ISSN 1749-3889 (print), 1749-3897 (online)
International Journal of Nonlinear Science
Vol.8(2009) No.3,pp.313-317

Adaptive Control of the Complex Network

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(Received 18 February 2009, accepted 15 September 2009)

Abstract: This paper studies the synchronization of a complex dynamical network. Adaptive controllers have been designed to make the dynamical network synchronize. Analytical results show that the states of the dynamical network can globally asymptotically synchronize onto a desired orbit under the designed controllers. A numerical example is given to demonstrate the validity of the proposed method, in which the famous Lorenz system is chosen as the node of the network.

Keywords: Synchronization; Adaptive control; complex dynamical networks

1 Introduction

Complex networks have attracted increasing attention from various fields of science and engineering [1] since the pioneering work of Watts and Strogatz [2], Barabási and Albert [3]. In general, a complex network is a large set of interconnected nodes, where a node is fundamental unit with detailed contents. In the past two decades the synchronization in large-scale complex networks has been a focus in various fields of science and engineering [4-10]. Many researchers have investigated the problem of controlling complex dynamical networks [11-18]. In [12] and [16], Wang and Chen pinned a complex dynamical network to its equilibrium by using local injections. Ref [6] showed several robust adaptive controllers for complex dynamical network with unknown but bounded nonlinear couplings. In [20] Zhou et al. investigated adaptive synchronization of uncertain complex dynamical networks. In [18], Li et al. introduced a linear state feedback controller to synchronize a complex network to a desired orbit. In most of these studies, the controllers have been designed based exactly on the information of the networked systems containing network topology, coupling strength, and output function of the isolated node.

Inspired by the above discussions, in this paper, adaptive method is employed to realize the synchronization of complex network and efficient controller is designed which ensures the states of each node to reach the desired manifold. Our results reveal that synchronization can be achieved by the proposed scheme.

The rest of the paper is organized as following. In Sec. 2, the coupled dynamical network model is introduced, and some necessary preliminaries are given. Then, in Sec. 3, an adaptive controller is designed, and synchronization is proved by construction of Lyapunov-Krasovskii functional. In Sec. 4, a numerical example is given to verify the theoretical results. Finally, conclusion remarks are given in Sec. 5.

2 Network model and preliminaries

Suppose a complex network consists of N identical adaptively coupled nodes, with each node being an n-dimensional dynamical system. The state equation of this dynamical network are given by

$$\dot{x_i} = f(x_i) + c\sum_{j=1}^{N} G_{ij}x_j, \quad i = 1, 2, ..., N, \tag{1}$$

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IJNS.2009.12.15/283

where $f(\cdot) \in R^n$is a given nonlinear vector valued function describing the dynamic of the nodes, $x_i = (x_{i1}, x_{i2}, ..., x_{in}) \in R^n$are the state variables of node i, the constant $c > 0$ represents the coupling strength; $G = (G_{ij})_{N\times N}$ is the coupling configuration matrix representing the topological structure of the network, in which G_{ij} is defined as follows: if there is a connection between node $i$and $j\,(j \neq i)$, then $G_{ij} = G_{ji} > 0$; otherwise, $G_{ij} = G_{ji} = 0\,(i \neq j)$, and the diagonal elements of matrix G are defined by

$$G_{ii} = -\sum_{\substack{j=1 \\ j\neq i}}^{N} G_{ij}, \quad i = 1, 2, ..., N \tag{2}$$

Suppose that the network (1) is connected in the sense that there are no isolated clusters. Then G is a symmetric and irreducible matrix, its eigenvalues are $0 = \lambda_1 > \lambda_2 \geq \lambda_3 \geq ... \geq \lambda_N$[19].

Our goal is to synchronize the network (1) onto a desired evolution, i.e.,

$$\lim_{t\to\infty} \|x_i(t) - s(t)\| = 0, i = 1, 2, ..., N, \tag{3}$$

where $\|\cdot\|$ stands for the Euclidean vector norm, synchronous evolution $s(t)$ is an arbitrary desired state, which is also an isolated node of the network (1) such that $\dot{s}(t) = f(s(t))$.

Through out the Letter, we have the following hypothesis:

Hypothesis1. For $i = 1, 2, ..., N$, there exists non-negative constants γ_i such that

$$\|f(x_i) - f(s)\| < \gamma_i \|x_i - s\|. \tag{4}$$

3 Adaptive controller design

To achieve the control goal (3), we shall add the following simple controllers

$$u_i = k_i x_i \tag{5}$$

to the network nodes, and then we obtain the following controlled dynamical network:

$$\dot{x}_i = f(x_i) + c\sum_{j=1}^{N} G_{ij}x_j + k_i x_i \tag{6}$$

$$\dot{k}_i = -\partial_i \|x_i(t) - s(t)\|^2, \quad i = 1, 2, ..., N, \tag{7}$$

where $\partial_i > 0$, $i = 1, 2, ..., N$,are arbitrary constants.

Define

$$e_i(t) = x_i(t) - s(t),$$

Then we write down the error equation of Eq. (6)

$$\dot{e}_i = f(x_i) - f(s) + c\sum_{j=1}^{N} G_{ij}e_j + k_i e_i. \tag{8}$$

It is obvious that the stability of the synchronous solution $s(t)$ of the network (6) is equivalent to the stability of the errors e_i for all $i = 1, 2, ..., N$ about its zero solution. And we have the following result.

Theorem 1 *The complex dynamical network (6) globally asymptotically synchronizes with the desired evolution $s(t)$ under the adaptive controllers (5) and (7).*

Proof. Selecting a Lyapunov-Krasovskii function as

$$V = \frac{1}{2}\sum_{i=1}^{N} e_i^T e_i + \frac{1}{2}\sum_{i=1}^{N} \frac{1}{\partial_i}(k_i + l)^2 , \tag{9}$$

where l is a positive constant to be determined.

Thus one gets

$$\begin{aligned} \dot{V} &= \sum_{i=1}^{N} e_i^T \dot{e}_i + \sum_{i=1}^{N} \frac{1}{\partial_i}(k_i + l)\dot{k}_i \\ &= \sum_{i=1}^{N} e_i^T \left[f(x_i) - f(s) + c\sum_{j=1}^{N} G_{ij}e_j + k_i e_i \right] - \sum_{i=1}^{N} (k_i + l)\|e_i\|^2 \\ &= \sum_{i=1}^{N} e_i^T \left[f(x_i) - f(s) + c\sum_{j=1}^{N} G_{ij}e_j \right] - l\sum_{i=1}^{N} \|e_i\|^2 \end{aligned}$$

the maximal eigenvalues of quadric form $\sum_{i=1}^{N} e_i^T \sum_{j=1}^{N} G_{ij}e_j$ is zero, and by Hypothesis 1,we have

$$\begin{aligned} \dot{V} &\le \gamma\sum_{i=1}^{N} e_i^T e_i - l\sum_{i=1}^{N} \|e_i\|^2 \\ &= (\gamma - l)\sum_{i=1}^{N} \|e_i\|^2 \le 0 \end{aligned}$$

if the constant $\gamma > l$, where $\gamma = \max\{\gamma_i\}$. It is obvious that $\dot{V} = 0$ if and only if $e_i = 0$ for $i = 1, 2, ...N$. The orbits of the network (8) are globally asymptotically stable at $e_i = 0$. That is, the states of nodes in coupled complex dynamical network (6) converge to the desired evolution $s(t)$ under the adaptive controllers (5) and (7). ■

4 Illustrative examples

In this section, a numerical example will be given to demonstrate the effectiveness of the proposed method in the previous section. We consider an example of the controlled dynamical network (6), with the Lorenz system as a node of coupled network. The number of nodes is chosen as$N = 3$. A single Lorenz system, as the desired orbit, is described by

$$\dot{s}_1 = a(s_2 - s_1), \dot{s}_2 = cs_1 - s_1 s_3 - s_2, \dot{s}_3 = s_1 s_2 - bs_3 \tag{10}$$

where $a = 10$, $b = \frac{8}{3}$, $c = 28$.

The state equations of the network as (1) are given by

$$\begin{cases} \dot{x}_{i1} = 10(x_{i2} - x_{i1}) + c\sum_{j=1}^{3} G_{ij}x_j + k_1 x_1 \\ \dot{x}_{i2} = 28x_{i1} - x_{i2} - x_{i1}x_{i3} + c\sum_{j=1}^{3} G_{ij}x_j + k_2 x_2 \\ \dot{x}_{i3} = -\frac{8}{3}x_{i3} + x_{i1}x_{i2} + c\sum_{j=1}^{3} G_{ij}x_j + k_3 x_3 \end{cases} \tag{11}$$

and

$$\dot{k}_i = -\partial_i \|x_i\|^2 , \quad i = 1, 2, 3.$$

The synchronization error signals are $e_i = x_i - s$, $e_i = (e_{i1}, e_{i2}, e_{i3})^T$, $i = 1, 2, 3$. Then the synchronization error system of network (12) is given:

$$\begin{cases} \dot{e}_{i1} = 10\left(e_{i2} - e_{i1}\right) + c\sum\limits_{j=1}^{3} G_{ij}e_j + k_1e_1 \\ \dot{e}_{i2} = 28e_{i1} - e_{i2} - x_{i1}x_{i3} + s_1s_3 + c\sum\limits_{j=1}^{3} G_{ij}e_j + k_2e_2 \\ \dot{e}_{i3} = -\frac{8}{3}e_{i3} + x_{i1}x_{i2} - s_1s_2 + c\sum\limits_{j=1}^{3} G_{ij}e_j + k_3e_3 \end{cases} \tag{12}$$

and

$$\dot{k}_i = -\partial_i \left\| e_i \right\|^2 \quad i = 1, 2, 3. \tag{13}$$

In the following simulations, we choose the coupling matrix G is as follows:

$$G = \begin{pmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 0 & 1 & -1 \end{pmatrix}.$$

Since the Lorenz attractor is bounded, Hypothesis 1 holds. According to Theorem 1, the network (11) can be synchronized by applying the adaptive controllers (5) and (7). Fig. 1 shows the error evolutions under the designed controllers. The coupling strength is chosen as$c = 0.1$,∂_1=∂_2=$\partial_3 = -3$ and $k_1(0) = -1$,$k_2(0) = -3$,$k_3(0) = -4$. We can clearly see that the states of the network (11) globally asymptotically synchronize with the states of the desired orbit (**??**).

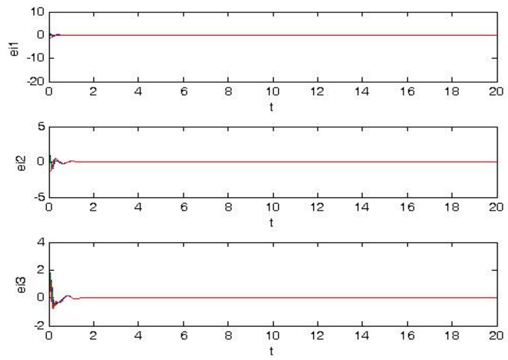

Fig.1. Synchronization errors $e_i(t) = (e_{i1}, e_{i2}, e_{i3})$ between the state $s(t) = (s_1, s_2, s_3)$ and the state $x_i = (x_{i1}, x_{i2}, x_{i3})\,(i = 1, 2, 3)$ of the network (11).

5 Conclusion

The synchronization in adaptive coupled dynamical network is studied in this paper. A roust adaptive controller has been designed to ensure the global asymptotical synchronization with an desired orbit. By construction of Lyapunov-Krasovskii functional, we obtain the analytical results that global synchronization of the controlled networks can be achieved under the designed adaptive controllers. The Lorenz chaotic system is chosen as the node of dynamical network in the numerical simulation. And the example shows the effectiveness of our method.

6 Acknowledgement

Research was supported by the National Nature Science Foundation of China (No: 10771088) and Foundation for Advanced Researchers of Jiangsu University (No: 06JDG080)

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