ISSN 1749-3889 (print), 1749-3897 (online)
International Journal of Nonlinear Science
Vol.8(2009) No.1,pp. 3-10

Synchronization of Identical and Non-identical 4-D Chaotic Systems via Lyapunov Direct Method

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(Received 2 July 2008, accepted 19 Februry 2009)

Abstract: This paper demonstrates the use of Lyapunov direct method in synchronizing two identical Lorenz-Stenflo (LS) systems, two identical Qi systems and two non-identical systems comprising the LS and Qi chaotic systems.The designed controllers enable the state variables of the slave system to globally synchronize with the state variables of the master system in both the identical and non-identical systems.The results are validated using numerical simulations.

Keywords: synchronization; 4-D chaotic system; Lyapunov direct method
PAC: 05.45.-a,05.45Pq,05.45.Ac

1 Introduction

It has been shown that the synchronization of chaotic systems has potential applications in physical systems [1-3], chemical reactor, ecological systems [4, 5], biomedical systems, secure communications [6-8] to mention but a few.

Resulting from the seminal work of Pecora and Caroll [9] on the synchronization of chaotic systems and the potential application thereof is a new body of research activities which is at the fore front of recent application topics in Nonlinear Dynamics [1,10-12]. Progress in these research activities has given birth to various methods of synchronization. Notable among these methods are linear feedback [2, 3, 13-16], adaptive synchronization [17-18], backstepping nonlinear control [19-24], sliding mode control [25], and active control [26-35]. The Lyapunov direct method has been used for stabilization of systems [36,37] and in the construction of some of the methods of synchronization [19-24], however it has not been applied directly for synchronization. In this paper we apply the Lyapunov method directly to achieve synchronization between identical and non-identical chaotic systems.

The problem of chaos synchronization is related to the observer problem in control theory. In general the designed controller makes the trajectories of the state variables of the response system to track the trajectories of the drive system. The two cases of 4-D chaotic systems to be synchronized are the Lorenz-Stenflo (LS) system [38] and a new 4-D chaotic system referred to as the Qi system [39]. These systems have been synchronized in very recent papers [34, 35] via active control. In addition chaos control in these systems has been examined based on recursive backstepping approach [24]. Also reduced-order synchronization of LS system (4-D) with Lorenz system (3-D) has been carried out using adaptive control [40].

However, considering the calculations involved in the active control and backstepping methods we propose the synchronization of these systems via Lyapunov direct method. This method is shorter and easier to manipulate than the methods of active control and backstepping.

In this paper Lyapunov direct method is applied to synchronize two identical LS systems, two identical Qi systems and two non-identical chaotic systems comprising the LS and Qi systems. The rest of the paper is organized as follows. Section 2 studies the synchronization behaviour of two identical LS systems, section 3 deals with the synchronization behaviour of two identical Qi systems, section 4 considers the synchronization between the LS and Qi systems and section 5 concludes the paper.

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IJNS.2009.08.15/237

2 Synchronization of two identical Lorenz-Stenflo systems

2.1 The Lorenz-Stenflo system

The Lorenz-Stenflo (LS) system was formulated by Stenflo [38] from a low-frequency short-wavelength gravity wave equation. It comprises the following system of first order differential equations.

$$\begin{aligned} \dot{x} &= \alpha(y-x)+\gamma w \\ \dot{y} &= x(r-z)-y \\ \dot{z} &= xy-\beta z \\ \dot{w} &= -x-\alpha w \end{aligned} \tag{1}$$

where the dots denote derivatives with respect to time, the parameters r,α,γ,β (all positive) are respectively the Rayleigh number, Prandtl number, rotational number, and geometric parameter. The LS system (1) is similar to the famous Lorenz equation, but differs from it by the introduction of the new control parameter γ, and a new state variable w describing the flow rotation. Thus system (1) reduces to the Lorenz system if γ and w are set to zero. With r =26.0, α=2.0, γ=1.5 and β=0.7, the LS system exhibits the chaotic attractor shown in Fig.1. The attractor is different from those obtained in [34,38] with $\alpha = 1$.

2.2 Formulation

We consider an LS system given by

$$\begin{aligned} \dot{x}_1 &= \alpha(y_1-x_1)+\gamma w_1 \\ \dot{y}_1 &= x_1(r-z_1)-y_1 \\ \dot{z}_1 &= x_1y_1-\beta z_1 \\ \dot{w}_1 &= -x_1-\alpha w_1 \end{aligned} \tag{2}$$

which drives a similar LS system given as

$$\begin{aligned} \dot{x}_2 &= \alpha(y_2-x_2)+\gamma w_2+u_1(t) \\ \dot{y}_2 &= x_2(r-z_2)-y_2+u_2(t) \\ \dot{z}_2 &= x_2y_2-\beta z_2+u_3(t) \\ \dot{w}_2 &= -x_2-\alpha w_+u_4(t) \end{aligned} \tag{3}$$

where $u(t) = [u_1(t), u_2(t), u_3(t), u_4]^T$ is the control function which will be determined such that system (3) can synchronize with system (2). Now, let the error states between the state variables of systems (3) and (2) be

$$e_x = x_2 - x_1, \quad e_y = y_2 - y_1, \quad e_z = z_2 - z_1, \quad e_w = w_2 - w_1 \tag{4}$$

$$\Rightarrow x_2 = x_1 + e_x, \quad y_2 = y_1 + e_y, \quad z_2 = z_1 + e_z, \quad w_2 = w_1 + e_w \tag{5}$$

Subtracting (2) from (3) and replacing $i_2 - i_1$ with e_i and i_2 with $i_1 + e_i$, $i = x, y, z, w$ we obtain the error dynamics between (3) and (2) as

$$\begin{aligned} \dot{e}_x &= \alpha(e_y-e_x)+\gamma e_w+u_1(t) \\ \dot{e}_y &= e_x(r-z_1)-e_y-e_z(e_x+x_1)+u_2(t) \\ \dot{e}_z &= e_x(e_y+y_1)+e_yx_1-\beta e_z+u_3(t) \\ \dot{e}_w &= -e_x-\alpha e_w+u_4(t) \end{aligned} \tag{6}$$

By the Lyapunov direct method we consider a Lyapunov function

$$V(e_x, e_y, e_z, e_w) = \frac{1}{2}\sum_i k_i e_i^2 \tag{7}$$

[where $k_i(> 0)$, $i = x, y, z, w$, are constant coefficients], differentiate it with respect to t and choose the controller $u\,(t)$ such that $\dot{V}\,(e_x, e_y, e_z, e_w)$ is negative definite in order to stabilize the error dynamics (6) and hence to achieve synchronization between the two systems under the chosen controller. The time derivative of (7) is

$$\dot{V}\,(e_x, e_y, e_z, e_w) = k_x e_x \dot{e}_x + k_y e_y \dot{e}_y + k_z e_z \dot{e}_z + k_w e_w \dot{e}_w \tag{8}$$

Substituting the $\dot{e}_i$, $i = x, y, z, w$, in (8) with the corresponding expressions in (6) and choosing $u\,(t)$ to be

$$\begin{aligned} u_1\,(t) &= -e_x - \alpha\,(e_y - e_x) - \gamma e_w \\ u_2\,(t) &= e_z\,(e_x + x_1) + e_x\,(z_1 - r) \\ u_3\,(t) &= -e_z\,(1 - \beta) - e_x\,(e_y + y_1) - e_y x_1 \\ u_4\,(t) &= e_x - e_w\,(1 - \alpha) \end{aligned} \tag{9}$$

equation (8) becomes

$$\dot{V}\,(e_x, e_y, e_z, e_w) = -k_x e_x^2 - k_y e_y^2 - k_z e_z^2 - k_w e_w^2 \tag{10}$$

which is negative definite thereby guaranteeing stability of the error dynamics (6) at the origin and hence the synchronization of systems (2) and (3) by the controller defined in (9).

2.3 Numerical results

Using the fourth order Runge-Kutta algorithm with initial conditions (x_1, y_1, z_1, w_1) =(0.01,0.02,0.03,0.04), (x_2, y_2, z_2, w_2) =(0.05,0.06,0.07,0.08), a time step of 0.005 and fixing the parameters values of r, α, γ, β as in Fig.1, to ensure chaotic motion we solved the systems (2) and (3) with the controller $u\,(t)$ as defined in (9). The results obtained show that the error states oscillate chaotically with time when the controller is turned off (Fig.2) and when the controller is switched on at t =100 the error states converge exponentially to zero (Fig.3). We thus see that the master-slave system is synchronized by the designed controller. This is also confirmed by the convergence of the synchronization quality defined by the error propagation on the error states as

$$e = \sqrt{e_x^2 + e_y^2 + e_z^2 + e_w^2} \tag{11}$$

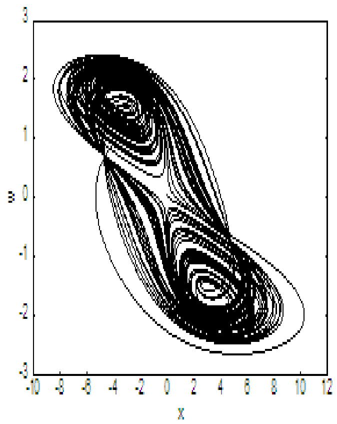

Figure 1: 2-Dimensional (x-w) view of the Lorenz-Stenflo attractor for parameter values α=2.0, β=0.7, γ=1.5 and r=26.0.

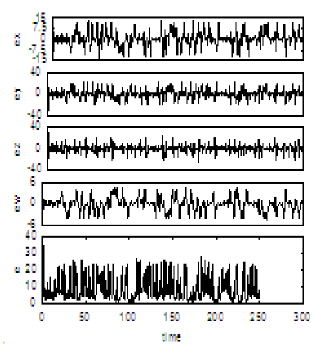

Figure 2: Time series of the error states (e_x, e_y, e_z, e_w) and the error propagation e of the coupled Lorenz-Stenflo systems when the controller is deactivated.

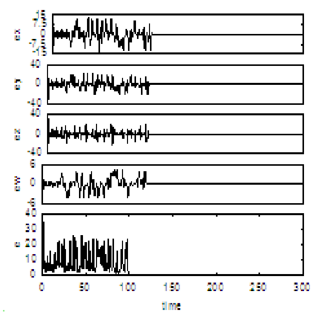

Figure 3: Time series of the error states (e_x, e_y, e_z, e_w) and the error propagation e of the coupled Lorenz-Stenflo systems when the controller is activated at t=100.

3 Synchronization of two identical Qi systems

3.1 The Qi system

The Qi system [39] comprises the following 4-D autonomous system of equations with three cross products.

$$\begin{aligned} \dot{x} &= a(y-x) + yzw \\ \dot{y} &= b(x+y) - xzw \\ \dot{z} &= -cz + xyw \\ \dot{w} &= -dw + xyz \end{aligned} \quad (12)$$

where x, y, z and w are the state variables of the system and a, b, c and d are all positive real constant parameters. System (12) has been found to exhibit complex dynamics leading to chaos [39]. For instance in Fig.4, we show a chaotic attractor in the $x - w$ plane for a =30, b =10, c =1 and d =10.

3.2 Formulation

Again, as in section 2, we choose a master Qi system given by

$$\begin{aligned} \dot{x}_1 &= a(y_1 - x_1) + y_1 z_1 w_1 \\ \dot{y}_1 &= b(x_1 + y_1) - x_1 z_1 w_1 \\ \dot{z}_1 &= -cz_1 + x_1 y_1 w_1 \\ \dot{w}_1 &= -dw_1 + x_1 y_1 z_1 \end{aligned} \quad (13)$$

which drives a slave Qi system given by

$$\begin{aligned} \dot{x}_2 &= a(y_2 - x_2) + y_2 z_2 w_2 + u_1(t) \\ \dot{y}_2 &= b(x_2 + y_2) - x_2 z_2 w_2 + u_2(t) \\ \dot{z}_2 &= -cz_2 + x_2 y_2 w_2 + u_3(t) \\ \dot{w}_2 &= -dw_2 + x_2 y_2 z_2 + u_4(t) \end{aligned} \quad (14)$$

where $u(t) = [u_1(t), u_2(t), u_3(t), u_4(t)]^T$ is the control function to be determined. Subtracting (13) from (14) and using (4) and (5) we obtain the following system of error dynamics.

$$\begin{aligned} \dot{e}_x &= a(e_y - e_x) + (e_w + w_1)(e_y e_z + e_y z_1 + e_z y_1) - e_w y_1 z_1 + u_1(t) \\ \dot{e}_y &= b(e_x + e_y) - (e_w + w_1)(e_x e_z + e_z x_1 + e_x z_1) - e_w x_1 z_1 + u_2(t) \\ \dot{e}_z &= -ce_z + (e_w + w_1)(e_x e_y + e_x y_1 + e_y x_1) + e_w x_1 y_1 + u_3(t) \\ \dot{e}_w &= -de_w + (e_z + z_1)(e_x e_y + e_x y_1 + e_y x_1) + e_z x_1 y_1 + u_4(t) \end{aligned} \quad (15)$$

Again we apply the Lyapunov direct method outlined in subsection 2.2 by substituting (15) in (8) and choosing the controller $u(t) = [u_1(t), u_2(t), u_3(t), u_4(t)]^T$ as

$$\begin{aligned} u_1(t) &= -e_x - a(e_y - e_x) - (e_w + w_1)(e_y e_z + e_y z_1 + e_z y_1) - e_w y_1 z_1 \\ u_2(t) &= -e_y - b(e_x + e_y) + (e_w + w_1)(e_x e_z + e_x z_1 + e_z x_1) + e_w x_1 z_1 \\ u_3(t) &= e_z(c-1) - (e_w + w_1)(e_x e_y + e_x y_1 + e_y x_1) - e_w x_1 y_1 \\ u_4(t) &= e_w(d-1) - (e_z + z_1)(e_x e_y + e_x y_1 + e_y x_1) - e_z x_1 y_1 \end{aligned} \quad (16)$$

so as to make $\dot{V}(e_x, e_y, e_z, e_w)$ in (8) negative definite as in (10). This guarantees the stability of the error dynamics (15) at the origin and hence the synchronization of systems (13) and (14).

3.3 Numerical results

Using the fourth order Runge-Kutta algorithm with initial conditions (x_1, y_1, z_1, w_1) =(0.01,0.02,0.03,0.04), (x_2, y_2, z_2, w_2) =(0.05,0.06,0.07,0.08), a time step of 0.0005 and fixing the parameter values of a, b, cdas in Fig.4, to ensure chaotic motion we solved the systems (13) and (14) with the controller $u\,(t)$ as defined in (16). The results obtained show that the error states oscillate chaotically with time when the controller is switched off (Fig.5) and when the control is switched on at t =12 the error states converge to zero (Fig.6). We thus see that the two systems are synchronized by the designed controller. This is also confirmed by the convergence of the synchronization quality defined in (11).

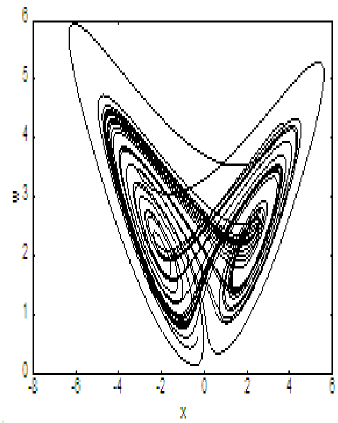

Figure 4: 2-Dimensional (x-w) view of the Qi attractor for parameter values a=30, b=10, c=1 and d=10.

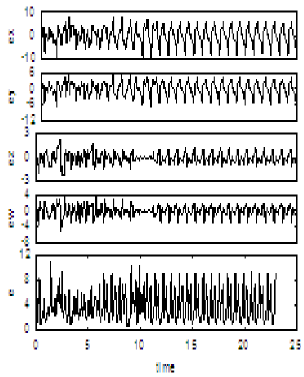

Figure 5: Time series of the error states (e_x, e_y, e_z, e_w) and the error propagation e of the coupled Qi systems when the controller is deactivate.

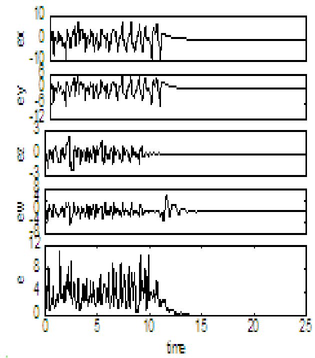

Figure 6: Time series of the error states (e_x, e_y, e_z, e_w) and the error propagation e of the coupled Qi systems when the controller is activated at t=12.

4 Synchronization between LS and Qi systems

4.1 Formulation

Here we choose the LS system (2) as the drive system and the Qi system (14) as the response system. This implies that when the two systems are synchronized the Qi system will track the LS system. To achieve this we proceed as in subsections 2.2 and 3.2, that is, we subtract (2) from (14) and apply (4) and (5) to obtain the following error dynamics.

$$
\begin{aligned}
\dot{e}_x &= a\,(e_y - e_x) - (a-\alpha)\,(x_1 - y_1) - \gamma w_1 + (e_w + w_1)\,(e_y e_z + e_y z_1 + e_z y_1 + y_1 z_1) + u_1\,(t)\\
\dot{e}_y &= b\,(e_x + e_y + y_1) - (r - b - z_1)\,x_1 - (e_w + w_1)\,(e_x e_z + e_x z_1 + e_z x_1 + x_1 z_1) + u_2\,(t)\\
\dot{e}_z &= \beta z_1 - c\,(e_z + z_1) + (e_w + w_1)\,(e_x e_y + e_x y_1 + e_y x_1 + x_1 y_1) + u_3\,(t)\\
\dot{e}_w &= \alpha w_1 - d\,(e_w + w_1) + (e_z + z_1)\,(e_x e_y + e_x y_1 + e_y x_1 + x_1 y_1) + u_4\,(t)
\end{aligned} \tag{17}
$$

As before we apply the Lyapunov direct method outlined in subsection 2.2 by substituting (17) in (8) and choosing the controller $u\,(t) = [u_1\,(t)\,, u_2\,(t)\,, u_3\,(t)\,, u_4\,(t)]^T$ as

$$
\begin{aligned}
u_1 &= (a-1)\,e_x - a e_y + (a-\alpha)\,(x_1 - y_1) + \gamma w_1 - (e_w + w_1)\,(e_y e_z + e_y z_1 + e_z y_1 + y_1 z_1)\\
u_2 &= (r - b - z_1)\,x_1 - (b+1)\,(e_y + y_1) - b e_x + (e_w + w_1)\,(e_x e_z + e_x z_1 + e_z x_1 + x_1 z_1)\\
u_3 &= (c-1)\,e_z + (c-\beta)\,z_1 - (e_w + w_1)\,(e_x e_y + e_x y_1 + e_y x_1 + x_1 y_1)\\
u_4 &= (d-1)\,e_w + (d-\alpha)\,w_1 - (e_z + z_1)\,(e_x e_y + e_x y_1 + e_y x_1 + x_1 y_1)
\end{aligned} \tag{18}
$$

If (17), with $u\,(t)$ as defined in (18), is substituted in (8) the time derivative of the Lyapunov function, $\dot{V}\,(e_x, e_y, e_z, e_w)$, is negative definite as in (10) which implies stability of the error dynamics (17) at the origin and hence the synchronization of the LS and the Qi systems by the controller (18).

4.2 Numerical results

Using the fourth order Runge-Kutta algorithm with initial conditions (x_1, y_1, z_1, w_1) =(0.01,0.02,0.03,0.04), (x_2, y_2, z_2, w_2) =(0.05,0.06,0.07,0.08), a time step of 0.0001 and fixing the parameter values of r, α, γ, β as in Fig.1 and a, b, cdas in Fig.4, to ensure chaotic motion we solved the systems (2) and (14) with the controller $u(t)$ as defined in (18). The results obtained show that the error states oscillate chaotically with time when the controller is switched off (Fig.7) and when the control is switched on at t =6 the error states converge to zero (Fig.8). We thus see that the LS and Qi systems are synchronized by the designed controller. This, again, is confirmed by the convergence of the synchronization quality defined in (11)

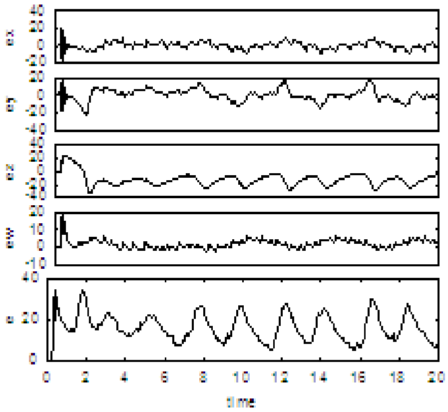

Figure 7: Time series of the error states (e_x, e_y, e_z, e_w) and the error propagation e of the coupled LS and Qi systems when the controller is deactivate.

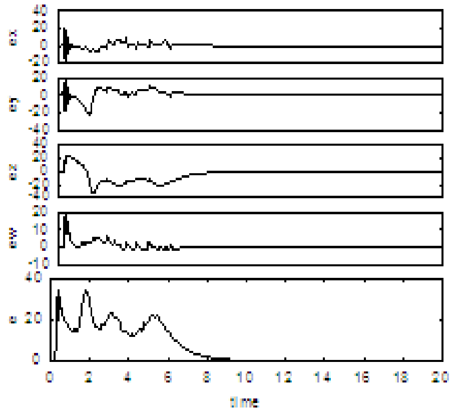

Figure 8: Time series of the error states (e_x, e_y, e_z, e_w) and the error propagation e of the coupled LS and Qi systems when the controller is activated at t=6.

5 Conclusion

This paper has examined the use of Lyapunov direct method in the synchronization of 4-D chaotic systems, consisting of two identical Lorenz-Stenflo (LS) systems, two identical Qi systems and two non-identical systems comprising the LS and Qi systems. The designed controllers were capable of making the time derivative of the Lyapunov function negative definite in each case. This guarantees stability of the error dynamics at the origin and hence synchronization of the identical and non-identical systems. Numerical simulations were also carried out to illustrate the effectiveness of the approach. The Lyapunov direct method of designing chaotic controllers is shorter and easier to carry out than other methods in the same category, and the designed controllers are effective in chaos synchronization.

Acknowledgement

The authors are grateful to U.E. Vincent for reading the manuscripts and making invaluable suggestions.

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