

Identification of Nonlinear Coupling Delays in Dynamical Networks via Synchronization and Control

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(Received 28 August 2008, accepted 19 February 2009)

Abstract: In this paper, chaotic dynamical networks with coupling delays are considered. These coupling delays are viewed as the unknown parameters, which need to be identified. To achieve this goal, some auxiliary systems (also called as network estimators) are designed. Linear feedback control and adaptive strategy are both applied in designing these network estimators. The sufficient conditions for the achievement of parameter identification are given for several coupling configurations. Illustrative examples are provided to show the effectiveness of this method.

Keywords: complex dynamical network; coupling delay; linear feedback control; network estimator; adaptive strategy

1 Introduction

Chaotic dynamical networks, as an interesting subject, has been thoroughly investigated for decades. These networks show very complicated behaviors and can be used to model and explain many complex systems in nature [1, 2, 6, 7], such as biological systems, economic systems, traffic systems and so on. For many real systems, we want to control the chaotic behavior to a desired state. Many control and synchronization approaches are developed to realize this aim, while most of which are based on the assumption that we know in advance all the parameters in these systems. However, not all parameters in these systems can be well determined beforehand because of various difficulties in practical applications, i.e., there may exist unknown parameters or parameter perturbations. Therefore estimating the unknown parameters in systems becomes an important topic in studying chaotic dynamical networks using observed data. There have been many methods developed to identify parameters. For examples, in [3], Yu et al suggested a method for estimating the topology structure of a network, i.e., the coupling matrix is unknown. In [5], Zhou et al identified the topologies of weighted complex dynamical networks. In [9–11], adaptive controllers were applied to identify unknown parameters. In [14], Creveling et al estimated parameter using balanced synchronization.

In many scientific and engineering fields, such as biology [12], economy [13] and so on, delay differential equations (DDEs) are often used to model complex real systems. In these systems, there also exist unknown parameters. And many effective methods have been proposed to identify these unknown parameters. In [8], Rakshit et al discussed parameter estimation of a time delay chaotic system through synchronization and a least square approach. In [4] Wu identified the topology of weighted general complex dynamical networks with time-varying coupling delay via synchronization.

In this paper, we will consider chaotic dynamical networks with unknown coupling delays. Firstly, we construct its corresponding auxiliary network (network estimator). And then, through the achievement of synchronization between the original network and its auxiliary network, we can identify these unknown delays. To reach this goal, linear feedback control and adaptive strategy are both adopted.

The rest of this paper is organized as follows. Section 2 addresses the problem formulation and studies chaotic dynamical network with single coupling delay. Section 3 considers chaotic dynamical networks

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with multi-coupling-delays. Section 4 illustrates some numerical examples to show the method is effective. Section 5 concludes the paper.

2 Problem formulation and parameter identification

Consider the following system with its state equation

$$\dot{x}_i(t) = f(x_i(t)) + \varepsilon \sum_{j=1}^N a_{ij} h(x_j(t - \tau)), \quad i = 1, 2, \dots, N, \quad (1)$$

where $x_i(t) = (x_{i1}(t), x_{i2}(t), \dots, x_{in}(t)) \in R^n$, $h : R^n \rightarrow R^n$ is inner coupling function, $A = (a_{ij})_{N \times N}$ is the adjacency matrix. If node i and node j ($i \neq j$) are connected, then $a_{ij} = a_{ji} = 1$; otherwise, $a_{ij} = a_{ji} = 0$ ($i \neq j$). The degree of node i is the number of its connections, denoted by d_i , which is defined as follows:

$$d_i = -a_{ii} = \sum_{j=1, j \neq i}^N a_{ij} = \sum_{j=1, j \neq i}^N a_{ji}, \quad i = 1, 2, \dots, N.$$

We suppose that the coupling delay τ is unknown, and the goal for us is to estimate it.

In order to estimate the delay τ in (1), we design a network estimator as follows:

$$\dot{y}_i(t) = f(y_i(t)) + \varepsilon \sum_{j=1}^N a_{ij} h(y_j(t - \hat{\tau})) + u_i, \quad i = 1, 2, \dots, N, \quad (2)$$

$$\dot{\hat{\tau}} = \gamma \sum_{i=1}^N \|y_i(t) - x_i(t)\|, \quad (3)$$

where $\hat{\tau}$ is the estimation of τ and u_i is a feedback controller. We set $\gamma > 0$, that is to say, $\hat{\tau}$ will converge to τ increasedly if these two networks are prone to synchronizing. And because of the boundedness of state spaces of above systems, we can let $\gamma \ll 1$ such that $\dot{\hat{\tau}} < \mu$, where the constant $\mu < 1$. We take the controllers as

$$u_i = -k(y_i(t) - x_i(t)), \quad i = 1, 2, \dots, N,$$

where $k > 0$ is the feedback gain. It is easy to see that the control u_i will be absent after the synchronization of above two networks. Letting $e_i(t) = y_i(t) - x_i(t)$, then we can obtain the error system as follows:

$$\begin{aligned} \dot{e}_i(t) &= f(y_i(t)) - f(x_i(t)) \\ &+ \varepsilon \sum_{j=1}^N a_{ij} [h(y_j(t - \hat{\tau})) - h(x_j(t - \tau))] + u_i, \quad i = 1, 2, \dots, N. \end{aligned} \quad (4)$$

Now, we assume that the functions f and h are Lipchitzian, i.e., there exist positive constants L_1 and L_2 such that $\|f(y) - f(x)\| \leq L_1 \|y - x\|$ and $\|h(y) - h(x)\| \leq L_2 \|y - x\|$ for all $x, y \in R^n$. Moreover, we assume that the systems considered in this paper are diffusive eventually, so the state variables are bounded, i.e., there exists positive constant L such that $\|x_i(t)\| \leq L$ for all i and t . Then, there exists positive constant L_3 such that $\|x_i(t_1) - x_i(t_2)\| \leq L_3 |t_1 - t_2|$ for all i and $t_1, t_2 \in [0, \infty)$. An useful lemma will be adopted as follow:

Lemma 1 (*J. Cao et al. [15]*) For any vectors $x, y \in R^m$ and positive definite matrix $Q \in R^{m \times m}$, the following matrix inequality holds:

$$2x^T y \leq x^T Q x + y^T Q^{-1} y$$

Then we choose the Lyapunov function as follows:

$$V = \frac{1}{2} \sum_{i=1}^N \|e_i(t)\|^2 + \frac{A}{2\gamma} (\hat{\tau} - \tau)^2 + C \sum_{i=1}^N \int_{t-\hat{\tau}}^t \|e_i(\theta)\|^2 d\theta,$$

where $A > 0$, $C > 0$ are two arbitrary positive constants. The derivative of V along with the solution of Eqs. (4) and (3) is

$$\begin{aligned} \dot{V} &= \sum_{i=1}^N e_i^T(t) \dot{e}_i(t) + \frac{A}{\gamma} (\hat{\tau} - \tau) \dot{\hat{\tau}} + C \sum_{i=1}^N \|e_i(t)\|^2 \\ &\quad - C(1 - \dot{\hat{\tau}}) \sum_{i=1}^N \|e_i(t - \hat{\tau})\|^2 \\ &= \sum_{i=1}^N e_i^T(t) \{f(y_i(t)) - f(x_i(t)) - ke_i(t) + Ce_i(t)\} \\ &\quad + A(\hat{\tau} - \tau) \sum_{i=1}^N \|e_i(t)\| - C(1 - \dot{\hat{\tau}}) \sum_{i=1}^N \|e_i(t - \hat{\tau})\|^2 \\ &\quad + \varepsilon \sum_{i=1}^N \sum_{j=1}^N a_{ij} e_i^T(t) \{h(y_j(t - \hat{\tau})) - h(x_j(t - \hat{\tau}))\} \\ &\quad + \varepsilon \sum_{i=1}^N \sum_{j=1}^N a_{ij} e_i^T(t) \{h(x_j(t - \hat{\tau})) - h(x_j(t - \tau))\}. \end{aligned}$$

From Lemma 1, we have

$$\begin{aligned} \dot{V} &\leq (N(N - 1)L_2L_3\varepsilon - A)(\tau - \hat{\tau}) \sum_{i=1}^N \|e_i(t)\| \\ &\quad + \left\{ \frac{N(N-1)}{2} L_2^2 \varepsilon - C(1 - \dot{\hat{\tau}}) \right\} \sum_{i=1}^N \|e_i(t - \hat{\tau})\|^2 \\ &\quad + (L_1 + C + \frac{N(N-1)}{2} \varepsilon - k) \sum_{i=1}^N \|e_i(t)\|^2. \end{aligned}$$

Then we can take $A \geq N(N - 1)L_2L_3\varepsilon$, $C \geq \frac{N(N-1)L_2^2\varepsilon}{2(1-\mu)}$ and $k > L_1 + C + \frac{N(N-1)}{2}\varepsilon$ such that $\dot{V} \leq 0$. Obviously, $\dot{V} = 0$ if and only if $e(t) = 0$, thus $E = \{\dot{V} = 0\} = \{e(t) = 0\}$ and the largest invariant set M contained in E is $M = \{e(t) = 0, \hat{\tau} = \tau\}$. According to LaSalle's invariance principle([16]), starting with arbitrary initial values, the trajectory asymptotically converges to the set M , i.e., $e(t) \rightarrow 0$ and $\hat{\tau} \rightarrow \tau$ as $t \rightarrow \infty$. This indicates that the coupling delay $\hat{\tau}$ of systems (2) and (3) can identify the unknown coupling delay τ of system (1). So systems (2) and (3) may be called as a network estimator.

3 Dynamical networks with multi-coupling delays

In section 2, we have considered the dynamical network with single coupling delay. In many real systems, however, there may exist multi-coupling-delays. So, in this section, we want to investigate the chaotic dynamical networks with multi-coupling-delays. Recall the system (1)

$$\dot{x}_i(t) = f(x_i(t)) + \varepsilon \sum_{j=1}^N a_{ij} h(x_j(t - \tau)), \quad i = 1, 2, \dots, N,$$

where $x_j(t - \tau) = (x_{j1}(t - \tau_1), x_{j2}(t - \tau_2), \dots, x_{jn}(t - \tau_n)) \in R^n$ in above equations. We take the network estimator as

$$\dot{y}_i(t) = f(y_i(t)) + \varepsilon \sum_{j=1}^N a_{ij} h(y_j(t - \hat{\tau})) + u_i, \quad i = 1, 2, \dots, N,$$

where $y_j(t - \hat{\tau}) = (y_{j1}(t - \hat{\tau}_1), y_{j2}(t - \hat{\tau}_2), \dots, y_{jn}(t - \hat{\tau}_n)) \in R^n$, and

$$\dot{\hat{\tau}}_j = \gamma \sum_{i=1}^N |y_{ij}(t) - x_{ij}(t)|, \quad j = 1, 2, \dots, n.$$

We choose the Lyapunov function as

$$V = \frac{1}{2} \sum_{i=1}^N \|e_i(t)\|^2 + \frac{A}{2\gamma} \sum_{j=1}^n (\hat{\tau}_j - \tau_j)^2 + C \sum_{i=1}^N \sum_{j=1}^n \int_{t-\hat{\tau}_j}^t e_{ij}^2(\theta) d\theta.$$

Similar to section 2, we also can show that $\dot{V} \leq 0$ for $A \geq N(N-1)L_2L_3\varepsilon$, $C \geq \frac{N(N-1)L_2^2\varepsilon}{2(1-\mu)}$ and $k > L_1 + C + \frac{N(N-1)}{2}\varepsilon$. Then we can identify the delays respectively.

4 Numerical examples

In this section, two simulation examples are given to show that the method is effective. We consider the following Lorenz system:

$$\begin{cases} \dot{x}_1 = a(x_2 - x_1) \\ \dot{x}_2 = cx_1 - x_1x_3 - x_2 \\ \dot{x}_3 = x_1x_2 - bx_3 \end{cases}$$

When $a = 10$, $b = \frac{8}{3}$, $c = 28$, it has a chaotic attractor. We discuss a network with 9 nodes. Its topological structure is a simple-center network, the dynamics of each node is modeled by the above Lorenz system and the coupling function $h(\cdot)$ is identity mapping, i.e.,

$$\dot{x}_i(t) = f(x_i(t)) + \varepsilon \sum_{j=1}^9 a_{ij}x_j(t - \tau), \quad i = 1, 2, \dots, 9 \quad (5)$$

where ε is the coupling strength, and

$$A = (a_{ij})_{9 \times 9} = \begin{pmatrix} -8 & 1 & 1 & \cdots & 1 \\ 1 & -1 & 0 & \cdots & 0 \\ 1 & 0 & -1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \\ 1 & 0 & 0 & \cdots & -1 \end{pmatrix}.$$

Firstly, we consider the dynamical network with simple coupling delay. Then its associated network estimator is

$$\dot{y}_i(t) = f(y_i(t)) + \varepsilon \sum_{j=1}^9 a_{ij}y_j(t - \hat{\tau}) - k(y_i(t) - x_i(t)), \quad i = 1, 2, \dots, 9 \quad (6)$$

$$\dot{\hat{\tau}} = \gamma \sum_{i=1}^9 \|y_i(t) - x_i(t)\| \quad (7)$$

In order to simulate, we choose $\tau = 0.8$, $\varepsilon = 0.6$ in (5). We take $k = 50$, $\gamma = 0.04$ in (6) and (7), Fig.1 shows the parameter identification of single coupling delay. Secondly, we consider the dynamical network with multi-coupling-delays, i.e., $\tau = (\tau_1, \tau_2, \tau_3)$ in (5). Then its associated network estimator is

$$\dot{y}_i(t) = f(y_i(t)) + \varepsilon \sum_{j=1}^9 a_{ij}h(y_j(t - \hat{\tau})) - k(y_i(t) - x_i(t)), \quad i = 1, 2, \dots, 9 \quad (8)$$

$$\dot{\hat{\tau}}_j = \gamma \sum_{i=1}^9 |y_{ij}(t) - x_{ij}(t)|, \quad j = 1, 2, 3 \quad (9)$$

In order to simulate, we choose $\tau_1 = 0.6$, $\tau_2 = 0.8$, $\tau_3 = 1.0$, $\varepsilon = 0.6$ in (5). We take $k = 50$, $\gamma = 0.01$ in (8) and (9). Fig.2 shows the parameter identification of multi coupling delays.

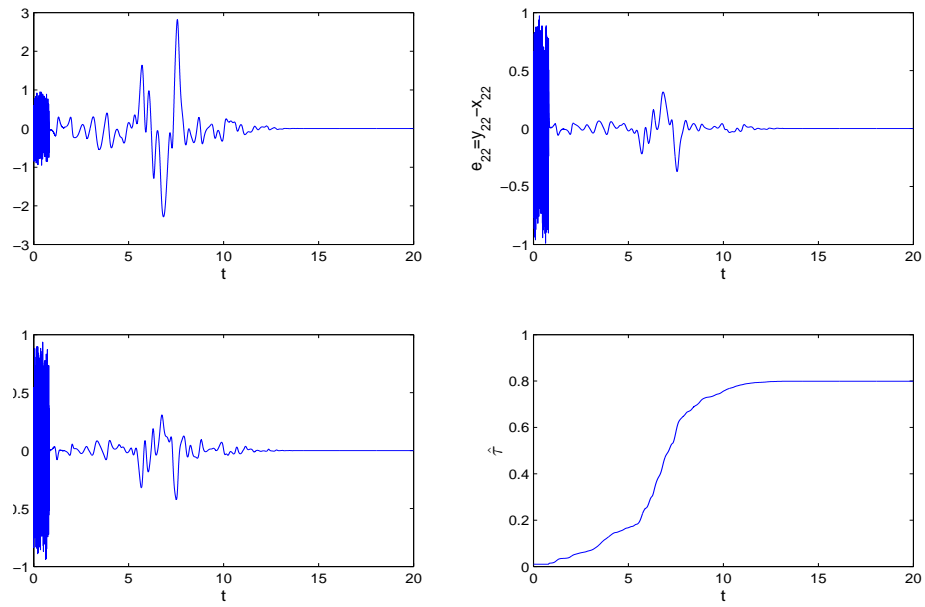


Figure 1: Single coupling delay.

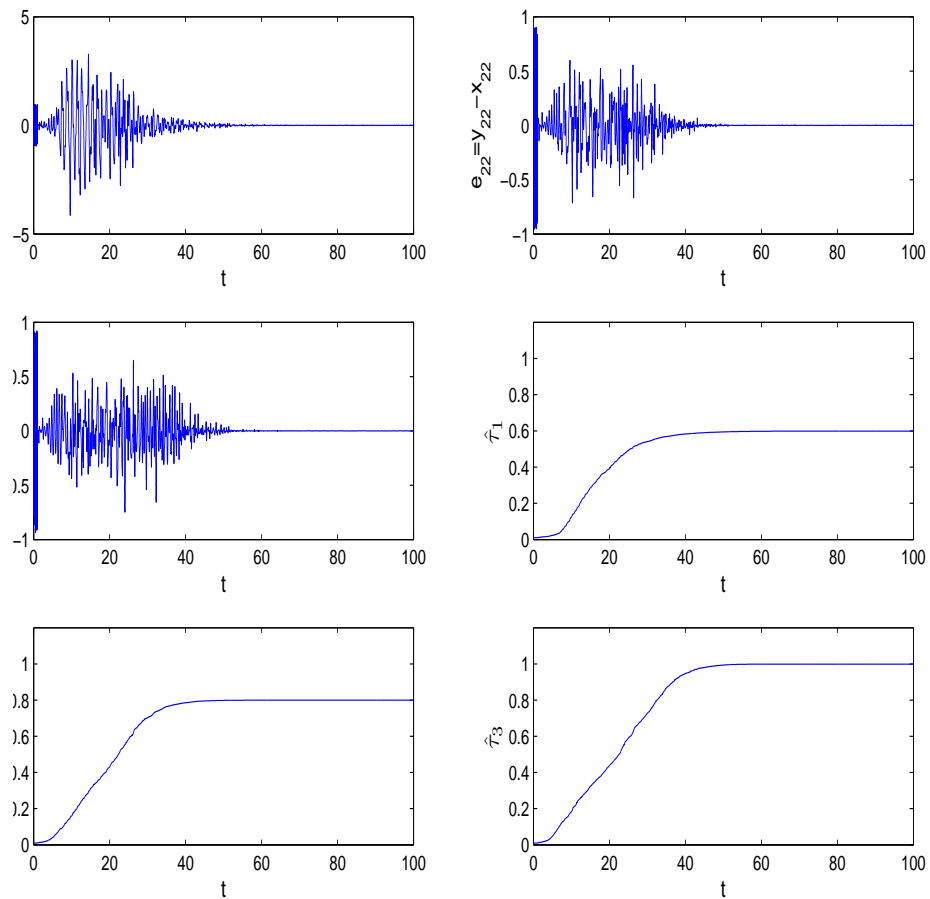


Figure 2: Multi coupling delays.

5 Conclusions

In this paper, we have considered the identification of coupling delays in chaotic dynamical networks with single coupling delay or with multi coupling delays via designing corresponding network estimators using feedback control and adaptive strategy. We have obtained the sufficient conditions for the identification, and given two numerical examples for single coupling delay and multi coupling delays to verify the effectiveness of the proposed method.

Acknowledgement

This research was jointly supported by the NSFC grant 10672146 and the Shanghai Leading Academic Discipline Project, Project Number: S30104.

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