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Adaptive Control for Modified Projective Synchronization of Four-dimensional Qi Chaotic System with Dispersive Term *

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Abstract: This paper is concerned with the modified projective synchronization problem for the four-dimensional Qi chaotic system with uncertain parameters .The designed controller ensures that the state variables of the state variables of the master systems respectively. The results are validated using numerical simulation.

Keywords: Four-dimensional chaotic system; Adaptive control; Lyapunov method

1 Introduction

In the last few years, synchronization of chaotic dynamical systems has received considerable interest among scientists in various fields. The results of chaos synchronization are utilized in biological science, chemical reaction, secret communication and cryptography, nonlinear oscillation synchronization and some other fields. The first idea of synchronizing two identical chaotic systems with different initial conditions is introduced by Pecora and Carroll[3-5] and the method is realized in electronic circuits. Synchronization techniques have been improved in recent years, and many different methods are applied theoretically and experimentally to synchronize the chaotic systems. Synchronization of hyper-chaotic systems were investigated and a generalized method for synchronization of chaotic systems. Various active and nonlinear control methods were used for chaos synchronization of two identical systems. Also synchronization between two different chaotic systems using different nonlinear control schemes . Parametric adaptive control for chaos synchronization has been proposed generalized synchronization method using parametric adaptive control is introduced. Using adaptive observer design and Lyapunov stability theorem, a nonlinear adaptive synchronization technique was developed for a class of chaotic systems. Most of the mentioned works are applied for two identical chaotic systems. In practice it is difficult to find two exactly identical chaotic systems. Hence, the synchronization of two different chaotic systems plays a significant role in practical applications and this problem will be more challenging and difficult if the parameters of two chaotic systems are unknown and time varying.

The problem of chaos synchronization is directly related to the observer problem in control theory[2]. In general, the designed controller with the state variable of the master will make the trajectories of the state variables of the slave system to track the trajectories of the state variables of the driver system. Notable among the various methods for achieving this goal include the linear feed-back[3-7], adaptive synchronization[8-10], back stepping nonlinear control[11], sliding mode control, active control[12-14] and projective synchronization[15].

In the present paper, the active control is applied to synchronize two identical new four-dimensional systems, hereafter called the Qi system. The paper is organized as follows: synchronization behavior of two identical Qi systems are studied in Sections 3 respectively. In Section 4, we conclude the paper.

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2 Adaptive control for modified projective synchronization

Projective synchronization means that drive the response system to change with a drive-scale α, that is, correspond variables of the system are proportional. Consider the following forms of chaotic systems:

$$\begin{cases} \dot{x} = f(x) \\ \dot{y} = g(x,y) + u(x,y) \end{cases}$$

In this system, $x, y \in R^n$;$f : R^n \to R^n, g : R^{2n} \to R^n$ are differentiable function, $u = (u_1, u_2, \cdots, u_n)^T$ is designed control law. The first formula is drive system, the second is response system. For a system of two coupled chaotic oscillators, the master ($\dot{x} = f(x,y)$) and the slave ($\dot{y} = g(x,y)$) where x and y are phase space variables, and f and g are the corresponding nonlinear functions, synchronization in a direct sense implies $|x(t) - y(t)| \to 0$ as $t \to \infty$. When this occurs, the coupled system is said to be completely synchronized, other forms of synchronization that have been observed includes phase synchronization, lag synchronization, generalized synchronization and sequential synchronization.

3 Synchronization of two identical Qi systems

3.1 The Qi system

The Qi system is a four-dimensional chaotic system, its equations have thrice non-linear across product subjects. The system will come into being complicated dynamical behaviors. Here we consider the following four coupled nonlinear autonomous first order differential equations:

$$\begin{cases} \dot{x} = a(y - x) + yzw \\ \dot{y} = b(x + y) - xzw \\ \dot{z} = -cz + xyw \\ \dot{w} = -dw + xyz \end{cases} \tag{1}$$

where x, y, z and w are state variables and a, b, c and d are all positive real constant parameters. Qi has introduced the system detailed[22]. When $a = 35, b = 10, c = 1, d = 10$, the four Lyaponov index of this system are: $\lambda_1 = 3.3152, \lambda_2 = 0, \lambda_3 = -4.1591, \lambda_4 = -35.1674$. At this time, the dynamic properties such as chaotic behavior, and its chaotic attract as Fig. (1).

Our goal is to make MPS between two 4D chaotic systems by using adaptive control scheme when the parameter of the master system is unknown and different with those of the slave system. For the 4D chaotic system (1), the master (or drive) and slave (or response) systems are defined below, respectively,

$$\begin{cases} \dot{x_1} = a(y_1 - x_1) + y_1 z_1 w_1 \\ \dot{y_1} = b(x_1 + y_1) - x_1 z_1 w_1 \\ \dot{z_1} = -cz_1 + x_1 y_1 w_1 \\ \dot{w_1} = -dw_1 + x_1 y_1 z_1 \end{cases} \tag{2}$$

and

$$\begin{cases} \dot{x_2} = a_1(y_2 - x_2) + y_2 z_2 w_2 + u_1 \\ \dot{y_1} = b_1(x_2 + y_2) - x_2 z_2 w_2 + u_2 \\ \dot{z_1} = -c_1 z_2 + x_2 y_2 w_2 + u_3 \\ \dot{w_1} = -d_1 w_2 + x_2 y_2 z_2 + u_4 \end{cases} \tag{3}$$

where x_i and y_i stand for state variables of the master system and the slave one, respectively, a_1, b_1, c_1, and d_1 are uncertain parameters of the slave system which needs to be estimated. Our goal is make the apt $u_i (i = 1, 2, 3, 4)$:

$$\begin{cases} \lim\limits_{t \to 0} \|x_1 - \alpha_1 x_2\| = 0 \\ \lim\limits_{t \to 0} \|y_1 - \alpha_2 y_2\| = 0 \\ \lim\limits_{t \to 0} \|z_1 - \alpha_3 z_2\| = 0 \\ \lim\limits_{t \to 0} \|w_1 - \alpha_4 w_2\| = 0 \end{cases} \tag{4}$$

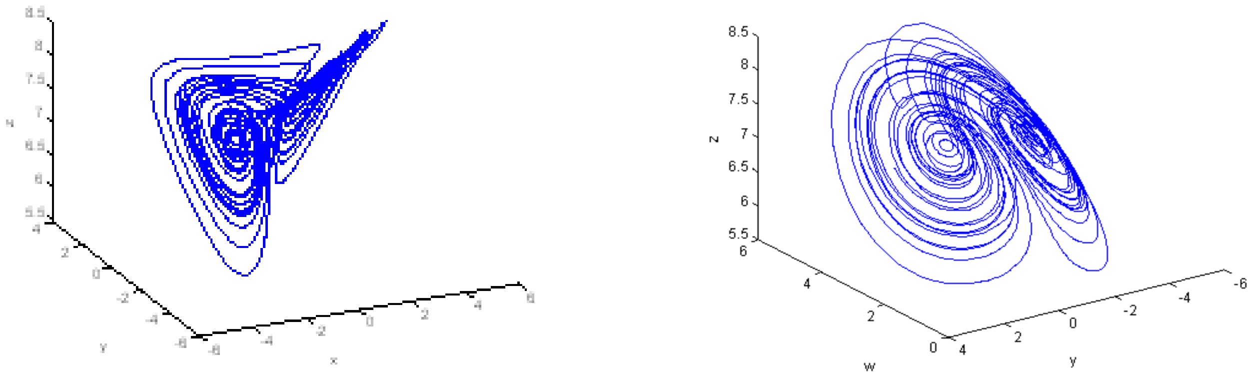

Figure 1: Chaotic attracts of Qi system

$u_i(i = 1, 2, 3, 4)$ are the nonlinear control laws such that two chaotic systems can be synchronized in the sense of MPS, i.e.

Now, define the error signals as

$$\begin{cases} e_1(t) = x_1 - \alpha_1 x_2 \\ e_2(t) = y_1 - \alpha_2 y_2 \\ e_3(t) = z_1 - \alpha_3 z_2 \\ e_4(t) = w_1 - \alpha_4 w_2 \end{cases} \tag{5}$$

From Eq. (5), we have the following error dynamics:

$$\begin{cases} \dot{e_1}(t) = a(y_1 - x_1) - \alpha_1 a_1 (y_2 - y_1) + y_1 z_1 w_1 - \alpha_1 y_2 z_2 w_2 - \alpha_1 u_1 \\ \dot{e_2}(t) = b(x_1 + y_1) - \alpha_2 b_1 (x_2 + y_2) - x_1 z_1 w_1 + \alpha_2 y_2 z_2 w_2 - \alpha_2 u_2 \\ \dot{e_3}(t) = -c z_1 + \alpha_3 c_1 z_2 + x_1 y_1 w_1 - \alpha_3 y_2 z_2 w_2 - \alpha_3 u_3 \\ \dot{e_4}(t) = -d w_1 + \alpha_4 d_1 w_2 + x_1 y_1 z_1 - \alpha_4 y_2 z_2 z_2 - \alpha_4 u_4 \end{cases} \tag{6}$$

3.2 Theorem

Theorem 1 *For two identical chaotic systems without control ($u_i = 0$), if the initial condition of two systems is different, i.e. $x_i(0) \neq y_i(0)$, the trajectories of the two identical systems will quickly separate each other and become irrelevant. However, for the two controlled chaotic systems, the two systems will approach synchronization for any initial condition by appropriate control gain and update laws for uncertain parameters. For this goal, the following control laws and update laws for system (3) are designed:*

$$\begin{cases} u_1 = \frac{1}{\alpha_1}[y_1 z_1 w_1 - \alpha_1 y_2 z_2 w_2 - a_1 y_2(\alpha_1 - \alpha_2) + a_1 e_2 + (k_1 - a_1) e_1] \\ u_2 = \frac{1}{\alpha_2}[-x_1 z_1 w_1 + \alpha_2 y_2 z_2 w_2 + b_1 e_1 + b_1 x_2(\alpha_1 - \alpha_2) + (b_1 + b_2) e_2] \\ u_3 = \frac{1}{\alpha_3}[x_1 y_1 w_1 - \alpha_3 y_2 z_2 w_2 + (k_3 - c_1) e_3] \\ u_4 = \frac{1}{\alpha_4}[x_1 y_1 z_1 - \alpha_4 y_2 z_2 z_2 + (k_4 - d_1) e_4] \end{cases} \tag{7}$$

and

$$\begin{cases} \dot{a_1} = (y_1 - x_1) e_1 \\ \dot{b_1} = (x_1 + y_1) e_2 \\ \dot{c_1} = -z_1 e_3 \\ \dot{d_1} = -w_1 e_4 \end{cases} \tag{8}$$

where $k_i > 0, (i = 1, 2, 3, 4)$ are the control gains of positive scalars.

Proof. For given nonzero scalars $\alpha_i(i = 1, 2, 3, 4)$ MPS between two systems (2) and (3) will occur by the adaptive control law (7) and update law (8). Define a Lyapunov candidate

$$V = \frac{1}{2}(e_1^2 + e_2^2 + e_3^2 + e_4^2 + e_a^2 + e_b^2 + e_c^2 + e_d^2) \tag{9}$$

where $e_a = a_1 - a, e_b = b_1 - b, e_c = c_1 - c, e_d = d_1 - d$, the time derivative of the Lyapunov function along the trajectory of error system (5) is

$$\begin{aligned} \frac{dv}{dt} &= \dot{e_1}e_1 + \dot{e_2}e_2 + \dot{e_3}e_3 + \dot{e_4}e_4 + \dot{e_a}e_a + \dot{e_b}e_b + \dot{e_c}e_c + \dot{e_d}e_d \\ &= e_1[a(x_2 - x_1) - \alpha_1 a_1(y_2 - y_1) + x_2x_3x_4 - \alpha_1 y_2y_3y_4 - \alpha_1 u_1] \\ &\quad + e_2[b(x_1 + x_2) - \alpha_2 b_1(y_1 + y_2) - x_1x_3x_4 + \alpha_2 y_1y_3y_4 - \alpha_2 u_2] \\ &\quad + e_3[-cx_3 + \alpha_3 c_1 y_3 + x_1x_2x_4 - \alpha_3 y_1y_2y_4 - \alpha_3 u_3] \\ &\quad + e_4[-dx_4 + \alpha_4 d_1 y_4 + x_1x_2x_3 - \alpha_4 y_1y_2y_3 - \alpha_4 u_4] \\ &\quad + \dot{a_1}(a_1 - a) + \dot{b_1}(b_1 - b) + \dot{c_1}(c_1 - c) + \dot{d_1}(d_1 - d) \end{aligned} \tag{10}$$

By substituting Eq. (7) and Eq. (8) into Eq. (10), we have $\frac{dv}{dt} = -e^T Pe$, where $e = (e_1, e_2, e_3, e_4)^T, P = diag(k_1, k_2, k_3, k_4)$, Since $\dot{V}$ is negative semi definite, we cannot immediately obtain that the origin of error system (5) is asymptotically stable. In fact, as $\dot{V} \le 0$ then $e_1, e_2, e_3, e_4, e_a, e_b, e_c, e_d \in L_\infty$. From the error system (5), we have $\dot{e_1}, \dot{e_2}, \dot{e_3}, \dot{e_4} \in L_\infty$. Since $\dot{V} = -e^T Pe$, and P are a positive-definite matrix, and then we have $\int_0^t \lambda_{min}\|e\|^2 dt \le \int_0^t -e^T Pedt \le \int_0^t -\dot{V}dt = V(0) - V(t) \le V(0)$. where $\lambda_{min}(P)$ is the minimum eigenvalue of positive-definite matrix P. Thus $e_1, e_2, e_3, e_4 \in L_2$. According to the Barbalat's lemma, we have $e_1(t), e_2(t), e_3(t), e_4(t) \to 0$ as $t \to \infty$. Therefore, the slave system (3) synchronizes the master system (2) in the sense of MPS. This completes the proof. ■

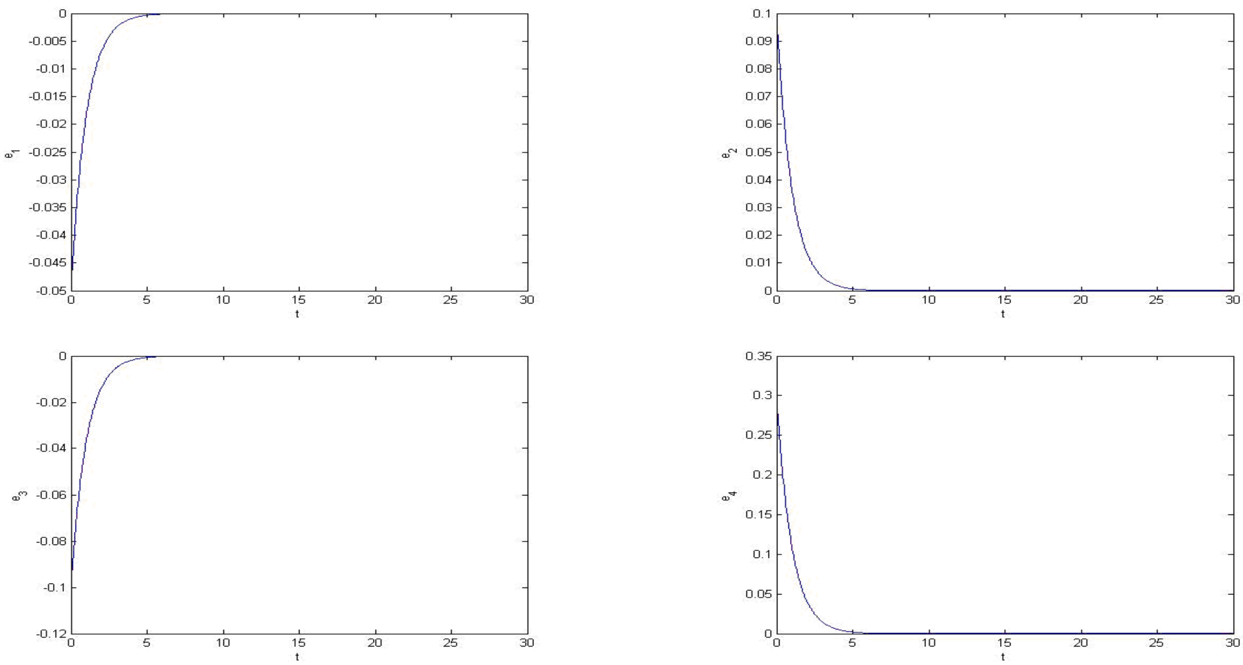

Figure 2: Synchronization errors

3.3 Numerical example

In this section, to verify and demonstrate the effectiveness of the proposed method, we discuss the simulation results for the 4D chaotic system (1). In the numerical simulations, the fourth-order Runge–Kutta method is used to solve the systems with time step size 0.001. For this numerical simulation, we assume that the initial condition $x_1(0), y_1(0), z_1(0), w_1(0) = (5, -5, -3, 2)$ and $x_2(0), y_2(0), z_2(0), w_2(0) = (-5, 5, 5, -5)$ control gains, $k_1, k_2, k_3, k_4 = (15, 1, 1, 1)$ are employed. As a test for verification of MPS of the system, let us take $\alpha_1 = 1, \alpha_2 = 0.5, \alpha_3 = -2, \alpha_4 = -1$. Hence the error system has the initial values $e_1(0) = 10, e_2(0) = -7.5, e_3(0) = 7, e_4(0) = -3$. The four unknown parameters are chosen as $a = 25, b = 10, c = 1$ and $d = 10$ in simulations so that system (1) exhibits a chaotic behavior. Synchronization of systems(2) and (3) via adaptive control law (7) and (8) with the initial estimated parameters $a_1(0) = 20, b_1(0) = 15, c_1(0) = 5$ and $d_1(0) = 5$ are shown in Fig.1 and Fig.2. Fig.1 displays the synchronization errors between systems (2) and (3).

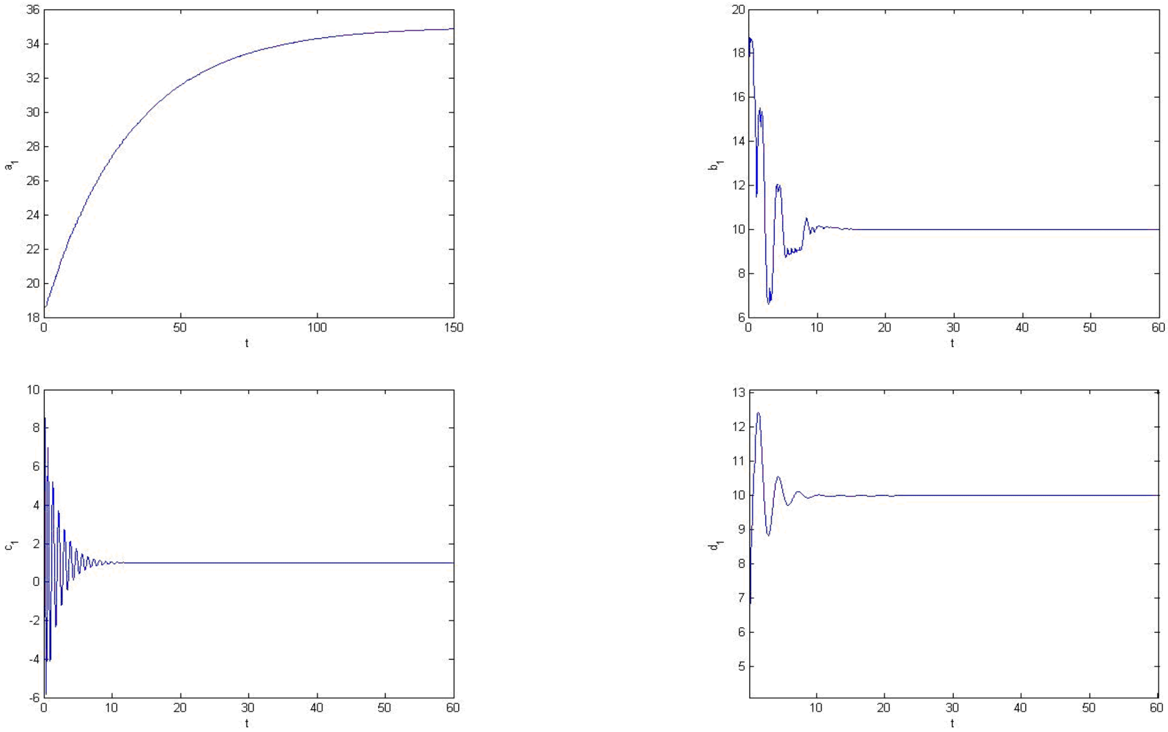

Figure 3: Estimation of uncertain parameters

4 Conclusion

This paper has examined the synchronization of identical Qi system using the technique of active control.The designed active controller ensures a stable synchronization between the drive-response systems. Numerical simulations also employed to illustrate the effective of the approach.

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