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Pinning Control of Complex Delayed Dynamical Networks for Projective Synchronization

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Abstract: The objective of this paper is to realize projective synchronization in complex delayed dynamical networks. With assuming irreducibility and symmetry of the coupling matrix, we prove that the complex delayed dynamical networks can be pinned to a homogenous solution by controllers. Sufficient conditions are presented to guarantee the convergence of the pinning process. And we verify our conclusion by numerical simulation of the lorenz network.

Keywords: Complex delayed dynamical networks; Projective synchronization;Pinning control

1 Introduction

Chaotic systems, a dynamical system for which its evolution sensitively depends on the initial conditions [1], are common in nature. Synchronization, a process where in two or many systems (either equivalent or nonequvilent) adjust a given property of their motion due to a suitable coupling scheme or to an external forcing, is a universal phenomenon in nature and society [2,3]. And since the synchronization of chaotic dynamical systems has been observed by Paccora and Carroll [3] in 1990, chaos synchronization has become a topic of interesting [4-8]. Synchronization phenomenas have been reported in the recent literature. Till now, different types of synchronization have been found in interacting chaotic systems, sunch as complete synchronization [9], generalized synchronization [10-11], anti-phase synchronization [12]phase synchronization [13] and projective synchronization[14-15] , etc.

Complex networks [16-19], large ensembles of interconnected nodes in which a node is a basic unit with specific contents, are ubiquitous in various fields of the real world. The early study of projective synchronization reported that the projective synchronization, was usually observable only in the coupled partially linear systems. Later , some researchers have extended it to a general class of chaotic systems without the limitation of partial-linearity, such as non-partially -linear systems. And in reality, there usually are some time delays in spreading and response due to the finite speeds of transmission and spreading as well as traffic congestions. Therefore, time delays should be modelled in order to simulate more realistic networks.

The objective of this paper is to realize projective synchronization in drive-response dynamical networks. With assuming irreducibility and symmetry of the coupling matrix, the theorem is proposed for continuous-time case. From the theorem, we can see that the synchronization of such networks is determined by the dynamics of each node, the coupling matrix, and the feed-back gain matrix of the network. Sufficient conditions are presented to guarantee the convergence of the pinning process and at last we prove that the whole network can be controlled to stable state by controllers. We verify our conclusion by numerical simulation of the Lorenz system.

The rest of this paper is organized as followings. Model description and some preliminaries are introduced in section 2. In section 3 pinning control of complex delayed dynamical network for projective synchronization is studied with controllers. And one theorem is obtained for the continuous-time complex network. We give a numerical simulation of the Lorenz system in section 4. Finally, in section 5 conclusions are presented.

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2 Model description and preliminaries

A chaotic system (such as a Lorenz system) is called a partially linear system, if the state vector can be broken into two parts (u, z), where the equation for z is nonlinearly related to the other variable, while the equation for the rate of change of the vector u is linearly related to z through a matrix M that can be depended on the variable z :

$$\begin{array}{l} \dot{u}(t) = M(z) \bullet u \\ \dot{z} = f(u, z) \end{array} \tag{1}$$

Projective synchronization may occur when the two system (1) are linked through the variable z in the form as

$$\begin{cases} \dot{u}^d(t) = M(z) \bullet u^d \\ \dot{z} = f(u^d, z) \\ \dot{u}^r(t) = M(z) \bullet u^r \end{cases} \tag{2}$$

The superscritpts d and r stand for the drive and response systems, respectively. In Ref.[15], we studied the projective synchronization in a DRSNs model which is described by the following equation:

$$\begin{cases} \dot{u}^d(t) = M(z) \bullet u^d \\ \dot{z} = f(u^d, z) \\ \dot{u}^r(t) = M(z) \bullet u^r + c\sum\limits_{j=1}^{N} g_{ij}u_j^r, j = 1, 2, ...N \end{cases} \tag{3}$$

where $u_i^r, j = 1, 2, ...N$ denote the multi-dimensional state variables of the response networks sustems. N is the number of coupled nodes and the constant $c > 0$ is the coupling strength. $G = (g_{ij})_{N\times N}$ is a real matrix reflecting the topology of the network and g_{ij} can be defined as follows : if there is a connection between the node i and node $j(i \neq j)$ at time t, $g_{ij} > 0$; else $g_{ij} = 0$; and the diagonal entries of the matrix are defined by $g_{ii} = -\sum\limits_{j=1, j\neq i}^{N} g_{ij}$. So we can find out that the matrix G is a symmetric matrix.

3 Pinning control of DRDNs for projective synchronization

In this paper we will mainly study the PCS in the three-dimension drive-response dynamical networks with time-delay which is described:

$$\begin{cases} \dot{x} = m_{11}x + m_{12}y \\ \dot{y} = m_{21}x + m_{22}y \\ \dot{x}_i = m_{11}x_i + m_{12}y_i + c\sum\limits_{j=1}^{N} g_{ij}x_j(t-\tau) \\ \dot{y}_i = m_{21}x_i + m_{22}y_i + c\sum\limits_{j=1}^{N} g_{ij}y_j(t-\tau) \end{cases} \tag{4}$$

where x, y, z, x_i, y_i are state variables, $m_{ij}(i, j = 1, 2)$ are the functions of z, coupling strength $c > 0$ and coupling matrix $G = (g_{ij})_{N\times N}$ has the form of (3). In this section, our goal is to realize $(x_i, y_i)^T \longrightarrow \alpha_i(x, y)^T, i = 1, 2, ...N$ as $t \longrightarrow \infty$. To achieve this goal, we apply the pinning control strategy on a small fraction $\delta_i(0 < \delta_i \ll 1)(i = 1, 2, ...n)$ in response networks of model (4). Sppose that nodes $i_1, i_2, ..., i_{l_1}, i_{l_1+1}, ..., i_{l_{n-1}}, ..., i_l$ are selected to be under pinning control and $i_{l+1}, i_{l+2}, ..., i_N$ are the unselected ones, where $l = l_1 + l_2 + ...l_n, l_i = [\delta_i m_i], i = 1, 2, ..., n$, where function $[x]$ denotes the nearest lower integer to the real number x. Thus the controlled DRDNs can be described as

$$\begin{cases} \dot{x} = m_{11}x + m_{12}y \\ \dot{y} = m_{21}x + m_{22}y \\ \dot{x}_i = m_{11}x_i + m_{12}y_i + c\sum\limits_{j=1}^{N} g_{ij}x_j(t-\tau) - ck_i(x_i - \alpha_i x) \\ \dot{y}_i = m_{21}x_i + m_{22}y_i + c\sum\limits_{j=1}^{N} g_{ij}y_j(t-\tau) - ck_i(y_i - \alpha_i y), i = 1, 2, ...N \end{cases} \tag{5}$$

where α_i can be any desided value. If $i \in \{i_1, i_2, ..., i_l\}$ then feedback gain $k_i > 0$; otherwise , $k_i = 0$.

Denote $e_i = \begin{pmatrix} x_i - \alpha x \\ y_i - \alpha y \end{pmatrix}$, $\dot{e}_i^1(t) = \dot{x}_i(t) - \alpha_i \dot{x}(t), \dot{e}_i^2(t) = \dot{y}_i(t) - \alpha_i \dot{y}(t)$

Then

$\dot{e}_i^1(t) = \dot{x}_i(t) - \alpha_i \dot{x}(t) = m_{11}x_i + m_{12}y_i + c\sum\limits_{j=1}^{N} g_{ij}x_j(t-\tau) - ck_i(x_i - \alpha_i x) - \alpha_i(m_{11}x + m_{12}y)$

$= (m_{11} - ck_i)e_i^1 + m_{12}e_i^2 + c\sum\limits_{j=1}^{N} g_{ij}e_j^1(t-\tau)$

$\dot{e}_i^2(t) = \dot{y}_i(t) - \alpha_i \dot{y}(t) = m_{21}x_i + m_{22}y_i + c\sum\limits_{j=1}^{N} g_{ij}y_j(t-\tau) - ck_i(y_i - \alpha_i y) - \alpha_i(m_{21}x + m_{22}y)$

$= m_{21}e_i^1 + (m_{22} - ck_i)e_i^2 + c\sum\limits_{j=1}^{N} g_{ij}e_j^2(t-\tau)$

$\dot{e}_i(t) = \begin{pmatrix} m_{11} - ck_i & m_{12} \\ m_{21} & m_{22} - ck_i \end{pmatrix} \begin{pmatrix} e_i^1(t) \\ e_i^2(t) \end{pmatrix} + c\sum\limits_{j=1}^{N} g_{ij} \begin{pmatrix} e_j^1(t-\tau) \\ e_j^2(t-\tau) \end{pmatrix}$,

and we denote $M(z) = \begin{pmatrix} m_{11} - ck_i & m_{12} \\ m_{21} & m_{22} - ck_i \end{pmatrix}$, then we have $\dot{e}_i(t) = M(z)e_i(t) + c\sum\limits_{j=1}^{N} g_{ij}e_j(t-\tau)$.

Theorem 1 *Let $u(t)$ be the maximum eigenvalue of the matrix $\frac{1}{2}(M(z) + M^T(z))$. If $u(t) < 0$ for all $t > 0$, then PCS with desired scaling factors $\alpha_i, i = 1, 2, ..., N$ can be exponentially synchronized in the drive-response network (5).*

Proof. We can construct a Lyapunov-Krasovskii of the form

$$V_i(t) = \frac{1}{2}e_i^T e_i(t) + c\int_{t-\tau}^{t} \sum_{j=1}^{N} g_{ij}e_j^T(\eta)e_j(\eta)d\eta$$

Differentiating $V(t)$ with respect to time , we have

$\dot{V}(t) = \frac{1}{2}\dot{e}_i^T(t)e_i(t) + \frac{1}{2}e_i^T(t)\dot{e}_i(t) + c\sum\limits_{j=1}^{N} g_{ij}e_j^T(t)e_j(t) - c\sum\limits_{j=1}^{N} g_{ij}e_j^T(t-\tau)e_j(t-\tau)$

$= \frac{1}{2}[e_i^T(t)M^T(z) + c\sum\limits_{j=1}^{N} g_{ij}e_j^T(t-\tau)]e_i(t) + \frac{1}{2}e_i^T(t)[M(z)e_i(t) + c\sum\limits_{j=1}^{N} g_{ij}e_j(t-\tau)]$

$+c\sum\limits_{j=1}^{N} g_{ij}e_j^T(t)e_j(t) - c\sum\limits_{j=1}^{N} g_{ij}e_j^T(t-\tau)e_j(t-\tau)$

$= e_i^T(t)[\frac{1}{2}(M^T(z)+M(z))]e_i(t) + c\sum\limits_{j=1}^{N} g_{ij}e_j^T(t-\tau)e_j(t) + c\sum\limits_{j=1}^{N} g_{ij}e_j^T(t)e_j(t) + c\sum\limits_{j=1}^{N} g_{ij}e_j^T(t-\tau)e_j(t-\tau)$

$\leq e_i^T[\frac{1}{2}(M^T(z) + M(z))]e_i(t) + c\sum\limits_{j=1}^{N} g_{ij}(e_j^T(t-\tau)e_j(t-\tau) + e_i^T(t)e_i(t))$

$+c\sum\limits_{j=1}^{N} g_{ij}e_j^T(t)e_j(t) - c\sum\limits_{j=1}^{N} g_{ij}(e_j^T(t-\tau)e_j(t-\tau)$

$= e_i^T[\frac{1}{2}(M^T(z) + M(z)) + 2c\sum\limits_{j=1}^{N} g_{ij}]e_i(t)$

Because the matrix $G = (g_{ij})_{N\times N}$ is symmetric and $u(t) < 0$ for all $t > 0$, then we have $\dot{V} = e_i^T(t)[\frac{1}{2}(M^T(z) + M(z))]e_i(t) \leq e_i^T(t)u(t)e_i(t) < 0$.

The theorem is proved. ■

4 Simulation

As an application of the above theoretical criteria, we consider a complex network with the well-known lorenz system and time-varying delay.

For simplicity, we first consider a eight-nodes network

$$\dot{x}(t) = -\sigma x(t) + \sigma y(t)$$

$$\dot{y}(t) = (\gamma - z)x(t) - y(t)$$

$$\dot{z}(t) = x(t)y(t) - \rho z(t)$$

$$\dot{x}_i(t) = -\sigma x_i(t) + \sigma y_i(t) + c\sum_{j=1}^{N} g_{ij}x_j(t-\tau)$$

$$\dot{y}_i(t) = (\gamma - z)x_i(t) - y_i(t) + c\sum_{j=1}^{N} g_{ij}y_j(t-\tau), i = 1, 2, ..., 8$$

where σ, γ, β are parameters. When $\sigma = 10, \gamma = 28, \rho = \frac{8}{3}$, the Lorenz system has a chaotic attractor.

Here we want to stabilize the network by controllers, which is described as follows

$$\dot{x}(t) = -\sigma x(t) + \sigma y(t)$$

$$\dot{y}(t) = (\gamma - z)x(t) - y(t)$$

$$\dot{z}(t) = x(t)y(t) - \rho z(t)$$

$$\dot{x}_i(t) = -\sigma x_i(t) + \sigma y_i(t) + c\sum_{j=1}^{N} g_{ij}x_j(t-\tau) - ck_i(x_i - \alpha_i x)$$

$$\dot{y}_i(t) = (\gamma - z)x_i(t) - y_i(t) + c\sum_{j=1}^{N} g_{ij}y_j(t-\tau) - ck_i(y_i - \alpha_i y), i = 1, 2, ..., 8$$

In numerical simulation the coupling matrix is

$$G = \begin{bmatrix} -3 & 3 & -1 & 1 & 0 & 0 & 0 & 0 \\ 3 & -3 & 1 & -1 & 0 & 0 & 0 & 0 \\ -1 & 1 & -10 & 5 & 5 & -2 & 1 & 1 \\ 1 & -1 & 5 & -10 & 5 & 1 & -2 & 1 \\ 0 & 0 & 5 & 5 & -10 & 1 & 1 & -2 \\ 0 & 0 & -2 & 1 & 1 & -6 & 3 & 3 \\ 0 & 0 & 1 & -2 & 1 & 3 & -6 & 3 \\ 0 & 0 & 1 & 1 & -2 & 3 & 3 & -6 \end{bmatrix}$$

The coupling strength c is set to 10, α_i are all set to -1 ,k_i are all set to 40 , and $\tau = 1$. Under these conditions, the maximum eigenvalue of the matrix $\frac{1}{2}(M(z) + M^T(z))$ is less than zero. We use the Matlab to describe the errors $e_i(t), i = 1, 2, ..., 8$, which are described on the same picture as Fig 1.

From the picture we can see that the controllers can pin the complex delayed dynamical network to synchronization.

5 Conclusion

The objective of this paper is to realize projective synchronization in complex delayed dynamical networks. With assuming irreduciblity and symmetry of the coupling matrix, we prove that the complex delayed dynamical networks can be pinned to a homogenous solution by controllers. Sufficient conditions are presented to guarantee the convergence of the pinning process. And we verify our conclusion by numerical simulation of the lorenz network.

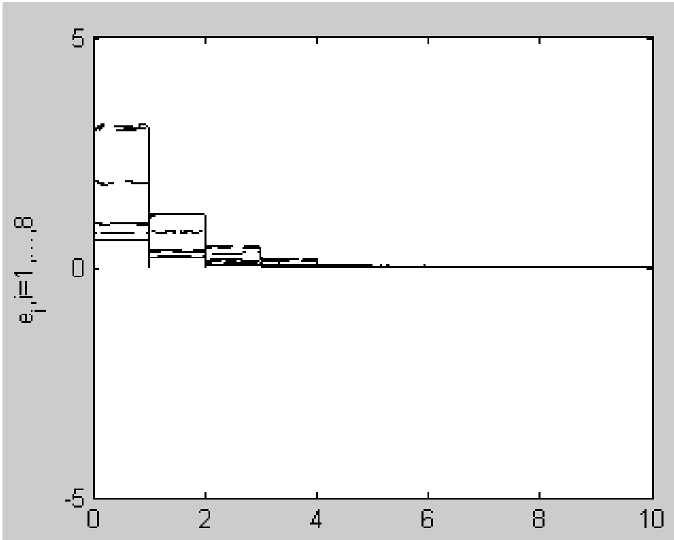

Fig 1 Synchronization errors for the network with $c = 10, \alpha_i = -1, k_i = 40, \tau = 1$

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