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Slip Boundary Layer Flow of non-Newtonian Fluid over a Flat Plate with Convective Thermal Boundary Condition

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Abstract: The flow and heat transfer of a power-law fluid over a flat plate with convective thermal and slip boundary conditions is studied. A similarity transformation is used to reduce the system of coupled boundary layer equations into a system of non-linear ordinary differential equations. These equations are solved numerically by the RKF45(Runge-Kutta-Fehlberg fourth-fifth) method. Solutions showing the effects of the Knudsen number(Kn_x) and heat transfer(ϕ) parameters on the velocity(f'), temperature(θ), wall skin friction ($f''(0)$) and Nusselt number(N_u) for some values of power-law index(n) have been obtained. The solutions are discussed with the aid of tables and graphs.

Keywords: Power-law fluid, flat plate, boundary layer flow, convective thermal condition, slip boundary

1 Introduction

In response to the pioneering papers of Sakiadis [1], several attempts for further developments in flow and heat transfer analysis have been reported in literature[[2]-[12]]. In recent years, the study of boundary layer flows of non-Newtonian fluids has increased considerably due to their relevance in scientific and technological applications such as oil recovery, material processing, soil, ceramics, lungs and kidney. In all these situations, one or more extensive quantities are transported through the solid and/or the fluid phases that together occupy a medium[3,9,10,11].

The important experiment by Beavers and Joseph(Nield, 2009) established that when a fluid flows in a parallel plate porous channel, then a velocity slip at the porous wall is proportional to the wall velocity gradient. These observations have led to many publications in non-Newtonian heat and mass transfer, especially the pseudoplastic fluids[$8-16$].

The purpose of this present work is to extend the flow and heat transfer analysis to power-law fluids in a boundary layer over a flat plate using a combination of slip boundary condition and the convective thermal boundary conditions[6]. We examined the effect of the Knudsen number(Kn_x) and the convective heat transfer parameter(ϕ) on the temperature (θ), velocity(f'), wall skin friction($f''(0)$), the wall temperature($\theta(0)$) and the wall heat transfer($\theta'(0)$) for various power-law index(n), pseudoplastic($0 < n < 1$) and Newtonian fluids($n = 1$). The case of dilatant fluids($n > 1$) is omitted because of space limitations and also because they are pratically less common[12].

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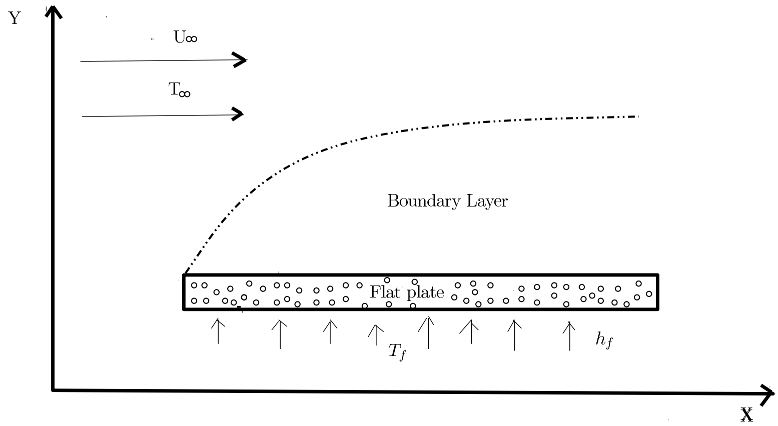

Figure 1: Sketch of the physical model

2 The Problem formulation

A power law fluid is a type of generalized Newtonian fluid given by(Ostwald-de Waele rheological model)

$$\tau = \mu\left(\frac{du}{dy}\right) = \mu_0\left(\frac{du}{dy}\right)^n, \quad \text{where} \quad \mu = \mu_0\left(\frac{du}{dy}\right)^{n-1}, \tag{1}$$

where n is the power-law index. The schematic diagram of the boundary layer flows over a flat plate in a stream of cold fluid at temperature T_∞ moving over the top surface of the flat plate with a uniform velocity U_∞ is as shown in Figure 1. Thus, the dimensional continuity, momentum and energy equations describing the flow can be written as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{2}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right)^n + g\beta(T - T_\infty) - \frac{\sigma B_0^2}{\rho}u, \tag{3}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2} + \frac{Q}{\rho c_p}(T - T_\infty) - \frac{1}{\rho c_p}\frac{\partial q_r}{\partial y}. \tag{4}$$

The flow velocity boundary conditions associated with this problem can be expressed as

$$u(x,0) = \frac{2 - F_M}{F_M}\lambda\frac{\partial u}{\partial y}, \quad v(x,0) = 0 \quad \text{and} \quad u(x,\infty) = U_\infty. \tag{5}$$

Similarly, assuming that the flat plate is heated from below by a hot fluid whose temperature is maintained at T_f, with heat transfer coefficient h_f, then the boundary condition at the plate surface and beyond the boundary layer may be written as

$$-k\frac{\partial T}{\partial y}(x,0) = h_f\left[T_f - T(x,0)\right] \quad \text{and} \quad T(x,\infty) = T_\infty, \tag{6}$$

where u and v are velocities along the x-axis(along the plate) and the y-axis(normal to the plate) components respectively, T is the temperature, ν is the kinematic viscosity of the fluid, and k is the thermal diffusivity of the fluid, q_r is the radiative heat flux, ρ is the density and c_p is the specific heat capacity, Q is the heat source coefficient, B_0 is the magnetic field strength, σ is the electric conductivity, β is the coefficient of thermal expansion, g is the acceleration due to gravity, F_M is the momentum accomodation factor and U_∞ is the velocity outside the boundary area. It is obvious that the momentum equation accounts for natural convection and the presence of magnetic field, while the energy equation accounts for the heat and radiative sources. By using the Rosseland approximation for radiation, the radiative heat flux may be simplified to be

$$q_r = -\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y}, \tag{7}$$

where σ^* and k^* are the Stefan-Boltzmann constant and the mean absorption coefficient, respectively. By expressing the term T^4 as a linear function of temperature using the Taylor series expansion about T_∞ and neglecting higher-order terms, we get

$$q_r = -\frac{16\sigma^* T_\infty^3}{3k^*}\frac{\partial T}{\partial y}. \tag{8}$$

Equations (1) - (7) can be made dimensionless by introducing a similarity variable η and a dimensionless stream function $f(\eta)$ defined as

$$\begin{aligned} &\eta = y\left(\tfrac{U_\infty^{2-n}}{\nu x}\right)^{\frac{1}{n+1}} = \tfrac{y}{x} Re_x{}^{\frac{1}{n+1}},\ u = U_\infty \tfrac{\partial f}{\partial \eta} = U_\infty f'(y), \\ &v = \tfrac{1}{n+1} x^{\frac{-n}{n+1}} \left(U_\infty^{2n-1}\nu\right)^{\frac{1}{n+1}} \left(\eta f'(y) - f(y)\right), \quad \theta(\eta) = \tfrac{T-T_w}{T_\infty - T_w}. \end{aligned} \tag{9}$$

Thus, the dimesionless ordinary differential equations and boundary conditions become

$$n(f'')^{n-1} f'''(\eta) + \frac{1}{n+1} f(\eta) f''(\eta) + G_r\theta(\eta) - Mf'(\eta) = 0, \tag{10}$$

$$\left(1 + \frac{4}{3}R_d\right)\theta''(\eta) + \frac{1}{n+1}P_r f(\eta)\theta'(\eta) + \gamma\theta(\eta) = 0, \tag{11}$$

$$f(0) = 0,\ \ f'(0) = \alpha K f''(0),\ \ f'(\infty) = 1, \tag{12}$$

and

$$\theta(\infty) = 0, \quad \theta'(0) = -\phi\left[1 - \theta(0)\right], \tag{13}$$

where

$$Re_x = \frac{x^n U_\infty^{2-n}}{\nu},\ G_r = \frac{g\beta(T_\infty - T_w)x}{U_\infty^2},\ \ M = \frac{\sigma B_0^2 x}{\rho U_\infty},\ \ R_d = \frac{4\sigma^* T_\infty}{kk^*}, \gamma = \frac{Qx}{\rho c_p U_\infty},$$

$$\phi = \frac{h_f}{k}\sqrt{\frac{\nu x}{U_\infty}}, Pr = \frac{\nu\rho c_p}{kx}\left(\frac{\nu x}{U_\infty^{2-n}}\right), Kn_x = \frac{\lambda}{x},\ \ K = Kn_x Re_x^{-1/2},\ \ \text{and}\ \ \alpha = \frac{F_M - 2}{F_M}.$$

The dimensionless quantities, G_r is the Grashoff number, P_r is the Prandtl number, M is the magnetic parameter, Kn_x is the Knudsen number, α is the ratio of accomodation factor, ϕ is the heat transfer coefficient, γ is the heat source parameter and R_d is the radiation parameter.

The physical quantities of most interest in such problems are the skin friction coefficient and Nusselt number. The shearing stress on the surface is defined by

$$\tau_w = \mu \left.\frac{\partial u}{\partial y}\right|_{y=0}. \tag{14}$$

Thus, the skin friction coefficient is defined by

$$C_f = \frac{2\tau_w}{\rho U_\infty} = 2Re_x^{\frac{-1}{n+1}}\left[f''(0)\right]^n, \tag{15}$$

while the local Nusselt number for heat transfer is defined by

$$Nu = \frac{\nu q_w}{k(T_\infty - T_w)} = -\nu x^{-1} Re_x^{\frac{1}{n+1}}\theta'(0), \tag{16}$$

where the heat flux at the wall is given by $q_w = -k\frac{\partial T}{\partial y}|_{y=0}$.

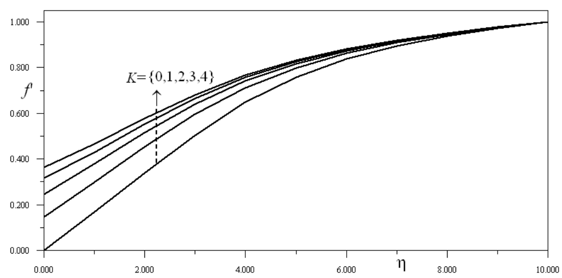

Figure 2: $f'(\eta)$ vs η, $Rd = 0$ and $\phi = 1$.

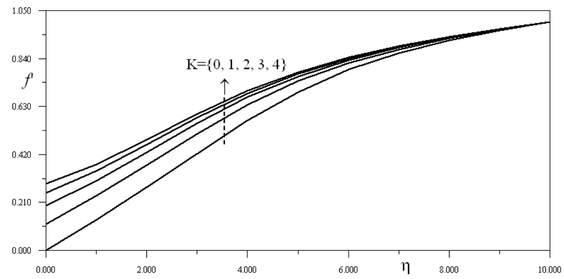

Figure 3: $f'(\eta)$ vs η, $Rd = 10$, $\phi = 1$

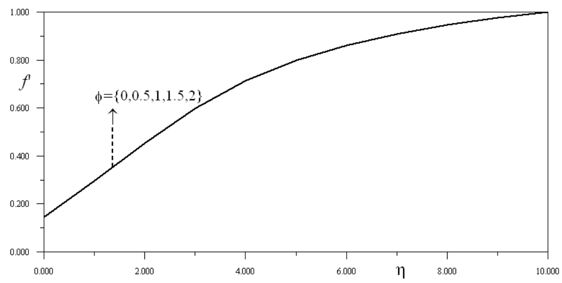

Figure 4: $f'(\eta)$ vs η, $Rd = 0$ and $K = 1$.

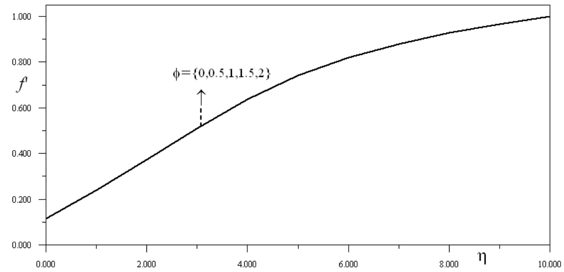

Figure 5: $f'(\eta)$ vs η, $Rd = 10$, $K = 1$

3 Results and Conclusion

In an attempt to solve the systems of equations $(10) - (13)$, we replace $\eta = \infty$ by $\eta = 10$, and use the symbolic Maple 12 software package. The software has RFK45 integration procedure built into it. The accuracy robustness of the method is established as we have recovered some known results. For example, the results of Cortell [7] for the constant temperature boundary condition was obtained by replacing $\phi = \infty$ by $\phi = 1000$, and for $P_r = 5$ and $\theta'(0)$=0.57667. Furthermore, we also recovered the results of Aziz [6] to a very high accuracy.

It is shown in Figures 2 and 3 that velocity increases monotonically with the Knusen number(K); however, in the presence of radiation($Rd = 10$), it appears to reduce the velocity considerably. On the contrary, Figures 4 and 5, show that the velocity is not influenced by the heat transfer parameter(ϕ). The plots of temperature profiles($\theta(\eta)$) against η are shown in Figures $6 - 9$. Although, θ decreases as K increases in Figures 6 and 7, in the presence of radiation($Rd = 10$), θ has higher values compared with the values when radiation is absent. Similarly, Figures 8 and 9 show that θ decreases monotonically with heat transfer parameter, and the presence of radiation leads to a further reduction in the temperature. The behaviour of solutions as the fluid changes from pseudoplastic to Newtonian is shown in Figures 10 and 11. The velocity($f'(\eta)$) decreases with the power index(n), while the temperature($\theta(\eta)$) increases with n. This behaviour is in line with the physics of the problem since pseudoplastic fluids have a lower apparent viscosity at higher shear rates[12].

Tables $1 - 4$ show the variations of wall skin friction($f''(0)$), temperature($\theta(0)$) and the temperature gradient($\theta'(0)$) with respect to K and ϕ. Tables 1 and 3 show that $f''(0)$ decreases with K, whereas no remarkable change is recorded in the variation of ϕ(Tables 2 and 4). It is also observed that the pseudoplastic fluids exhibit higher wall skin friction than the Newtonian fluids. The behaviour of the wall temperature with varying K and ϕ are also shown. Tables 1 and 3 show that the wall temperature($\theta(0)$) decreases with K, while Tables 2 and 4 show that in the absence of radiation($Rd = 0$), $\theta(0)$ increases with the heat transfer parameter(ϕ) for pseudoplastic fluid($n = 0.5$), whereas the reverse is the case for the Newtonian fluid($n = 1$). The behaviour of the wall temperature gradient($\theta'(0)$) with varying K and ϕ have also been observed. For $Rd \neq 0$, $\theta'(0)$ decreases with increasing K and ϕ. However, in the absence of radiation($Rd = 0$), $\theta'(0)$ fluctuates with K and decreases with increasing heat transfer parameter(ϕ).

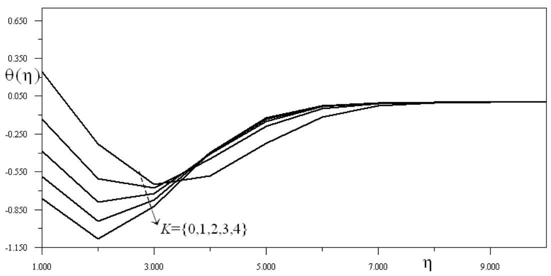

Figure 6: $\theta(\eta)$ vs η, $Rd = 0$, $\phi = 1$.

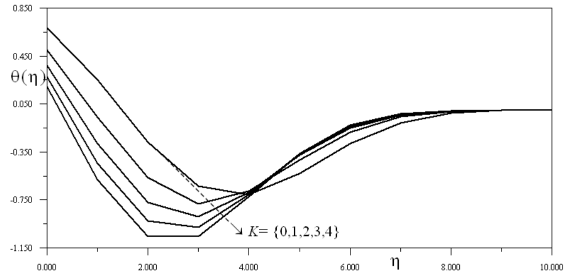

Figure 7: $\theta(\eta)$ vs η, $Rd = 10$, $\phi = 1$

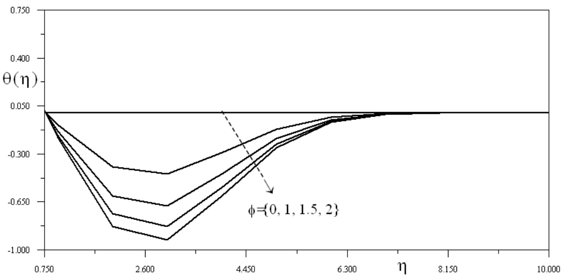

Figure 8: $\theta(\eta)$ vs η, $Rd = 0$, $K = 1$

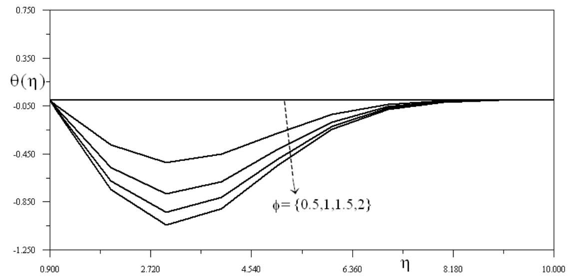

Figure 9: $\theta(\eta)$ vs η, $Rd = 10$, $K = 1$

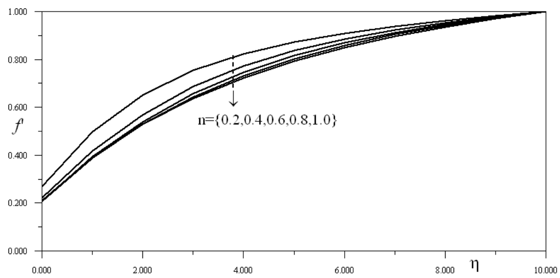

Figure 10: $f'(\eta)$ vs η, $Rd = 10$, $K = \phi = 1$

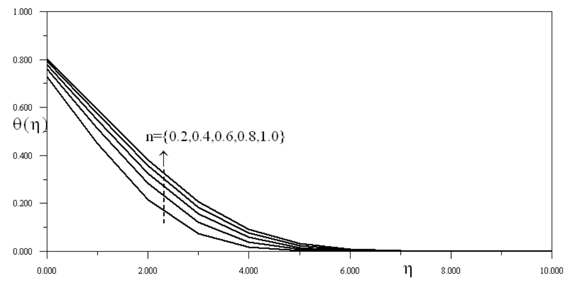

Figure 11: $\theta(\eta)$ vs η, $Rd = 10$, $K = \phi = 1$

Table 1: Values of $f''(0)$, $\theta(0)$ and $\theta'(0)$ for some values of K, $Gr = 0$, $M = \gamma = 0.1$ and $\phi = 1$,$n = 0.5$.

	$Rd = 0$, $Pr = 0.72$			$Rd = 10$, $Pr = 0.72$			$Rd = 10$, $Pr = 10$		
K	$f''(0)$	$\theta(0)$	$-\theta'(0)$	$f''(0)$	$\theta(0)$	$\theta'(0)$	$f''(0)$	$\theta(0)$	$-\theta'(0)$
0	0.165	0.703	0.297	0.165	1.039	0.039	0.165	0.832	0.168
1	0.146	0.467	0.533	0.146	1.029	0.029	0.146	0.794	0.206
2	0.123	0.310	0.690	0.123	1.023	0.023	0.123	0.773	0.227
3	0.105	0.184	0.816	0.105	1.019	0.019	0.105	0.761	0.239
4	0.091	0.074	0.926	0.091	1.017	0.017	0.091	0.753	0.247
5	0.080	-0.027	-1.027	0.080	1.016	0.016	0.080	0.747	0.253
6	0.071	-0.120	-1.120	0.071	1.014	0.014	0.072	0.742	0.257
7	0.065	-0.209	-1.209	0.065	1.013	0.013	0.065	0.739	0.261

Table 2: Values of $f''(0)$, $\theta(0)$ and $\theta'(0)$ for some values of K, $Gr = 0$, $M = \gamma = 0.1$ and $K = 1$, $n = 0.5$.

	$Rd = 0$, $Pr = 0.72$			$Rd = 10$, $Pr = 0.72$			$Rd = 10$, $Pr = 10$		
ϕ	$f''(0)$	$\theta(0)$	$-\theta'(0)$	$f''(0)$	$\theta(0)$	$\theta'(0)$	$f''(0)$	$\theta(0)$	$-\theta'(0)$
0.1	”	0.081	0.092	”	1.391	0.064	”	0.278	0.072
0.2	”	0.149	0.170	”	1.163	0.039	”	0.435	0.113
0.4	”	0.260	0.296	”	1.076	0.033	”	0.607	0.157
0.6	”	0.345	0.393	”	1.049	0.030	”	0.698	0.181
0.8	”	0.412	0.470	”	1.036	0.029	”	0.755	0.196
1.0	”	0.467	0.533	”	1.029	0.029	”	0.794	0.206
5.0	”	0.814	0.928	”	1.006	0.028	”	0.951	0.247

Table 3: Values of $f''(0)$, $\theta(0)$ and $\theta'(0)$ for some values of K, $Gr = 0$, $M = \gamma = 0.1$ and $\phi = 1$,$n = 1.0$.

	$Rd = 0$, $Pr = 0.72$			$Rd = 10$, $Pr = 0.72$			$Rd = 10$, $Pr = 10$		
K	$f''(0)$	$\theta(0)$	$-\theta'(0)$	$f''(0)$	$\theta(0)$	$\theta'(0)$	$f''(0)$	$\theta(0)$	$-\theta'(0)$
0	0.132	1.828	-0.828	0.132	1.058	0.058	0.131	0.869	0.131
1	0.099	1.092	-0.092	0.116	1.051	0.050	0.116	0.837	0.163
2	0.084	0.913	0.087	0.098	1.047	0.047	0.099	0.820	0.180
3	0.073	0.830	0.170	0.084	1.044	0.044	0.084	0.810	0.190
4	0.064	0.781	0.219	0.072	1.043	0.043	0.773	0.803	0.197
5	0.057	0.749	0.251	0.063	1.042	0.042	0.064	0.798	0.202
6	0.051	0.726	0.274	0.057	1.041	0.041	0.057	0.795	0.205
7	0.046	0.709	0.291	0.51	1.040	0.040	0.051	0.792	0.208

Table 4: Values of $f''(0)$, $\theta(0)$ and $\theta'(0)$ for some values of K, $Gr = 0$, $M = \gamma = 0.1$ and $K = 1$, $n = 1.0$.

	$Rd = 0$, $Pr = 0.72$			$Rd = 10$, $Pr = 0.72$			$Rd = 10$, $Pr = 10$		
ϕ	$f''(0)$	$\theta(0)$	$\theta'(0)$	$f''(0)$	$\theta(0)$	$\theta'(0)$	$f''(0)$	$\theta(0)$	$-\theta'(0)$
0.1	”	6.497	0.550	”	1.945	0.095	”	0.340	0.066
0.2	”	1.733	0.147	”	1.321	0.064	”	0.507	0.099
0.4	”	1.268	0.107	”	1.138	0.055	”	0.673	0.131
0.6	”	1.164	0.098	”	1.088	0.053	”	0.755	0.147
0.8	”	1.118	0.095	”	1.065	0.051	”	0.804	0.156
1.0	”	1.092	0.092	”	1.051	0.051	”	0.837	0.163
5.0	”	1.017	0.086	”	1.010	0.049	”	0.963	0.187

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