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On Deformed Algebra in Particle Physics

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Abstract: Hopf algebra is shown to lead , via infinitesimal deformation, to an explanation of confinement which is a standard feature in non-abelian gauge theories. We also mention that deformed algebra is related to fractional calculus (which is ,by definition, nonlocal). This motivates the study of nonlocal field theory. It is known that such a theory is finite, free of (the so far unobserved) Higgs particles and it agrees with the observed nonlocality of quantum mechanics.

Keywords: Nonlinear deformations; fractional order

1 Lie Algebra Deformation

Definition 1 *A Lie algebra is defined as a vector space "g" that satisfying the following condition:*
(i) $[x, y]$ *is a binary operation i.e. [x ,y] = x y -y x .*
(ii) $[x, y] = -[y, x]$
$(iii)[x, [y, z]] + [y, [z, x]] + [z, [x, y]] = 0$
such that: x , y , z *is in g.*

The condition (iii) is called Jacobi identity, Sweedler [10].

Let "g" be a Lie algebra of a Lie group G with dimension n and basis $\{x_1,x_2,...,x_n\}$ over a field $K = R$ or C defined by the relation :

$$[x_i, x_j] = C^k_{ij} x_k \tag{1}$$

where C^k_{ij} i,j,k=$1, 2, ..., n$ are the structure constants.

The deformed quantum bosonic oscillator algebra is given by:

$$[a, a^+] = q^{2N}, [N, a] = -a, [N, a^+] = a^+ \tag{2}$$

while the fermionic one is given by:

$$[b, b^+] = q^{2N} \ , \ [N, b] = -b \ , \ [N, b^+] = b^+ \ , \ b^2 = b^{+2} = 0$$

where N is the number operator. For the infinitesimally deformed oscillators one sets $q = 1+t$ and linearize in t to get:

$$\begin{aligned} [a, a^+] &= 1 + 2tN \ , \quad N = a^+a - ta^{+2}a^2 \\ [b, b^+] &= 1 + 2tN \ , \quad N = b^+b \end{aligned}$$

Using the standard definition of the Hamiltonian H=lN ,where l is the generic energy of the oscillator, and the definition of the coordinates and momentum we get Ahmed and Hegazi [1]

$$q = \hbar/(2mw)(a + a^+) \ , \ p = i\hbar mw/2(a + a^+) \ , \ [q, p] = i\hbar + ltH \tag{3}$$

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IJNS.2009.06.30/253

It is known Saavedra and Utreras [9] that this algebra can explain quark confinement since it corresponds to the modified uncertainty relation

$$\delta q \delta p \geqslant \hbar/2 + tE/2 \quad (4)$$

2 Deformation of algebras and fractional order calculus

Caputo's definition for derivative of order α , $0 < \alpha \leq 1$ is given by

$$D^{\alpha} f(t) = [1/\Gamma(1-\alpha)] \int_0^t f'(s)/(t-s)^{\alpha} ds$$

A correspondence between fractional order parameter α and deformation parameter q has been proposed via the eigenvalues of the oscillators as follows Herrmann [6]

$$\begin{aligned} E^q(n) &= (\hbar w/2)(([n]_q + [n+1]_q), [n]_q = (q^n - q^{-n})/(q - q^{-1}) \\ E^{\alpha}(n) &= (1/2)([n]_{\alpha} + [n+1]_{\alpha}) \\ [n]_{\alpha} &= 2^{1+\alpha} \pi^{\alpha/2} \{\alpha[\Gamma((1+\alpha)/(2\alpha)]/\Gamma(1/2\alpha)]\}^{\alpha} [\varsigma(-\alpha, n/2 + 1/4) - \varsigma(-\alpha, n/2 + 3/4)] \end{aligned}$$

where $\varsigma(-s, x)$ is Riemann zeta function defined by:

$$\varsigma(-s, x) = \sum_{k=0}^{\infty} (k+x)^{-s}$$

Therefore deformed theories corresponds to fractional order ones.

By definition fractional order is non-local. Non-locality is a phenomena which is known to exist in quantum systems Wikipedia [7]. A non-local electroweak theory has been constructed Moffat [8] and it has been shown to be finite. The observed W and Z bosons are derived. Moreover symmetry breaking does not need the existence of Higg's particles. It is known that the absence of Higgs particles is one of the important problems in the standard model which is currently searched for by the large hadron collider of CERN.

3 Conclusions

In this letter the following points have been shown:
(i) Infinitesimally deformed oscillator algebra corresponds to a Heisenberg relation which may explain confinement.
(ii) There is a correspondence between q-deformed oscillators and fractional order ones. The fractional order derivative is by definition nonlocal. Nonlocality is known to exist in quantum systems. Nonlocal field theory are known to be finite and does not need the (so far unobserved) Higg's particles.

Nonlinear science is expanding in many different fields e.g. algebra (our paper); Physics [5], game theory [2], nonlinear dynamics [3] and economics [4].

References

[1] E.Ahmed, A.S.Hegazi A.S.: Infinitesimally deformed field and string theories. *J.Math.Phys*.33(1):379-381(1992)

[2] E.Ahmed, H.A.Abdusalam: On dynamical games. *International Journal of Nonlinear Science*. 6: 59-63 (2008)

[3] E. M. ELabbasy, H. N. Agiza, H. EL-Metwally, A. A. Elsadany : Bifurcation Analysis, Chaos and Control in the Burgers Mapping. *International Journal of Nonlinear Science*. 4: 171-185(2007)

[4] E. M. Elabbasy, H. N. Agiza, A. A. Elsadany, H. EL-Metwally : The Dynamics of Triopoly Game with Heterogeneous Players . *International Journal of Nonlinear Science*. 3: 83-90 (2007)

[5] S. El Wakil, E.Abulwafa, M.Abdou: New PeriodicWave Solution to Nonlinear Evolution Equations Arising in Physics. *International Journal of Nonlinear Science*. 7:75-83 (2009)

[6] Richard Herrmann: q-deformed Lie algebras and fractional calculus.*Physics- General Physics*. arXiv.0711.3701.(2007)

[7] http://en.wikipedia.org/wiki/nonlocality.

[8] J.W. Moffat J.W.: Electroweak model without a Higg's particle. *High Energy Physics - Phenomenology* . arXiv:0709.4269v6(2008)

[9] A. Saavedra, C. Utreras: A generalization of quantum mechanics for high energy and quark physics. *phys. Lett. B*. 98: 74-76 (1981)

[10] M.E. Sweedler, Hopf Algebras,W.A. Benjamin:INC. *New York* (1969)