

Adaptive Synchronization for an Uncertain New Hyperchaotic Lorenz System

Jiang Xu¹, Guoliang Cai², Song Zheng^{2*}

¹School of Mathematics and Physics, Jiangsu university of Science and Technology,
Zhenjiang Jiangsu 212003, China

² Faculty of science, Jiangsu University, Zhenjiang Jiangsu 212013, China

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Abstract: Synchronization of an uncertain new hyperchaotic Lorenz system is studied in this paper. Based on Lyapunov stability theory and adaptive synchronization method, an adaptive control law and a parameter update rule for unknown parameters are given for self-synchronization of the hyperchaotic Lorenz systems. In addition, the synchronization of the hyperchaotic Lorenz system at speed with the unified hyperchaotic Rössler system is implemented. Numerical Simulations show the effectiveness of the presented methods.

Keywords: Hyperchaotic Lorenz system ; Hyperchaotic Lorenz system ; Lyapunov stability theory ; Adaptive synchronization

1 Introduction

Since Pecora and Carroll [1] introduced a method to synchronize two identical chaotic systems with different initial conditions, chaos synchronize has attracted a great deal of attention from various fields including secure communication, information science, chemical reactions, biological systems and some other fields. In the past two decades, many approaches and techniques have been proposed for the synchronization of chaotic systems such as PC method [1], OGY method [2], time-delay feedback approach [3], adaptive synchronization [4, 5], coupled synchronization [6, 7], and backstepping design technique [8, 9]. However, to our best knowledge, most of the methods mentioned above and many other existing synchronization methods mainly concern the synchronization of chaotic systems with low dimensional attractors, characterized by one positive Lyapunov exponent. This feature limits the complexity of chaotic dynamics. It is believed that chaotic systems with higher dimensional attractors have much wider application. In fact, the adopting of higher dimensional chaotic systems has been proposed for secure communication and the presence of more than one positive Lyapunov exponent clearly improves the security of communication scheme by generating more complex dynamics.

Recently, many authors have studied the control and synchronization for the hyperchaotic systems. Wang and Liu [10, 11] studied the synchronizations of hyperchaotic Lorenz systems by passive controllers. Jia et al. [12] designed two state feedback adaptive controllers to synchronize two identical hyperchaotic L systems. Wu et al. [13] proposed a scheme to synchronize hyperchaotic Chen system and generalized Henon-Heiles System of different structures via adaptive control.

In this paper synchronization of an uncertain new hyperchaotic Lorenz system is studied via adaptive control, and design of an adaptive control law and an update rule for parameters based on Lyapunov stability theory. The rest of the paper is organized as follows. Section 2 gives a brief description of the new hyperchaotic Lorenz system, in Section 3, we present chaos synchronization between two identical hyperchaotic Lorenz system via adaptive control, Section 4, we present chaos synchronization between the hyperchaotic

* Corresponding author. E-mail address: xjiang2008@yahoo.com.cn

Lorenz system and the hyperchaotic Rössler system via adaptive control, concluding remark is given in Section 5 finally.

2 Systems description

The new hyperchaotic system [14] is given by

$$\begin{cases} \dot{x} = a(y - x) + w \\ \dot{y} = cx - y - xz \\ \dot{z} = xy - bz \\ \dot{w} = -yz + rw \end{cases} \quad (1)$$

where x, y, z, w are state variables, and a, b, c, r are real constants.

When $a = 10, b = 8/3, c = 28$ and $r = -1$, system (1) is hyperchaotic and has two positive Lyapunov exponents $\lambda_1 = 0.3381$ and $\lambda_2 = 0.1586$. Some of the phase plane strange attractors of system (1) are show in Fig.1, the others can be seen in [14].

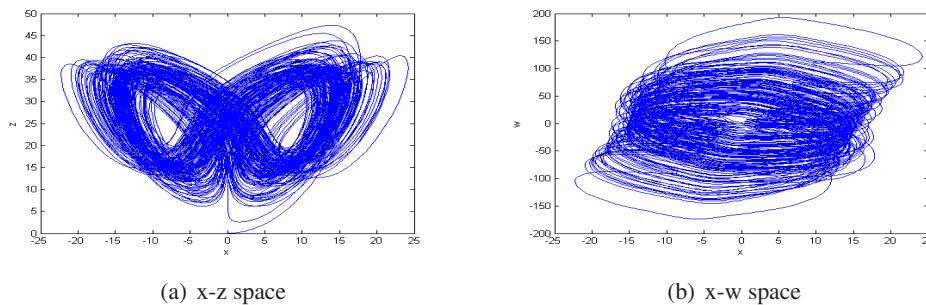


Figure 1: Hyperchaotic attractors of system (1)

3 Synchronization of two identical hyperchaotic Lorenz systems

In this section, based on the Lyapunov stability theory and adaptive control theory, synchronization between two identical hyperchaotic systems with uncertain parameters is achieved.

Let (1) be the drive system and the response systems be given as follow

$$\begin{cases} \dot{x} = a_1(y_1 - x_1) + w_1 + u_1 \\ \dot{y} = c_1x_1 - y_1 - x_1z_1 + u_2 \\ \dot{z} = x_1y_1 - b_1z_1 + u_3 \\ \dot{w} = -y_1z_1 + r_1w_1 + u_4 \end{cases} \quad (2)$$

where a, b, c, r are uncertain parameters of the drive system, and a_1, b_1, c_1, r_1 are uncertain parameters, which need to be estimated in the response system, u_1, u_2, u_3, u_4 are the controllers which are to be designed such that two hyperchaotic systems can be synchronized.

Subtracting the drive system (1) from the response system (2), we obtain the following error dynamical system

$$\begin{cases} \dot{e}_1 = e_a(y_1 - x_1) + a(e_2 - e_1) + e_4 + u_1 \\ \dot{e}_2 = e_cx_1 + ce_1 - e_2 - x_1e_3 - ze_1 + u_2 \\ \dot{e}_3 = -e_bz_1 - be_3 + x_1e_2 + ye_1 + u_3 \\ \dot{e}_4 = -z_1e_2 - ye_3 + e_rw_1 + re_4 + u_4 \end{cases} \quad (3)$$

where $e_1 = x_1 - x, e_2 = y_1 - y, e_3 = z_1 - z, e_4 = w_1 - w, e_a = a_1 - a, e_b = b_1 - b, e_c = c_1 - c, e_r = r_1 - r$.

To guarantee the error dynamical system converge to the origin asymptotically, we choose the following controllers

$$\begin{cases} u_1 = (a - k_1)e_1 - ae_2 - e_4 \\ u_2 = ze_1 - ce_1 + z_1e_4 - (k_2 - 1)e_2 \\ u_3 = (b - k_3)e_3 - ye_1 \\ u_4 = -(r + k_4)e_4 + ye_3 \end{cases} \quad (4)$$

and the parameters estimation updates law as follows

$$\begin{cases} \dot{a}_1 = (x_1 - y_1)e_1 - k_5e_a \\ \dot{b}_1 = z_1e_3 - k_6e_b \\ \dot{c}_1 = -x_1e_2 - k_7e_c \\ \dot{r}_1 = -w_1e_4 - k_8e_r \end{cases} \quad (5)$$

where $k_i > 0 (i = 1, 2, 3, 4, 5, 6, 7, 8)$.

Thus, we can establish the following theorem.

Theorem 1 *For any initial conditions, the system (2) and the system (3) are globally asymptotically synchronized by the control law (5) and the update law (6) of parameters.*

Proof. Choose the following Lyapunov function

$$V = \frac{1}{2}(e_1^2 + e_2^2 + e_3^2 + e_4^2 + e_a^2 + e_b^2 + e_c^2 + e_r^2)$$

The time derivative of V along the trajectory of error system (4) is

$$\begin{aligned} \dot{V} &= e_1\dot{e}_1 + e_2\dot{e}_2 + e_3\dot{e}_3 + e_4\dot{e}_4 + e_a\dot{e}_a + e_b\dot{e}_b + e_c\dot{e}_c + e_r\dot{e}_r \\ &= e_1(e_a(y_1 - x_1) + a(e_2 - e_1) + e_4 + u_1) + e_2(e_c x_1 + ce_1 - e_2 - x_1e_3 - ze_1 + u_2) + e_3(-e_b z_1 \\ &\quad - be_3 + x_1e_2 + ye_1 + u_3) + e_4(-z_1e_2 - ye_3 + e_r w_1 + re_4 + u_4) + e_a\dot{e}_a + e_b\dot{e}_b + e_c\dot{e}_c + e_r\dot{e}_r \end{aligned} \quad (6)$$

Substituting Eqs. (4) and (5) into Eq. (6) yields

$$\dot{V} = -k_1e_1^2 - k_2e_2^2 - k_3e_3^2 - k_4e_4^2 - k_5e_a^2 - k_6e_b^2 - k_7e_c^2 - k_8e_r^2 = -\mathbf{e}^T \mathbf{P} \mathbf{e}$$

where $\mathbf{e} = [e_1, e_2, e_3, e_4, e_a, e_b, e_c, e_r]^T$, $\mathbf{P} = \text{diag} \{k_1, k_2, k_3, k_4, k_5, k_6, k_7, k_8\}$.

Since $\dot{V} \leq 0$, we have $e_1, e_2, e_3, e_4, e_a, e_b, e_c, e_r \rightarrow 0$ as $t \rightarrow \infty$, i.e., $\lim_{t \rightarrow \infty} \|\mathbf{e}\| = 0$. This completes the proof. ■

4 Chaos synchronization between the hyperchaotic system and the hyperchaotic Lorenz Rössler system

In this section, synchronization of this new hyperchaotic system and the hyperchaotic Rössler system is realized. An effective adaptive controller and a parameter estimation update law are designed via adaptive control theory.

The hyperchaotic Rössler system is described by the following equations

$$\begin{cases} \dot{x} = -y - z \\ \dot{y} = x + ay + w \\ \dot{z} = b + xz \\ \dot{w} = -cz + dw \end{cases} \quad (7)$$

where a, b, c and d are constant parameters, when $a = 0.25, b = 3, c = 0.5$ and $d = 0.05$, this dynamical system shows hyperchaotic behavior.

We choose the new hyperchaotic Lorenz system (1) as the drive system, let the hyperchaotic Rössler system be the response one, that is

$$\begin{cases} \dot{x}_2 = -y_2 - z_2 + u_1 \\ \dot{y}_2 = x_2 + a_2 y_2 + w_2 + u_2 \\ \dot{z}_2 = b_2 + x_2 z_2 + u_3 \\ \dot{w}_2 = -c_2 z_2 + d_2 w_2 + u_4 \end{cases} \quad (8)$$

where u_1, u_2, u_3, u_4 are the controllers to be designed.

Supposing all the parameters $a, b, c, d, a_2, b_2, c_2$ and d_2 are uncertain. We will design an adaptive controller and a parameter update law to achieve the synchronization of the two different structure uncertain hyperchaotic systems.

Then, the error dynamical system between Eqs. (1) and (8) can be expressed by

$$\begin{cases} \dot{e}_1 = -e_2 - e_3 + ax - (a+1)y - z - w + u_1 \\ \dot{e}_2 = e_1 + a_2 e_2 + (1-c)x + (1+a_2)y + w_2 + xz + u_2 \\ \dot{e}_3 = ze_1 + xe_3 + e_1 e_3 + xz - xy + bz + b_2 + u_3 \\ \dot{e}_4 = -c_2 e_3 + d_2 e_4 + yz - c_2 z + (d_2 - r)w + u_4 \end{cases} \quad (9)$$

The goal of control is to find controllers $u_i (i = 1, 2, 3, 4)$ and a parameter estimation update law for Eq. (9) such that the states of the response system (8) and the states of the drive system (1) are globally synchronized asymptotically.

Then we obtain the following theorem.

Theorem 2 Let the adaptive control law be defined as

$$\begin{cases} u_1 = w + \hat{a}(y-x) + y_2 + z_2 - k_1 e_1 \\ u_2 = -xz + \hat{c}x - x_2 - w_2 - \hat{a}_2 y_2 - y - k_2 e_2 \\ u_3 = xy - \hat{b}z - x_2 z_2 - \hat{b}_2 - k_3 e_3 \\ u_4 = -yz + \hat{r}w + \hat{c}_2 z_2 - \hat{d}_2 w_2 - k_4 e_4 \end{cases} \quad (10)$$

and the parameters estimation updates law as follows

$$\begin{cases} \dot{\hat{a}} = (x-y)e_1 - k_5 e_a \\ \dot{\hat{b}} = ze_3 - k_6 e_b \\ \dot{\hat{c}} = -xe_2 - k_7 e_c \\ \dot{\hat{r}} = -we_4 - k_8 e_r \\ \dot{\hat{a}}_2 = y_2 e_2 - k_9 e_{a_2} \\ \dot{\hat{b}}_2 = e_3 - k_{10} e_{b_2} \\ \dot{\hat{c}}_2 = -z_2 e_4 - k_{11} e_{c_2} \\ \dot{\hat{d}}_2 = w_2 e_4 - k_{12} e_{d_2} \end{cases} \quad (11)$$

where $e_a = \hat{a} - a, e_b = \hat{b} - b, e_c = \hat{c} - c, e_r = \hat{r} - r, e_{a_2} = \hat{a}_2 - a_2, e_{b_2} = \hat{b}_2 - b_2, e_{c_2} = \hat{c}_2 - c_2, e_{d_2} = \hat{d}_2 - d_2$ and $k_i > 0 (i = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12)$, then the response system (8) is globally synchronous with the drive system (1).

Proof. Choose the following Lyapunov function

$$V = \frac{1}{2}(e_1^2 + e_2^2 + e_3^2 + e_4^2 + e_a^2 + e_b^2 + e_c^2 + e_r^2 + e_{a_2}^2 + e_{b_2}^2 + e_{c_2}^2 + e_{d_2}^2)$$

The time derivative of V along the trajectory of error system (9) is

$$\begin{aligned} \dot{V} &= e_1 \dot{e}_1 + e_2 \dot{e}_2 + e_3 \dot{e}_3 + e_4 \dot{e}_4 + e_a \dot{e}_a + e_b \dot{e}_b + e_c \dot{e}_c + e_r \dot{e}_r + e_{a_2} \dot{e}_{a_2} + e_{b_2} \dot{e}_{b_2} + e_{c_2} \dot{e}_{c_2} + e_{d_2} \dot{e}_{d_2} \\ &= e_1(-e_2 - e_3 + ax - (a+1)y - z - w + u_1) + e_2(e_1 + a_2 e_2 + (1-c)x + (1+a_2)y + w_2 \\ &\quad + xz + u_2) + e_3(ze_1 + xe_3 + e_1 e_3 + xz - xy + bz + b_2 + u_3) + e_4(-c_2 e_3 + d_2 e_4 + yz \\ &\quad - c_2 z + (d_2 - r)w + u_4) + e_a \dot{e}_a + e_b \dot{e}_b + e_c \dot{e}_c + e_r \dot{e}_r + e_{a_2} \dot{e}_{a_2} + e_{b_2} \dot{e}_{b_2} + e_{c_2} \dot{e}_{c_2} + e_{d_2} \dot{e}_{d_2} \end{aligned} \quad (12)$$

Substituting Eqs. (10) and (11) into Eq. (12) yields

$$\dot{V} = -k_1e_1^2 - k_2e_2^2 - k_3e_3^2 - k_4e_4^2 - k_5e_a^2 - k_6e_b^2 - k_7e_c^2 - k_8e_r^2 - k_9e_{a_2}^2 - k_{10}e_{b_2}^2 - k_{11}e_{c_2}^2 - k_{12}e_{d_2}^2 = -\mathbf{e}^T \mathbf{P} \mathbf{e}$$

where $\mathbf{e} = [e_1, e_2, e_3, e_4, e_a, e_b, e_c, e_r, e_{a_2}, e_{b_2}, e_{c_2}, e_{d_2}]^T$, $\mathbf{P} = \text{diag} \{k_1, k_2, k_3, k_4, k_5, k_6, k_7, k_8, k_9, k_{10}, k_{11}, k_{12}\}$

Since $\dot{V} \leq 0$, we have $e_1, e_2, e_3, e_4, e_a, e_b, e_c, e_r, e_{a_2}, e_{b_2}, e_{c_2}, e_{d_2} \rightarrow 0$ as $t \rightarrow \infty$, i.e., $\lim_{t \rightarrow \infty} \|\mathbf{e}\| = 0$.

This completes the proof. ■

5 Numerical simulations

In this section, to verify and demonstrate the effectiveness of the proposed methods, we consider two numerical examples. In the numerical simulations, the fourth-order Runge-Kutta method is used to solve the systems with time step size 0.001.

Example 1 Adaptive synchronization of the new hyperchaotic Lorenz system with uncertain parameters.

For this numerical simulation, we assume that the control gain $k_i = 10 (i = 1, 2, 3, 4, 5, 6, 7, 8)$, the initial values of the drive system and response system are taken as $x(0) = -1, y(0) = -1, z(0) = -1, w(0) = -1, x_1(0) = 1, y_1(0) = 1, z_1(0) = 1, w_1(0) = 1$ respectively. Hence, the initial errors are $e_1=2, e_2=2, e_3=2$ and $e_4=2$. The four uncertain parameters are chosen as $a_1(0) = -5, b_1(0) = 1/3, c_1(0) = -2$ and $r_1(0) = 5$, the parameters of the drive system are choose as $a = 10, b = 8/3, c = 28$ and $r = -1$ so that the system (2) exhibits a hyperchaos behavior.

The simulation results are illustrated in Fig.2 and 3. Fig. 2 displays the synchronization errors between systems (2) and (3); Fig.3 shows the estimates of parameters $a_1(t), b_1(t), c_1(t), r_1(t)$ as $t \rightarrow \infty$. From Fig. 2, We can see the synchronization errors converge asymptotically to zero and two systems are indeed achieved with chaos synchronization. Further more, the estimated values of parameters converge to $a = 10, b = 8/3, c = 28, r = -1$ as $t \rightarrow \infty$.

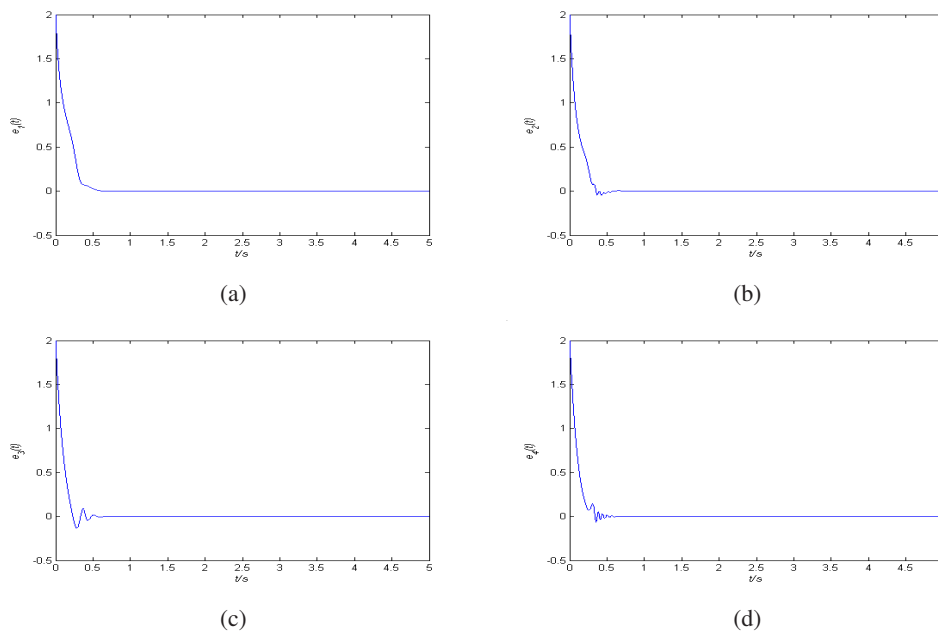


Figure 2: Time evolution of error valuable in self- synchronization of the hyperchaotic systems

Example 2 Adaptive synchronization of the hyperchaotic Lorenz system and the hyperchaotic Rössler system.

Let the parameters $a = 10, b = 8/3, c = 28$ and $r = -1$, thus the drive system (1) is hyperchaotic, choose parameters $a_2 = 0.25, b_2 = 3, c_2 = 0.5$ and $d_2 = 0.05$, then the response system (8) is also hyperchaotic. we assume that the control gain $k_i = 10 (i = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12)$, the initial values of the drive system and response system are taken as $x(0) = -1, y(0) = -1, z(0) = -1, w(0) = -1, x_1(0) = 1, y_1(0) = 1, z_1(0) = 1, w_1(0) = 1$ respectively. Hence, the initial errors are $e_1 = 2, e_2 =$

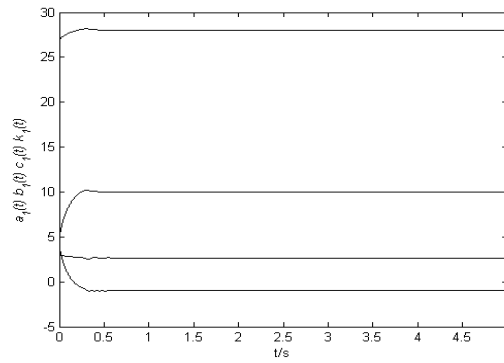


Figure 3: The response system parameters with time t

2, $e_3 = 2$ and $e_4 = 2$. The simulation results are illustrated in Fig.4 and 5. From the figures, it can be seen that the synchronization errors converge to zero and two different systems are indeed achieving chaos synchronization.

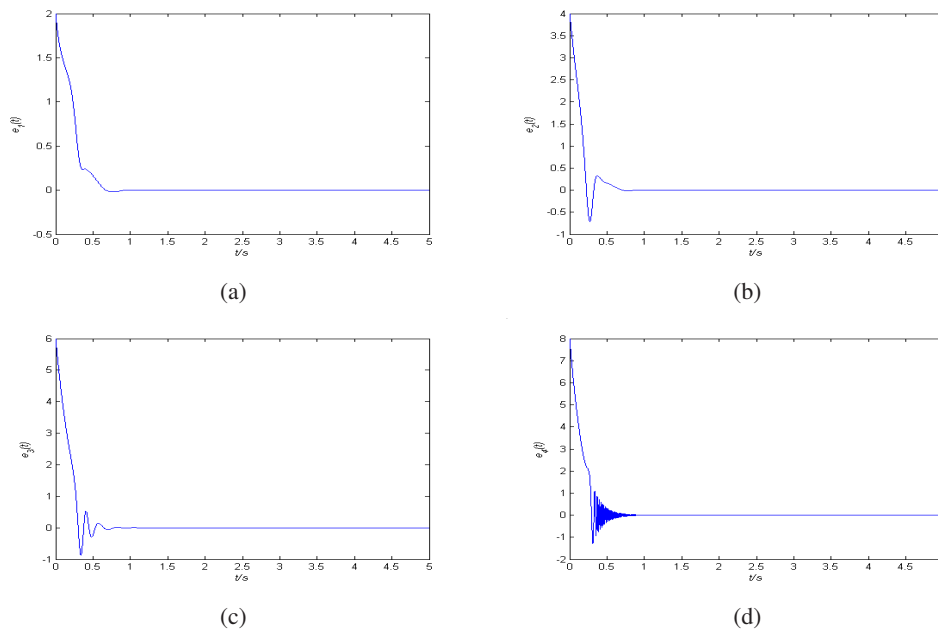


Figure 4: Time evolution of error valuable between the hyperchaotic Lorenz system and the hyperchaotic Rössler system

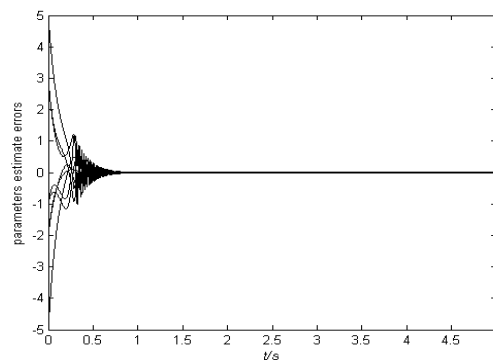


Figure 5: Evolutions of the parameter estimation errors with time t

6 Conclusion

This paper is concerned with adaptive control and synchronization of a new hyperchaotic Lorenz system with uncertain parameters. Adaptive controller has been derived for controlling hyperchaos to unstable equilibrium point of the uncontrolled system. Synchronizations not only between two identical hyperchaotic systems but also between two different hyperchaotic systems are achieved via the Lyapunov stability theory and adaptive control theory. Numerical simulations are given for the purpose of illustration and verification.

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