

Inventory-Transportation Integrated Optimization Problem:A Model of Product Oil Logistics

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Abstract: According to the delivery situation of product oil in China, this paper takes the example of R-System (Retailer System) to study the Inventory-Transportation Integrated Optimization problem, use the Lingo software to verify, and compare with the traditional optimization methods, to prove its superiority concise and practical.

Keywords: product oil; delivery; inventory; integrated optimization

1 Introduction

Compared with foreign big oil companies, the circulation costs of product oil business in China are very high, and the main part of the circulation costs come from the logistics costs. According to statistics from related departments, the average circulation costs of domestic product oil enterprises are 200 yuan, while the logistics costs (transportation and storage) accounted for about 80% . In 2007, domestic oil consumptions are about 192 million tons, including 300 billion yuan of logistics costs. If the logistics costs can be lowered by 10 percentage, then 3 billion yuan can be saved every year[1]. So the effective reduction of product oil's logistics costs will significantly enhance the cash flow rate and the return rate. Transportation and storage are the most important aspects in logistics system[2-5]. In the product oil logistics system, the strategy of inventory and transportation is extremely important, their anti-back interests relationship makes the optimization of them even more important[6].

When the transportation plan is made up, the volume discounts brought by large quantities of transportation should not be pursued excessively[7-8]. As this would bound to increase inventory costs throughout the system, also when the inventory strategy is determined, transportation costs can not be dealt as a fixed fee, but as a variable cost directly impacting on transportation frequency and inventory distribution[9-11]. Under the prerequisite of comprehensively balancing the transportation costs and inventory costs, the objectives that Inventory-Transportation Integrated Optimization problem (ITIO) are to optimize the logistics system, reduce logistics costs, and determine the transportation program and inventory strategy of the system[3]. The object studied in this paper is a system composed by the supply side (product oil distribution center) and the demand side (gas station). Traditional optimization methods often study inventory and transportation separately, in this paper, the two systems are integrated to study, the programming model is given, compared with the traditional optimization methods.

2 Establish Mathematic Models

2.1 Traditional Transportation Optimization Model

Let c_{ij} denote the transportation cost of unit goods from supplier i to demander j ; x_{ij} the volume of goods transport from supplier i to demander j ; b_j the demand volume of the demander j ; a_i the supply ability of

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supplier i . Then

$$\begin{aligned} \min & \sum_{i=1}^m \sum_{j=1}^n c_{ij} x_{ij} \\ \text{s.t.} & \begin{cases} \sum_{i=1}^m x_{ij} = b_j, & j = 1, 2, \dots, n \\ \sum_{j=1}^n x_{ij} \leq a_i, & i = 1, 2, \dots, m \\ x_{ij} \geq 0, & i = 1, 2, \dots, m; j = 1, 2, \dots, n. \end{cases} \end{aligned} \quad (1)$$

2.2 Traditional Inventory Optimal Model[12]

Assume that it doesn't allow shortage, the demand is continuous and even; the inventory is replenished every t time, the demand quantity must meet the requirement Rt during the cycle. Let Q denote the order volume, then $Q = Rt$; C_1 the unit inventory sustaining cost; C_2 the fixed order cost; K the price of the goods, then the order cost is $C_2 + KRt$, the average order cost in t time is $\frac{C_2}{t} + KR$, the average volume of inventory is $\frac{1}{2}Rt$, the average inventory sustaining cost is $\frac{1}{2}C_1Rt$.

Then we know the average total inventory cost in the cycle is expressed in mathematic formula as:

$$C(t) = \frac{C_2}{t} + KR + \frac{1}{2}C_1Rt.$$

Let the formula above be 0, and get the derivations of both sides, through solving the equation, we can get the value of t that makes $C(t)$ minimum:

$$t_0 = \sqrt{\frac{2C_2}{C_1R}}.$$

Then the order volume

$$Q_0 = Rt_0 = \frac{2C_2R}{C_1}.$$

While

$$\min C(t) = C(t_0) = \sqrt{2C_1C_2R}.$$

2.3 The Optional Model of ITIO

Assume that all demand points are managed by one person, the suppliers have no stock, it doesn't allow shortage, the demand among every demand point is even and independent to each other, the total demand volume of all demand volume of all demand points is certain.

Let m denote the amount of the supplier; n the amount of the demander; c_{ijk} the cost that demander j purchase goods k from supplier i ; x_{ijk} the volume of goods k transport from supplier i to demander j ; w_{jk} the requirement of goods k from demander j ; β_{jk} the initial inventory of demander j for goods k ; h_{jk} the unit storage cost of demander j for goods k ; s_{jk} the safe stock of demander j for goods k ; Q_k the total requirement of goods k ; r_{ijk} the unit transportation cost of goods k from supplier i to demander j ; B_{ik} the supply capacity of supplier i for goods k . Then we get the optimal model of ITIO as follows:

$$\begin{aligned} \min C &= \sum_{i=1}^m \sum_{j=1}^n \sum_{k=1}^o c_{ijk} x_{ijk} + \sum_{j=1}^n \sum_{k=1}^o \frac{1}{2} (w_{jk} + \beta_{jk}) h_{jk} + \sum_{i=1}^m \sum_{j=1}^n \sum_{k=1}^o r_{ijk} x_{ijk} \\ \text{s.t.} & \begin{cases} w_{jk} + \beta_{jk} \geq S_{jk}, & \forall j, k \\ \sum_{j=1}^n w_{jk} = Q_k, & \forall k \\ \sum_{i=1}^m x_{ijk} = w_{jk}, & \forall j, k \\ \sum_{j=1}^n x_{ijk} \leq B_{ik}, & \forall i, k \\ x_{ijk} \geq 0, & \forall j, k. \end{cases} \end{aligned} \quad (2)$$

Here, w_{jk} and x_{ijk} are the strategy variable. In the formula (2), the first term of the planning objective function is the order cost, the second one is the inventory cost, the third one is the total transportation cost;

the first constrain conclusion shows that the storage of any demand point for any goods is more than its safe stock, the second one shows that the sum requirement of all demand points for goods k is its total demand volume, the third one shows that the total transportation volume of demander j for goods k is its storage requirement, the forth one shows that the transportation volume of supplier i for goods k is less than its supply capacity.

3 Application

Here we will show that the model of ITIO is better than the traditional methods. The product oil distribution centers A_1 and A_2 deliver one kind oil to B_1, B_2, B_3 and B_4 four gas stations. The demand rate R_1 of B_1 is 1.5 ton every year, the unit inventory sustaining cost C_1 is 160 yuan every ton every year, the fixed order cost C_2 is 240 yuan every time, the good's unit price is 1000 yuan every ton, the safe stock S_1 is 1.4 ton; for gas station B_2 , the demand rate R_2 is 25 ton every year, the safe stock S_2 is 2.8 ton, the others are as the same as gas station B_1 ; for gas station B_3 , the demand rate R_3 is 20 ton every year, the safe stock S_3 is 2.2 ton, the others are as the same as gas station B_1 ; for gas station B_4 , the demand rate R_4 is 16 ton every year, the safe stock S_4 is 1.8 ton, the others are as the same as gas station B_1 .

3.1 The Solution of Traditional Method

For B_1 : $t_{01} = \sqrt{\frac{2C_2}{C_1R}} = \sqrt{\frac{2 \times 240}{160 \times 1.5}} \approx 5.4$, $Q_{01} = \sqrt{\frac{2C_2R}{C_1}} = \sqrt{\frac{2 \times 240 \times 1.5}{160}} \approx 6.7(t)$; B_2 : $t_{02} \approx 4.2$, $Q_{02} \approx 8.7(t)$; B_3 : $t_{03} \approx 4.7$, $Q_{03} \approx 7.7(t)$; B_4 : $t_{04} \approx 5.2$, $Q_{04} \approx 6.9(t)$.

As the delivery cycle of the four gas stations is different, in order to lower the transportation costs, we need to unite their cycle, regarding 5.4 as the standard cycle, then

The volume of B_2 after adjustment is $8.7 + 1.2 \div 12 \times 25 = 11.2(t)$; the volume of B_3 after adjustment is $8.9(t)$; the volume of B_4 after adjustment is $7.2(t)$; the inventory cost of B_1 is $240 / (5.4 / 12) + 1000 \times 15 + \frac{1}{2} \times 160 \times 15 \times (5.4 / 12) = 16073(\text{yuan})$; the inventory cost of B_2 is $26433(\text{yuan})$; the inventory cost of B_3 is $21253(\text{yuan})$; the inventory cost of B_4 is $17109(\text{yuan})$.

So, the total inventory cost is $16073 + 26433 + 21253 + 17109 = 80868(\text{yuan})$.

In every cycle, A_1 can supply 16 tons, A_2 can supply 18 tons, Table 1 shows the unit transportation cost between product oil distribution centers and gas stations.

Table 1: UTC

UTC	B_1	B_2	B_3	B_4
A_1	1300	1280	1050	1570
A_2	1160	1440	1310	1250

Then the transportation model is as follows:

$$\begin{aligned} & \min 1300x_{11} + 1280x_{12} + 1050x_{13} + 1570x_{14} + 1160x_{21} + 1440x_{22} + 1310x_{23} + 1250x_{24} \\ & s.t. \begin{cases} x_{11} + x_{12} + x_{13} + x_{14} = 16 \\ x_{21} + x_{22} + x_{23} + x_{24} = 18 \\ x_{11} + x_{21} = 6.7 \\ x_{12} + x_{22} = 11.2 \\ x_{13} + x_{23} = 8.9 \\ x_{14} + x_{24} = 7.2 \\ x_{ij} \geq 0, \quad i = 1, 2; j = 1, 2, 3, 4. \end{cases} \end{aligned} \quad (3)$$

To solve it with Lingo, we can get the results as follows:

$x_{11} = 0, x_{12} = 7.1, x_{13} = 8.9, x_{14} = 0, x_{21} = 6.7, x_{22} = 4.1, x_{23} = 0, x_{24} = 7.2$; the transportation cost is $41109(\text{yuan})$.

Then the total logistics cost is $80868 + 41109 = 121977(\text{yuan})$.

3.2 The Solution of ITIO Model

Because of the use of Inventory-Transportation Integrated Optimization, in accordance with the whole supply chain needs to make distribution decisions, rather than delivering when demander needs. So, we can ignore the order cost in the ITIO model.

$$\begin{aligned} \min & 1000 \sum_{i=1}^2 \sum_{j=1}^4 x_{ij} + \frac{1}{2}(w_1 + w_2 + w_3 + w_4) \times 160 + 1300x_{11} + 1280x_{12} + 1050x_{13} + 1570x_{14} \\ & + 1160x_{21} + 1440x_{22} + 1310x_{23} + 1250x_{24} \\ \text{s.t.} & \begin{cases} w_1 \geq 1.4 \\ w_2 \geq 2.8 \\ w_3 \geq 2.2 \\ w_4 \geq 1.8 \\ w_1 + w_2 + w_3 + w_4 = 34 \\ x_{11} + x_{21} - w_1 = 0 \\ x_{12} + x_{22} - w_2 = 0 \\ x_{13} + x_{23} - w_3 = 0 \\ x_{14} + x_{24} - w_4 = 0 \\ x_{11} + x_{12} + x_{13} + x_{14} = 16 \\ x_{21} + x_{22} + x_{23} + x_{24} = 18 \\ x_{ij} \geq 0, i = 1, 2; j = 1, 2, 3, 4. \end{cases} \end{aligned} \quad (4)$$

To solve it with Lingo, we get the results as follows:

$x_{11} = 0, x_{12} = 2.8, x_{13} = 13.2, x_{14} = 0, x_{21} = 16.2, x_{22} = 0, x_{23} = 0, x_{24} = 1.8$, the total logistics cost is 750206(yuan).

It can be seen that, through using the ITIO model, we can save the cost of $[(121977 - 75206)/121977] \times 100\% = 38.34\%$.

4 Conclusion

Compared with traditional optimization methods, ITIO method regards the transportation and inventory as a whole to consider, it is more closely reflect their relevance, thereby, to coordinate the logistics activities and reduce the total cost of logistics services, improve the efficiency of logistics systems, provide a basis for the strategy of the logistics programs.

In the view of the distribution system of product oil, the research of ITIO is very important for improving the distribution system of product oil in China, and breaking the bottlenecks of the product oil's existing high logistics cost.

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