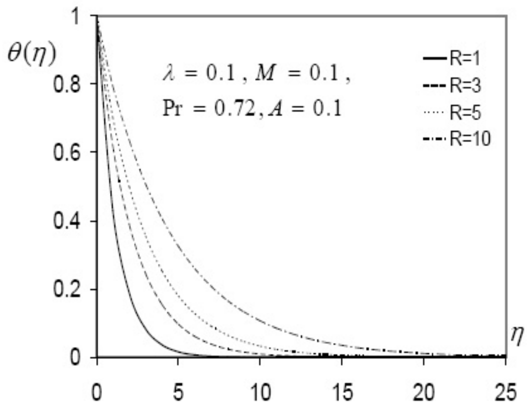

Figure 9: Temperature profiles for different values of radiation parameter R at $f_0 = 1.5$.

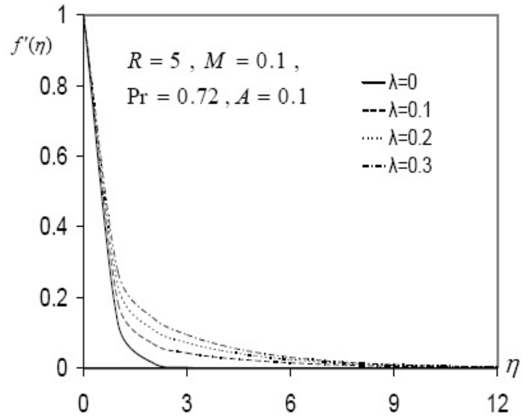

Figure 10: Velocity profiles for different values of mixed convection parameter λ at $f_0 = 1.5$.

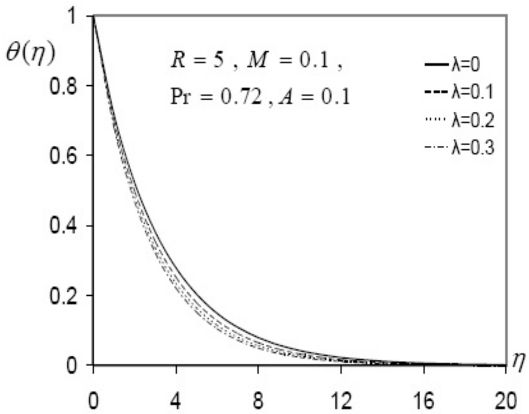

Figure 11: Temperature profiles for different values of mixed convection parameter λ at $f_0 = 1.5$.

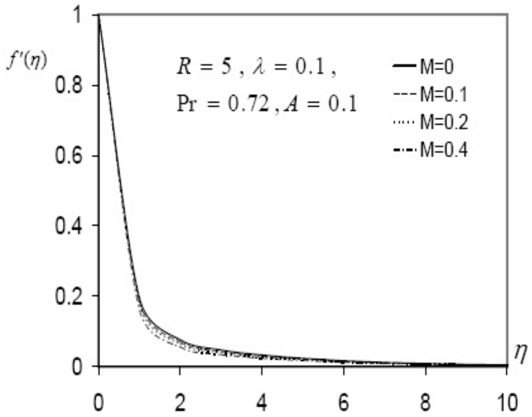

Figure 12: Velocity profiles for different values of magnetic parameter M at $f_0 = 1.5$.

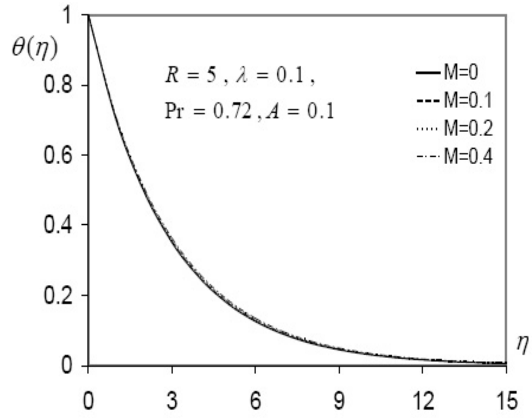

Figure 13: Temperature profiles for different values of magnetic parameter M at $f_0 = 1.5$.

4 Conclusions

The purpose of this study is to present numerical solutions of unsteady mixed convection flow and heat transfer over a porous stretching surface taking into consideration, the effects of unsteadiness parameter A, mixed convection parameter

λ, magnetic parameter M, radiation parameter R, suction parameter f_0 and Prandtl number Pr. With the help of similarity transformations, the governing time dependent boundary layer equations for momentum and thermal are reduced to couple ordinary differential equations which are then solved numerically using shooting method. The numerical solution indicated that:

1- The local Nusselt number $-\theta'(0)$ increases with suction parameter f_0, the unsteadiness parameter A and mixed convection parameter λ but decreases with the magnetic parameter M and thermal radiation parameter R.

2- The skin friction coefficient at the surface decreases with suction parameter f_0, mixed convection parameter λ and thermal radiation parameter R but increases with unsteadiness parameter A and the magnetic parameter M.

3- The velocity increases with an increase in the value of mixed convection parameter λ while decreases with increase of magnetic parameter M, suction parameter f_0 and unsteadiness parameter A.

4- The temperature decreases with an increase in the value of mixed convection parameter λ, suction parameter f_0 and unsteadiness parameter A while increases with increase of radiation parameter R.

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