

New Exact Solutions for MKdV-ZK Equation *

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Abstract: In this paper, we extended the Jacobian elliptic functions expansion method, and gave the unified form expression of the Jacobian elliptic functions form solutions. With the aid of mathematica software we obtained abundant new exact solutions of mKdV-ZK equation by using this method. These solutions are degenerated to the solitary wave solutions and the triangle function solutions in the cases when the modulus of the Jacobian elliptic functions degenerated to 1 and 0.

Keywords: mKdV-ZK equations; extended Jacobian elliptic functions expansion method; exact solutions; periodical solutions

1 Introduction

In recent years, the investigation of exact solutions to nonlinear evolution equations plays an important role in the nonlinear physical phenomena. Several powerful methods have been proposed to construct exact solutions for nonlinear partial differential equations, such as homogeneous balance method, the hyperbolic function method, the F-expansion method, inverse scattering method, projective Riccati equations method and so on [1-7]. By using these methods we obtained many valuable exact solutions for nonlinear evolution equations. In order to find more extended exact solutions for nonlinear evolution equations, Liu presented the Jacobi elliptic functions expansion method [8-9]. Due to this method can be realized with the aid of computer algebra system, it has an extensive promotion and application [10-11]. In this paper, based on previous we extended the Jacobian elliptic functions expansion method, and obtained a lot of new exact solutions of mKdV-ZK equations by using this method.

This paper is arranged as follows. In section 2, we briefly describe the new generalized Jacobian elliptic functions expansion method. In section 3, we obtain several families of solutions for mKdV-ZK equation. In section 4, some conclusions are given.

2 Method

For the given nonlinear evolution equations (NEEs) with two variables x and t

$$F(u, u_x, u_t, u_{tt}, u_{xt}, u_{xx} \dots) = 0, \quad (1)$$

where F is a polynomial with the variables u, u_t, u_x, \dots .

We make the gauge transformation

$$u(x, t) = u(\xi) \quad \xi = k(x - ct), \quad (2)$$

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where k and c are constants to be determined later. Substituting (2) into Eq.(1) yields a complex ordinary differential equation of $u(\xi)$, namely

$$F(u, u', u'', \dots) = 0, \tag{3}$$

We assume Eq. (3) has the following formal solutions :

$$u(\xi) = a_0 + \sum_{i=0}^n a_i F^i(\xi) + \sum_{i=0}^n b_i F^{i-1}(\xi) E(\xi) + \sum_{i=0}^n c_i F^{i-1}(\xi) G(\xi) + \sum_{i=0}^n d_i F^{i-1}(\xi) H(\xi), \tag{4}$$

where $a_0, a_i, b_i, c_i, d_i (i = 1, 2, \dots, n)$ are constants to be determined later. $\xi = \xi(x, t)$ is arbitrary function with the variable x and t . The positive integer n can be determined by considering the homogeneous balance between governing nonlinear terms and the highest order derivatives of u in Eq. (3). And $F(\xi), E(\xi), G(\xi), H(\xi)$ are an arbitrary array of the four functions e, f, g, h . The selection obey the principle which make the calculation more simple. Here we ansatz

$$e = e(\xi) = \frac{1}{p + j \operatorname{sn}[\xi] + r \operatorname{cn}[\xi] + s \operatorname{dn}[\xi]} \quad f = f(\xi) = \frac{\operatorname{sn}[\xi]}{p + j \operatorname{sn}[\xi] + r \operatorname{cn}[\xi] + s \operatorname{dn}[\xi]} \tag{5}$$

$$g = g(\xi) = \frac{\operatorname{cn}[\xi]}{p + j \operatorname{sn}[\xi] + r \operatorname{cn}[\xi] + s \operatorname{dn}[\xi]} \quad h = h(\xi) = \frac{\operatorname{dn}[\xi]}{p + j \operatorname{sn}[\xi] + r \operatorname{cn}[\xi] + s \operatorname{dn}[\xi]} \tag{6}$$

where p, j, r, s are arbitrary constants, the four functions e, f, g, h satisfy the following restricted relations

$$\begin{cases} e' = -jgh + rfh + sm^2fg, & f' = pgh + reh + seg, \\ g' = -pjh - jeh + s(m^2 - 1)ef, & h' = -m^2pjh - r(m^2 - 1)ef - jeg, \\ g^2 = e^2 - f^2, & h^2 = e^2 - m^2f^2 \end{cases} \tag{7}$$

where "′" denotes $\frac{d}{d\xi}$, m is the modulus of the Jacobian elliptic function ($0 \leq m \leq 1$).

Substituting (4) and (7) into Eq.(3) yields a polynomial equation for $F(\xi), E(\xi), G(\xi), H(\xi)$. Setting the coefficients of $F^i(\xi)E^j(\xi)G^k(\xi)H^s(\xi) (i = 0, 1, 2, \dots)(j = 0, 1)(k = 0, 1)(s = 0, 1)$ to zero yields a set of nonlinear algebraic equations (NAEs) in $a_0, a_i, b_i, c_i, d_i (i = 0, 1, 2, \dots, n)$ and ξ , solving the NAEs by Mathematica and Wu elimination, we obtain $a_0, a_i, b_i, c_i, d_i (i = 0, 1, 2, \dots, n)$. Substituting these results into(4), we can obtain the exact solutions of Eq.(1).

Obviously, the type (4), (5) and (6) are the general forms of the relevant expression in Ref. [9-12], if we choose special value of p, j, r and s , we can get the results in Ref. [9-12].

3 Exact solutions of the mKdV-ZK equation

Let us consider the mKdV-ZK equations with following form:

$$u_t + qu^2u_x + u_{xxx} + u_{xyy} + u_{xzz} = 0, \tag{8}$$

We introduce a gauge transformation

$$u = u(\xi) \quad \xi = k(x - ct + \lambda y + \eta z), \tag{9}$$

Substituting (9) into (8), we have

$$-cu + \frac{1}{3}qu^3 + k^2(1 + \lambda^2 + \eta^2)u'' = 0, \tag{10}$$

By balancing the governing nonlinear terms and the highest order derivatives in Eq.(10), we obtain $n = 1$. Thus we assume Eq.(8) has the following solutions

$$u(\xi) = a_0 + a_1F(\xi) + b_1E(\xi) + c_1G(\xi) + d_1H(\xi), \tag{11}$$

Substituting (7) and (11) into Eq.(10) and setting the coefficients of $F^i(\xi)E^j(\xi)G^k(\xi)H^s(\xi)$ ($i = 0, 1, 2, \dots$) ($j = 0, 1$) ($k = 0, 1$) ($s = 0, 1$) to zero yields nonlinear algebraic equations with respect to the unknowns $k, a_0, a_1, b_1, c_1, d_1$. We could determine the following solutions:

Family1:

When $s = j = 0$, we have $e = \frac{1-rq}{p}$, then select $F(\xi) = g(\xi), E(\xi) = e(\xi), G(\xi) = f(\xi), H(\xi) = h(\xi)$.

Case 1:

$$a_1 = b_1 = c_1 = 0, d_1 = \frac{\pm p\sqrt{3c}}{\sqrt{q(1+2m^4-3m^2)}}, r = \frac{\epsilon mp}{\sqrt{m^2-1}}, k_1 = \frac{\pm\sqrt{2c}}{\sqrt{(1-2m^2)(1+\eta^2+\lambda^2)}}$$

Case 2:

$$a_1 = b_1 = 0, c_1 = \frac{\pm mp\sqrt{3c}}{\sqrt{q(1+m^2)}}, d_1 = \frac{\epsilon p\sqrt{3c}}{\sqrt{q(1+m^2)}}, r = 0, k_2 = \frac{\pm\sqrt{2c}}{\sqrt{(1+m^2)(1+\eta^2+\lambda^2)}}$$

Case 3:

$$c_1 = b_1 = 0, a_1 = \pm \frac{\sqrt{-3c(r^2+m^2(p^2-r^2))}}{\sqrt{q(1-2m^2)}}, d_1 = \frac{\epsilon\sqrt{3c(p^2-r^2)}}{\sqrt{q(1-2m^2)}}, k_3 = k_1$$

Case 4:

$$b_1 = c_1 = d_1 = 0, a_1 = \pm \frac{p\sqrt{3c}}{\sqrt{q(2m^2-1)}}, r = \epsilon p, k_4 = k_1$$

Case 5:

$$b_1 = d_1 = 0, a_1 = \frac{\pm mp\sqrt{3c}}{\sqrt{q(2-m^2)}}, c_1 = \frac{mp\sqrt{3c}}{\sqrt{q(m^2-2)}}, r = 0, k_5 = \frac{\pm\sqrt{2c}}{\sqrt{(m^2-2)(1+\eta^2+\lambda^2)}}$$

where $\epsilon = \pm 1, i = \sqrt{-1}, a_0 = 0$ and p is a constant and not equal to zero.

Therefore, we obtain the following solutions to the Eq.(8).

$$u_1 = \pm \frac{\sqrt{3c}}{\sqrt{q(2m^2-1)}} \frac{dn\xi_1}{\sqrt{m^2-1} + \epsilon mcn\xi_1},$$

$$u_2 = \frac{\sqrt{3c}}{\sqrt{q(m^2+1)}} (\mp mcn\xi_2 + \epsilon dn\xi_2),$$

$$u_3 = \frac{\sqrt{3c}}{\sqrt{q(2m^2-1)}} \frac{\pm\sqrt{r^2+m^2(p^2-r^2)}sn\xi_3 + \epsilon\sqrt{r^2-p^2}dn\xi_3}{p + rcn\xi_3}$$

$$u_4 = \frac{\sqrt{3c}}{\sqrt{q(2m^2-1)}} \frac{sn\xi_4}{1 + \epsilon cn\xi_4}, u_5 = \frac{m\sqrt{3c}}{\sqrt{q}} \left(\pm \frac{cn\xi_5}{\sqrt{m^2-2}} + \frac{\epsilon sn\xi_5}{\sqrt{2-m^2}} \right)$$

where $\xi_i = k_i(x - ct + \lambda y + \eta z)$, ($i = 1, 2, 3, 4, 5$), c, λ, η are arbitrary constants.

Family2:

When $r = j = 0$, we have $h = \frac{1-pe}{s}$, then select $F(\xi) = e(\xi), E(\xi) = f(\xi), G(\xi) = g(\xi), H(\xi) = h(\xi)$.

Case 6:

$$b_1 = c_1 = d_1 = 0, a_1 = \frac{\pm m^2 p\sqrt{3c}}{\sqrt{q(2-m^2)}}, s = \epsilon p, k_6 = \frac{\pm\sqrt{2c}}{\sqrt{(m^2-2)(1+\eta^2+\lambda^2)}}$$

Case 7:

$$a_1 = b_1 = d_1 = 0, c_1 = \frac{\pm m^2 p\sqrt{3c}}{\sqrt{q(2-3m^2+m^4)}}, s = \frac{\epsilon p}{\sqrt{1-m^2}}, k_7 = k_6$$

Case 8:

$$b_1 = d_1 = 0, a_1 = \frac{\pm m\sqrt{-3c(p^2+(m^2-1)s^2)}}{\sqrt{q(m^2-2)}}, c_1 = \frac{\epsilon m\sqrt{3c(p^2-s^2)}}{\sqrt{q(m^2-2)}}, k_8 = k_6$$

where $\varepsilon = \pm 1, i = \sqrt{-1}, a_0 = 0$ and p is a constant and not equal to zero.

Therefore, we obtain the following solutions to the Eq.(8).

$$u_6 = \frac{\pm m^2 \sqrt{3c}}{\sqrt{q(2-m^2)}} \frac{sn\xi_6}{1 + \varepsilon dn\xi_6}, u_7 = \frac{\pm m^2 \sqrt{3c}}{\sqrt{q(2-m^2)}} \frac{cn\xi_7}{\sqrt{1-m^2 + \varepsilon dn\xi_7}},$$

$$u_8 = \frac{m\sqrt{3c}}{\sqrt{q(2-m^2)}} \frac{\pm \sqrt{p^2 + (m^2 - 1)sn\xi_8 + \varepsilon \sqrt{s^2 - p^2}cn\xi_8}}{p + sdn\xi_8}$$

where $\xi_i = k_i(x - ct + \lambda y + \eta z), (i = 6, 7, 8), c, \lambda, \eta$ are arbitrary constants.

Family3:

When $p = j = 0$, we have $h = \frac{1 - rg}{s}$, then select $F(\xi) = g(\xi), E(\xi) = e(\xi), G(\xi) = f(\xi), H(\xi) = h(\xi)$.

Case 9:

$$a_1 = c_1 = d_1 = 0, b_1 = \frac{\pm(m^2 - 1)\sqrt{3c}}{\sqrt{q(1 + m^2)}}, r = \varepsilon ms, k_9 = \frac{\pm\sqrt{2c}}{\sqrt{(1 + m^2)(1 + \eta^2 + \lambda^2)}}$$

Case 10:

$$c_1 = d_1 = 0, a_1 = \frac{\pm\sqrt{3c(m^2 - 1)(r^2 - m^2s^2)}}{\sqrt{q(1 + m^2)}}, b_1 = \frac{\varepsilon\sqrt{3c(m^2 - 1)(r^2 - s^2)}}{\sqrt{q(1 + m^2)}}, k_{10} = k_9$$

Case 11:

$$a_1 = d_1 = 0, b_1 = \frac{\pm s\sqrt{3c(m^2 - 1)}}{\sqrt{q(2m^2 - 1)}}, c_1 = \frac{\varepsilon ms\sqrt{3c}}{\sqrt{q(2m^2 - 1)}}, r = 0, k_{11} = \frac{\pm\sqrt{2c}}{\sqrt{(2m^2 - 1)(1 + \eta^2 + \lambda^2)}}$$

Case 12:

$$a_1 = b_1 = 0, c_1 = \frac{\pm s\sqrt{3c(1 + m^2)}}{\sqrt{2q}}, d_1 = \frac{\varepsilon s\sqrt{3c}}{\sqrt{q}}, r = \frac{\varepsilon s\sqrt{1 + m^2}}{\sqrt{2}}, k_{12} = ik_9$$

where $\varepsilon = \pm 1, i = \sqrt{-1}, a_0 = 0$ and s is a constant and not equal to zero.

Therefore, we obtain the following solutions to the Eq.(8).

$$u_9 = \frac{\pm(m^2 - 1)\sqrt{3c}}{\sqrt{q(1 + m^2)}} \frac{1}{\varepsilon mcn\xi_9 + dn\xi_9},$$

$$u_{10} = \frac{\sqrt{3c(m^2 - 1)}}{\sqrt{q(1 + m^2)}} \frac{\pm\sqrt{r^2 - m^2s^2}sn\xi_{10} + \varepsilon\sqrt{r^2 - s^2}}{rcn\xi_{10} + sdn\xi_{10}},$$

$$u_{11} = \frac{\sqrt{3c}}{\sqrt{q(2m^2 - 1)}} \frac{\pm\sqrt{m^2 - 1} + \varepsilon mcn\xi_{11}}{dn\xi_{11}}, u_{12} = \frac{\sqrt{3c}}{\sqrt{q}} \frac{sn\xi_{12} + \varepsilon dn\xi_{12}}{\varepsilon\sqrt{1 + m^2} + \sqrt{2}dn\xi_{12}},$$

where $\xi_i = k_i(x - ct + \lambda y + \eta z), (i = 9, 10, 11, 12), c, \lambda, \eta$ are arbitrary constants.

Family4:

When $s = p = 0$, we have $g = \frac{1 - jf}{r}$, then select $F(\xi) = f(\xi), E(\xi) = e(\xi), G(\xi) = g(\xi), H(\xi) = h(\xi)$.

Case 13:

$$a_1 = b_1 = c_1 = 0, d_1 = \frac{\pm rm\sqrt{3c}}{\sqrt{q(2 - m^2)}}, j = \varepsilon r\sqrt{m^2 - 1}, k_{13} = \frac{\pm\sqrt{2c}}{\sqrt{(m^2 - 2)(1 + \eta^2 + \lambda^2)}}$$

Case 14:

$$b_1 = c_1 = 0, a_1 = \frac{\pm r\sqrt{3c(1 - m^2)}}{\sqrt{q(m^2 - 2)}}, d_1 = \frac{\varepsilon r\sqrt{3c}}{\sqrt{q(2m^2 - 1)}}, j = 0, k_{14} = \frac{\pm\sqrt{2c}}{\sqrt{(1 - 2m^2)(1 + \eta^2 + \lambda^2)}}$$

Case 15:

$$a_1 = c_1 = 0, b_1 = \frac{\pm\sqrt{3c(j^2 - (m^2 - 1)r^2)}}{\sqrt{q(2 - m^2)}}, d_1 = \frac{\epsilon\sqrt{3c(j^2 + r^2)}}{\sqrt{q(2 - m^2)}}, j = 0, k_{15} = k_{13}$$

Case 16:

$$a_1 = c_1 = d_1 = 0, b_1 = \frac{\pm mr\sqrt{3c}}{\sqrt{q(m^2 - 2)}}, j = \epsilon ir, k_{16} = k_{13}$$

where $\epsilon = \pm 1, i = \sqrt{-1}, a_0 = 0$ and r is a constant and not equal to zero.

Therefore, we obtain the following solutions to the Eq.(8).

$$u_{13} = \frac{\pm m\sqrt{3c}}{\sqrt{q(2 - m^2)}} \frac{dn\xi_{13}}{\epsilon\sqrt{m^2 - 1}sn\xi_{13} + cn\xi_{13}},$$

$$u_{14} = \frac{\sqrt{3c}}{\sqrt{q}} \left(\frac{\pm\sqrt{1 - m^2}sn\xi_{14}}{\sqrt{m^2 - 2}cn\xi_{14}} + \frac{\epsilon dn\xi_{14}}{\sqrt{2m^2 - 1}cn\xi_{14}} \right),$$

$$u_{15} = \frac{\sqrt{3c}}{\sqrt{q(2 - m^2)}} \frac{\pm\sqrt{j^2 - (m^2 - 1)r^2} + \epsilon\sqrt{j^2 + r^2}dn\xi_{15}}{j sn\xi_{15} + rcn\xi_{15}},$$

$$u_{16} = \frac{\pm m\sqrt{3c}}{\sqrt{q(m^2 - 2)}} \frac{1}{\epsilon isn\xi_{16} + cn\xi_{16}},$$

where $\xi_i = k_i(x - ct + \lambda y + \eta z)$, ($i = 13, 14, 15, 16$), c, λ, η are arbitrary constants.

Family5:

When $s = r = 0$ we have $e = \frac{1 - jf}{p}$, then select $F(\xi) = f(\xi), E(\xi) = e(\xi), G(\xi) = g(\xi), H(\xi) = h(\xi)$.

Case 17:

$$a_1 = b_1 = 0, c_1 = \frac{\sqrt{3c(m^2 p^2 - j^2)}}{\sqrt{q(1 + m^2)}}, d_1 = \frac{\epsilon\sqrt{3c(p^2 - j^2)}}{\sqrt{q(1 + m^2)}}, k_{17} = k_9$$

Case 18:

$$a_1 = b_1 = d_1 = 0, c_1 = \frac{\pm p\sqrt{3c(m^2 - 1)}}{\sqrt{q(1 + m^2)}}, j = \epsilon p, k_{18} = k_9$$

Case 19:

$$a_1 = b_1 = c_1 = 0, d_1 = \frac{\pm p\sqrt{3c(1 - m^2)}}{\sqrt{q(2m^2 - 1)}}, j = \epsilon mp, k_{19} = k_9$$

where $\epsilon = \pm 1, i = \sqrt{-1}, a_0 = 0$ and p is a constant and not equal to zero.

Therefore, we obtain the following solutions to the Eq.(8).

$$u_{17} = \frac{m\sqrt{3c}}{\sqrt{q(1 + m^2)}} \frac{\pm\sqrt{m^2 p^2 - j^2}cn\xi_{17} + \epsilon\sqrt{p^2 - j^2}dn\xi_{17}}{p + j sn\xi_{17}},$$

$$u_{18} = \frac{\pm\sqrt{3c(m^2 - 1)}}{\sqrt{q(m^2 + 1)}} \frac{cn\xi_{18}}{1 + \epsilon sn\xi_{18}}, u_{19} = \frac{\pm\sqrt{3c(1 - m^2)}}{\sqrt{q(2m^2 - 1)}} \frac{dn\xi_{19}}{1 + \epsilon m sn\xi_{19}},$$

where $\xi_i = k_i(x - ct + \lambda y + \eta z)$, ($i = 17, 18, 19$), c, λ, η are arbitrary constants.

Family6:

When $p = r = 0$, we have $h = \frac{1 - jf}{s}$, then select $F(\xi) = f(\xi), E(\xi) = e(\xi), G(\xi) = g(\xi), H(\xi) = h(\xi)$.

Case 20:

$$b_1 = d_1 = c_1 = 0, a_1 = \frac{\pm m^2 s\sqrt{6c}}{\sqrt{q(2m^2 - 1)}}, j = 0, k_{20} = \frac{\sqrt{c}}{\sqrt{(2m^2 - 1)(1 + \eta^2 + \lambda^2)}}$$

Case 21:

$$a_1 = d_1 = 0, b_1 = \frac{\pm s\sqrt{3c}}{\sqrt{2q(1 - 2m^2)}}, c_1 = \frac{\epsilon s\sqrt{3c}}{\sqrt{2q(2m^2 - 1)}}, j = \frac{\epsilon s\sqrt{1 - 2m^2}}{\sqrt{2}}, k_{21} = k_{14}$$

where $\varepsilon = \pm 1$, $i = \sqrt{-1}$, $a_0 = 0$ and s is a constant and not equal to zero.

Therefore, we obtain the following solutions to the Eq.(8).

$$u_{20} = \frac{\pm m^2 \sqrt{6c}}{\sqrt{q(2m^2 - 1)}} \frac{sn\xi_{20}}{dn\xi_{20}}, u_{21} = \frac{\sqrt{3c}}{\sqrt{q(1 - 2m^2)}} \frac{\pm 1 + \varepsilon cn\xi_{21}}{\varepsilon \sqrt{1 - 2m^2} sn\xi_{21} + \sqrt{2} dn\xi_{21}},$$

where $\xi_i = k_i(x - ct + \lambda y + \eta z)$, ($i = 20, 21$), c, λ, η are arbitrary constants.

When the module of the Jacobian elliptic function $m \rightarrow 1$ or $m \rightarrow 0$, these solutions degenerated to the solitary wave solutions and the triangle function solutions.

4 Conclusion

By using this extended Jacobian elliptic functions method we obtained many new exact solutions of mKdV-ZK equation, for example $u_3, u_8, u_{10}, u_{15}, u_{17}$. By using our method all results of Ref.[12] can be obtained. The method in this article contained traditional Jacobian elliptic functions method. This method applies to a wider range. By using this method we can get abundant solutions of nonlinear evolution equations.

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