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Exact Travelling Wave Solutions of Generalized Zakharov Equations with Arbitrary Power Nonlinearities

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Abstract: An extended F-expansion method with a computerized symbolic computation for constructing a new exact travelling wave solutions for generalized Zakharov equations with arbitrary power nonlinearitie.As a result, many exact travelling wave solutions are obtained which include new periodic wave solution,trigonometric function solutions and rational solutions.The method is straightforward and concise,and it can also be applied to other nonlinear evolution equations arising in mathematical physics. It is worthwhile to mention that the method is straightforward and concise, and it can also be applied to other nonlinear evolution equations in physics.

Keywords:generalized Zakharov equations;F-expansion method;new travelling wave solutions;solitary wave solutions

1 Introduction

The investigation of the travelling wave solutions for nonlinear evolution equations arising in mathematical physics plays an important role in the study of nonlinear physical phenomena.The nonlinear evolution equations are major subjects in physical science, appears in various scientific and engineering fields,such as fluid mechanics,plasma physics,optical fibers,biology,solid state physics, chemical kinematics, chemical physics and ochemistry.In the past several decades,new exact solutions may help to find new phenomena.A variety of powerful methods for obtaining the exact solutions of nonlinear evolution equations have been presented $[1-15]$.

The application of computer algebra to science has a bright future.In the field of nonlinear science,to find as many and general as possible exact solutions for a nonlinear system is one of the most fundamental and significant study. In the line with the development of computerized symbolic computation,much work has been focused on the various extensions and application of the known algebraic methods to construct the solutions of nonlinear evolution equations.

In recent years,numerical analysis [16] has considerably been developed to be used for nonlinear partial equations such as Ginzburg-Landau equation,which is a class of a Schrodinger equation with a nonlinear term [17].This equation governs the finite amplitude evolution of instability waves in a large variety of dissipative systems which are close to criticality. Various forms of Ginzburg-Landau equation arise in hydrodynamic instability theory: the development of Tollmien-Schlichting waves in plane Poiseuille flows, the nonlinear growth of convection rolls in the Rayleigh-Bnard problem, and appearance of Taylor vortices in the flow between counter rotating circular cylinders [18, 19],

The rest of this paper is arranged as follows. Section 2 contains the description of the problem of generalized Zakharov equations.In Section 3, we simply provide the mathematical framework of the extended

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F-expansion method is described in Section 4 and also used to give the solution of the generalized Zakharov equations with arbitrary power nonlinearities which include new soliton like solutions,trigonometric function solutions and exponentional function solutions.And we conclude the paper in the last section.

2 Formulating the problem

Let us first consider the generalized Zakharov equations as [20]

$$H_{tt} - H_{xx} = (|u|^{2m})_{xx}, \tag{1}$$

$$iu_t + u_{xx} = Hu + \alpha|u|^{2m}u + \beta|u|^{4m}u, \tag{2}$$

where $m > 0$,α,β are constants.If $m = 1$,$\alpha = \beta = 0$,Eqs.(1) and (2) reduces to the famous Zakhrov equation as

$$H_{tt} - H_{xx} = (|u|^2)_{xx}, \tag{3}$$

$$iu_t + u_{xx} = Hu \tag{4}$$

Making use the following transformation

$$u(x,t) = v(x,t)^{\frac{1}{2m}} e^{i(kx+wt)} \tag{5}$$

Substituting Eq.(5) into Eqs.(1) and (2),we have

$$H_{tt} - H_{xx} = v_{xx}, \tag{6}$$

$$i[v_t + 2kv_x] - 2mwv + v_{xx} + [\frac{1}{2m} - 1]\frac{v_x^2}{v} - 2mk^2v - 2m[Hv + \alpha v^2 + \beta v^3] = 0 \tag{7}$$

By introducing a complex variation defined as $\eta = \mu(x - 2kt)$,then Eqs.(6) and (7) reduces to

$$\mu^2[4k^2 - 1]H^{''} = \mu^2 v^{''} - 2mwv + \mu^2 v^{''} + [\frac{1}{2m} - 1]\mu^2\frac{v^{'2}}{v} - 2mk^2v - 2m[Hv + \alpha v^2 + \beta v^3] = 0 \tag{8}$$

Solving Eq.(6) for H ,we obtain

$$H = \frac{v}{4k^2 - 1} \tag{9}$$

Inserting Eq.(9) into (8) yields

$$-2mwv + \mu^2 v^{''} + [\frac{1}{2m} - 1]\mu^2\frac{v^{'2}}{v} - 2mk^2v - 2m[\frac{v^2}{4k^2 - 1} + \alpha v^2 + \beta v^3] = 0 \tag{10}$$

The motivation in this paper,we purpose to present implementation the extended F expansion method for solving the reduced Eq.(10) and many exact travelling wave solutions are obtained which include new periodic wave solution, trigonometric function solutions,exponentional solutions and rational solutions.

3 Basic idea of generalized F-expansion method

For a given nonlinear evolution equations,in two independent variables x and t as follows

$$M(\phi, \phi_x, \phi_t, \phi_{xx}, ...) = 0, \tag{11}$$

where M is in general a polynomial in ϕ and its various partial derivatives. Seeking its travelling wave solution of Eq.(11) by taking

$$\phi(x,t) = \phi(\eta), \eta = k(x + \lambda t), \tag{12}$$

where k and λ are constants to be determined later,inserting (12) into (11) yields an ODE for $\phi(\eta)$

$$\psi(\phi, k\phi^{'}, \lambda k\phi^{'}, k^2\phi^{''}, ...) = 0 \tag{13}$$

The next crucial step is that solution we are looking for is expressed in the general form

$$\phi(\eta) = a_0 + \sum_{i=1}^{N}[a_iF^i(\eta) + b_iF^{-i}(\eta) + c_iF^{i-1}(\eta)F^{'}(\eta) + d_iF^{-i}(\eta)F^{'}(\eta)], \tag{14}$$

where $a_0 = a_0(x)$,$a_i = a_i(x)$,$b_i = b_i(x)$,$c_i = c_i(x)$,$d_i = d_i(x)(i = 1, 2, .., n)$, N is a positive integer that can be determined by balancing the hightest order linear term with the nonlinear terms in the equation,and $F(\eta)$ and $F^{'}(\eta)$ satisfies the Riccati equation

$$F^{'}(\eta) = A + BF(\eta) + CF^2(\eta), \tag{15}$$

where A,B and C are constants to be determined.Inserting Eq.(14) into (13),with the aid of Eq.(15),the left hand side of Eq.(13) can be converted into a finite series in $F^{'i}(\eta)F^j(\eta)$,and equating each coefficient of $F^{'i}(\eta)F^j(\eta)(i = 0, 1; j = 0, \pm1, \pm2, ...)$ to zero yields a system of algebraic equations for a_0,a_i,b_i,c_i,$d_i(i = 1, 2, .., n)$.Solve the system of algebraic equations for a_0,a_i,b_i,c_i,$d_i(i = 1, 2, .., n)$ can be expressed by A,B,C.Substituting these results into Eq.(14) we can obtain the general form for travelling wave solutions to Eqs.(1) and (2).With the aid of the $Appendix A$, from the general form of travelling wave solutions, we can give a series of soliton-like solutions,trigonometric function solutions and rational solutions of Eqs.(1) and (2).

4 New exact solutions of generalized Zakharov equations

Our gool in this paper is to solve Eq.(10) by extended F-expasion method mentioned above.Considering the homogeneous balance between $v^3(\eta)$ and $v^{''}(\eta)$ in Eq.(10),yields $N = 1$, we suppose that the solution of Eq.(10) can be expressed by

$$v(\eta) = a_0 + [a_1F(\eta) + b_1F^{-1}(\eta) + c_1F^{'}(\eta) + d_1F^{-1}(\eta)F^{'}(\eta)], \tag{16}$$

where a_0,a_1,b_1,c_1 and d_1 are constants to be determined later,$F(\eta)$,$F^{'}(\eta)$ satisfy Eq.(15).

Substituting Eq.(16) along with (15) into Eq.(10),the left hand side of Eq.(10) is converted into a polynomial of $F^{'i}(\eta)F^j(\eta)(i = 0, 1; j = 0, \pm1, \pm2, ...)$,then setting each coefficients to zero,we get a set of over determined algebraic system for a_0,a_1,b_1,c_1,d_1,$k$and w.Solving the system of over-determined algebraic equations using Maple, we get the following solution :

Case A:When $A = 0$,we have

$$b_1 = 0, c_1 = 0, d_1 = d_1, a_1 = -d_1C, a_0 = -d_1B, w = w, k = k \tag{17}$$

Case B:For $B = 0$,we have

$$w = w, b_1 = -d_1A, c_1 = 0, d_1 = d_1, a_0 = 0, k = k, a_1 = -d_1C \tag{18}$$

Case C:When $A = B = 0$,we have

$$b_1 = 0, c_1 = 0, d_1 = d_1, a_0 = 0, k = k, a_1 = -Cd_1, w = w \tag{19}$$

Substituting these solutions into (16),with the aid of Appendix A ,we can obtain many soliton like solutions,trigonometric function solutions and rational solutions of Eqs.(1) and (2) as follows:

(I):For $A = 0$,$B = 1$,$C = -1$, from the Appendix A,then $F(\eta) = [\frac{1}{2} - \frac{1}{2}tanh(\eta)]$. By case (A), we have soliton-like solutions of Eq.(1)

$$u_1(\eta) = [a_0 - a_0[\frac{1}{2} + \frac{1}{2}tanh(\eta)] - \frac{a_0[\frac{1}{2} - \frac{1}{2}tanh^2(\eta)]}{[\frac{1}{2} + \frac{1}{2}tanh(\eta)]}]^{\frac{1}{2m}}e^{i(kx+wt)},$$

$$H_1(\eta) = \frac{u_1(\eta)}{4k^2 - 1} \tag{20}$$

(II):In case of $A = 0$,$B = -1$,$C = 1$, from the Appendix A, then $F(\eta) = [\frac{1}{2} - \frac{1}{2}coth(\eta)]$.By case (A),we have soliton-like solutions of Eqs.(1) and (2)

$$u_2(\eta) = [a_0 - a_0[\frac{1}{2} - \frac{1}{2}coth(\eta)] + \frac{a_0[-\frac{1}{2} + \frac{1}{2}coth^2(\eta)]}{[\frac{1}{2} - \frac{1}{2}coth(\eta)]}]^{\frac{1}{2m}} e^{i(kx+wt)},$$

$$H_2(\eta) = \frac{u_2(\eta)}{4k^2 - 1} \quad (21)$$

(III):If $A = \frac{1}{2}$,$B = 0$,$C = -\frac{1}{2}$ from the Appendix A, then $F(\eta) = coth(\eta) + csch(\eta)$ or $F(\eta) = tanh(\eta) \pm isech(\eta)$.By case (B),we have soliton-like solutions of Eqs.(1) and (2)

$$u_3(\eta) = [\frac{d_1}{2}(coth(\eta)+csch(\eta)) - \frac{d_1}{2(coth(\eta) + csch(\eta))} + \frac{d_1[1 - coth^2(\eta) - csch(\eta)coth(\eta)]}{[coth(\eta) + csch(\eta)]}]^{\frac{1}{2m}} e^{i(kx+wt)},$$

$$u_4(\eta) = [\frac{d_1}{2}[tanh(\eta)+isech(\eta)] - \frac{d_1}{2[tanh(\eta) + isech(\eta)]} + \frac{d_1[1 - tanh^2(\eta) - isech(\eta)tanh(\eta)]}{[tanh(\eta) + isech(\eta)]}]^{\frac{1}{2m}} e^{i(kx+wt)},$$

$$H_3(\eta) = \frac{u_3(\eta)}{4k^2 - 1},$$

$$H_4(\eta) = \frac{u_4(\eta)}{4k^2 - 1}, \quad (22)$$

(IV):For $A = 1$,$B = 0$,$C = -1$ from the Appendix A,then $F(\eta) = tanh(\eta)$.By case (B),we have soliton-like solutions of Eqs.(1) and (2)

$$u_5(\eta) = [-b_1tanh(\eta) + \frac{b_1}{tanh(\eta)} - \frac{b_1(1 - tanh^2(\eta))}{tanh(\eta)}]^{\frac{1}{2m}} e^{i(kx+wt)},$$

$$H_5(\eta) = \frac{u_5(\eta)}{4k^2 - 1}, \quad (23)$$

(V):When $A = C = \frac{1}{2}$,$B = 0$,with Appendix A,then $F(\eta) = sec(\eta) + tan(\eta)$ or $csc(\eta) - cot(\eta)$.By case (B),we have trigonometric function solutions of Eqs.(1) and (2)

$$u_6(\eta) = [-\frac{1}{2}d_1[sec(\eta) + tan(\eta)] - \frac{d_1}{[2[sec(\eta) + tan(\eta)]]} + \frac{d_1[sec(\eta)tan(\eta) + 1 + tan^2(\eta)]}{sec(\eta) + tanh(\eta)}]^{\frac{1}{2m}} e^{i(kx+wt)},$$

$$u_7(\eta) = [-\frac{1}{2}d_1[csc(\eta) - cot(\eta)] - \frac{d_1}{[2[csc(\eta) - cot(\eta)]]} + \frac{d_1[-csc(\eta)cot(\eta) + 1 + cot^2(\eta)]}{csc(\eta) + cot(\eta)}]^{\frac{1}{2m}} e^{i(kx+wt)},$$

$$H_6(\eta) = \frac{u_6(\eta)}{4k^2 - 1},$$

$$H_7(\eta) = \frac{u_7(\eta)}{4k^2 - 1}, \quad (24)$$

(VI):In case of $A = C = -\frac{1}{2}$,$B = 0$, with the aid of Appendix A, then $F(\eta) = sec(\eta) - tan(\eta)$ or $csc(\eta) + cot(\eta)$.By case (B),we have trigonometric function solutions of Eqs.(1) and (2)

$$u_8(\eta) = [\frac{1}{2}d_1[sec(\eta) - tan(\eta)] + \frac{d_1}{[2[sec(\eta) - tan(\eta)]]} + \frac{d_1[sec(\eta)tan(\eta) - 1 - tan^2(\eta)]}{sec(\eta) - tanh(\eta)}]^{\frac{1}{2m}} e^{i(kx+wt)},$$

$$u_9(\eta) = [\frac{1}{2}d_1[csc(\eta) + cot(\eta)] - \frac{d_1}{[2[csc(\eta) + cot(\eta)]]} + \frac{d_1[-csc(\eta)cot(\eta) - 1 - cot^2(\eta)]}{csc(\eta) + cot(\eta)}]^{\frac{1}{2m}} e^{i(kx+wt)},$$

$$H_8(\eta) = \frac{u_8(\eta)}{4k^2 - 1},$$

$$H_9(\eta) = \frac{u_9(\eta)}{4k^2 - 1} \tag{25}$$

(VII):In the limiting case $A = C = 1$,$B = 0$, from the Appendix A, then $F(\eta) = tan(\eta)$. By case (B),admits to trigonometric function solutions of Eqs.(1) and (2)

$$u_{10}(\eta) = [-d_1 tan(\eta) - \frac{d_1}{tan(\eta)} + \frac{d_1[1 + tan^2(\eta)]}{tanh(\eta)}]^{\frac{1}{2m}} e^{i(kx+wt)}$$

$$H_{10}(\eta) = \frac{u_{10}(\eta)}{4k^2 - 1} \tag{26}$$

(VIII):When $A = C = -1$,$B = 0$,from the Appendix A, then $F(\eta) = cot(\eta)$. By case (B), we have trigonometric function solutions of Eqs.(1) and (2)

$$u_{11}(\eta) = [d_1 cot(\eta) + \frac{d_1}{cot(\eta)} + \frac{d_1[-1 - cot^2(\eta)]}{cot(\eta)}]^{\frac{1}{2m}} e^{i(kx+wt)},$$

$$H_{11}(\eta) = \frac{u_{11}(\eta)}{4k^2 - 1} \tag{27}$$

(IX):As long as $C = 0$,A,$B = 0$,from the Appendix A, then $F(\eta) = \frac{e^{Bt}-A}{B}$, admits to exponentional solution as follows

$$u_{12}(\eta) = [-d_1 B - c_1 A - c_1(e^{Bt} - A) - \frac{ABd_1}{(e^{Bt} - A)} + c_1 ln(e) e^{Bt} + \frac{d_1 e^{Bt} ln(e) B}{(e^{Bt} - A)}]^{\frac{1}{2m}} e^{i(kx+wt)},$$

$$H_{12}(\eta) = \frac{u_{11}(\eta)}{4k^2 - 1},$$
$$\eta = \mu(x - 2kt)$$

Appendix A

Relations between values of (A, B, C) and corresponding $F(\eta)$ in NODE $F^{'}(\eta) = A + BF(\eta) + CF^2(\eta)$.

A	B	C	F(ξ)
0	1	-1	$F(\eta) = \frac{1}{2} + \frac{1}{2}tanh(\frac{\eta}{2})$
0	-1	1	$F(\eta) = \frac{1}{2} - \frac{1}{2}coth(\frac{\eta}{2})$
$\frac{1}{2}$	0	$-\frac{1}{2}$	$F(\eta) = coth(\eta) \pm csch(\eta)$,$tanh(\eta) \pm isech(\eta)$
1	0	-1	$F(\eta) = tanh(\eta)$,$coth(\eta)$
$\frac{1}{2}$	0	$\frac{1}{2}$	$F(\eta) = sec(\eta) + tan(\eta)$,$csc(\eta) - cot(\eta)$
$-\frac{1}{2}$	0	$-\frac{1}{2}$	$F(\eta) = sec(\eta) - tan(\eta)$,$csc(\eta) + cot(\eta)$
$1(-1)$	0	$1(-1)$	$F(\eta) = tan(\eta)$,$cot(\eta)$
0	0	$\neq 0$	$F(\eta) = \frac{-1}{C\eta+\lambda}$
$constant$	0	0	$F(\eta) = A\eta$
$constant$	$\neq 0$	0	$F(\eta) = \frac{exp(B\eta)-A}{B}$

5 Conclusion

In the summary, an extended F-expansion method with a computerized symbolic computation has been proposed to obtain the new exact travelling wave solutions to nonlinear evolution equations arising in mathematical physics. The validity of this method has been tested by applying it successfully to generalized Zakharov equations with arbitrary power nonlinearities by introducing a new generalized ansatz Eq.(14).

The underlying mechanism for a series of fundamental solutions such as polynomial,exponential,solitary wave,rational,triangular periodic,Jacobi and Weierstrass doubly periodic solutions act to change wave forms in many nonlinear equations arsising in physics.It seems that the generalized F-expansion method is more effective and simple than other methods and a lot of solutions can be obtained in the same time.In our work,we use the Maple package.

It is worthwhile to mention that the method is straightforward and concise,and it can also be applied to other nonlinear evolution equations in physics. This is our task in the future work.

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References

[1] S Yu,Lix in Tian:Nonsymmetrical Kink solution of generalized KdV equation with variable coefficients. *International Journal of Nonlinear Science*. 5:71-78(2008)

[2] Chen Y,Wang Q.:Jacobi elliptic function rational expansion method and abundant families of Jacobi elliptic function solutions to dispersive long wave equation.*Chaos,Solitons and Fractals* 24:745(2005)

[3] M.A.Abdou, A.Elhanbaly:Construction of periodic and solitary wave solutions by the extended Jacobi elliptic function expansion method. *Commun.in Nonl.Sci.and Numer.Simu*.12:1229-1241(2007)

[4] S.A.El-Wakil,M A Abdou:New applications of the homotopy analysis method.*Zeitschrift fur Naturforschung*. 63A:1-8(2008)

[5] J.H.He:Non perturbative method for strongly nonlinear problems. *dissertation,de Verlag in Internt GmbH,Berlin* (2006)

[6] S.A.El-Wakil, M.A.Abdou:New exact travelling wave solutions of two nonlinear physical models.*Nonlinear Analysis*. 68:235-245(2008)

[7] M.A.Abdou:The extended F-expansion method and its application for a class of nonlinear evolution equations.*Chaos,Solitons and Fractals*. 31:95-104(2007)

[8] E.M.Abulwafa, M.A.Abdou,A.A.Mahmoud:The solution of nonlinear coagulation problem with mass loss.*Chaos,Solitons and Fractals*. 29:313-330(2006)

[9] P.J.Olver,Applications of Lie Groups to Differential Equations. *Springer,Berlin*(1986)

[10] He J H,Xu-Hong Wu:Exp-function method for nonlinear wave equations. *Chaos Solitons and Fractals*. 30:700-708(2006)

[11] M A Abdou:Further improved F-expansion and new exact solutions for nonlinear evolution equations.*J.of Nonlinear Dynamics*. 52(3):277-288(2007)

[12] M A Abdou:Exact periodic wave solutions for some nonlinear evolution equations.*International Journal of Nonlinear Science*. 5:1-9(2008)

[13] C.Rogers,W.F.Shadwich:Bulcklund Transformations and Their Application.*Academic Press,New York*.(1982)

[14] M A Abdou,S Zhang:New periodic wave solutions via extended mapping method.*Communcation in Nonlinear Science and Numerical Simulation*.14:2-11(2009)

[15] Y Peng Y.:Exact periodic wave solutions to a Hamiltonian Amplitude Equations. *J.Phys.Soc.Jpn*. 72:1356-1359(2003)

[16] R.L.Burden,J.D.Faires:Numerical Analysis,PWS Publishing Company. *Boston*(1993)

[17] R.Temam:Infinite-Dimensional Dynamical Systems in Mechanics and Physics. *second,Springer-Verlag*(2001)

[18] P.J.Blennerhassett,Philos.Trans.R.Soc.*London,Ser.A 298:451(1*980)

[19] H.T.Moon,P.Huerre,L.G.Redekopp.*Phys.Rev.Lett*.49:485(1982)

[20] Ya-Zhou Li,Kai-ming Li,Chang Lin.A*pplied Mathematics and Computation;in* press