Adaptive Control and Synchronization of a Four-Dimensional Energy Resources System of JiangSu Province

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Abstract: This paper investigates the control and synchronization of a new four-dimensional energy resources system of JiangSu province. Adaptive synchronization controllers and parameter update laws, which can make two systems with different unknown parameters asymptotically synchronized, are introduced. The sufficient conditions for the synchronization have been analyzed theoretically and all results are proved using a well-known Lyapunov stability theorem.

Keywords: synchronization; adaptive control; parameter update laws; energy resources system

1 Introduction

Chaos in control system and controlling chaos in dynamical systems have both attracted increasing attention in recent years. A chaotic system has complex dynamical behaviors that posses some special features, such as being extremely sensitive to tiny variations of initial conditions, having bounded trajectories in the phase space, and so on. Chaos synchronization has been a flurry of research activities for over a decade.

Many approaches have been presented for the synchronization of chaotic systems such as a feedback control, impulsive control and so on. Most of them are based on the exactly knowing of the system structure and parameters. But in practice, some or all of the system’s parameters may be not available to the designer of the synchronization device. A lot of works have been preceded to solve this problem using adaptive synchronization. Fradkov and cowokers [1-3] studied the adaptive synchronization problem based on speed gradient methods. Ref.[4] presents an adaptive synchronization design method via backstepping design. Liao [5] uses the Lyapunov method to design an adaptive synchronization controller for Lorenz system. Refs [6,7] give a scheme for the design of an adaptive synchronization which is based on the controller for the case when the drive system’s parameters are available.

This paper presents the adaptive synchronization of energy system. And we establish a four-dimensional energy resources system of JiangSu province described by the following differential equations:

\[
\begin{align*}
\frac{dx}{dt} &= a_1x(1 - \frac{x}{M}) - a_2(y + z) - d_3w \\
\frac{dy}{dt} &= b_1y(b_2x - b_3) \\
\frac{dz}{dt} &= c_1z - c_2xy - c_3z \\
\frac{dw}{dt} &= d_1x - d_2w
\end{align*}
\]

where \(x(t)\) the consumption of crude coal in JiangSu province; \(y(t)\) the consumption of oil gas in JiangSu province; \(z(t)\) the consumption of other energy resource of unrenewable resources in JiangSu province; and \(w(t)\) the renewable energy resources in JiangSu province; \(a_1, b_1, c_1, d_1, M\) are positive real constant. This system has three equilibria: \(O(0, 0, 0, 0), S_1(x_1, y_1, z_1, w_1), S_2(x_2, y_2, z_2, w_2)\). When \(a_1 = 0.06, a_2 = \)
0.15, b₁ = 0.05, b₂ = 0.3, b₃ = 0.25, c₁ = 0.02, c₂ = 0.2, c₃ = 0.22, and d₁ = 0.1, d₂ = 0.06, d₃ = 0.08, M = 1, O(0, 0, 0, 0) is an unstable saddle focus.

In this paper, the problem of adaptive control and adaptive synchronization of the four-dimensional energy resources system with unknown parameters is introduced. It is proved that the adaptive control and synchronization can be achieved using adaptive controllers and parameter update laws.

2 Controlling chaos via adaptive control methods

In this section, adaptive control methods are applied to control chaos of the system (1.1). This chaos is controlled to the equilibrium point O(0, 0, 0, 0). For simplicity, we fix parameters a₂ = 0.15, b₂ = 0.3, b₃ = 0.25, c₁ = 0.02, c₂ = 0.2, d₃ = 0.08, M = 1, while let parameters a₁, b₁, c₁, d₁ and d₂ be varied. Let a₄ = a, b₁ = b, c₁ = c, d₁ = d and d₂ = h. In this case, the system (1.1) becomes:

\[
\begin{align*}
\frac{dx}{dt} &= ax(1 - x) - 0.15(y + z) - 0.08w \\
\frac{dy}{dt} &= by(0.3x - 0.25) \\
\frac{dz}{dt} &= 0.02z - 0.2xy - cz \\
\frac{dw}{dt} &= dx - hw
\end{align*}
\] (2.2)

The controlled system of the system (2.1) is described as follows:

\[
\begin{align*}
\frac{dx}{dt} &= ax(1 - x) - 0.15(y + z) - 0.08w + u₁ \\
\frac{dy}{dt} &= by(0.3x - 0.25) + u₂ \\
\frac{dz}{dt} &= 0.02z - 0.2xy - cz + u₃ \\
\frac{dw}{dt} &= dx - hw + u₄
\end{align*}
\] (2.3)

where a, b, c, d and h are unknown parameters to be identified, and u₁, u₂, u₃ and u₄ are four controllers to be designed. We can pick a Lyapunov function for Eq.(2.2)

\[
V(x, y, z, w, \tilde{a}, \tilde{b}, \tilde{c}, \tilde{d}, \tilde{h}) = \frac{1}{2} x² + \frac{1}{2} y² + \frac{1}{2} z² + \frac{1}{2} w² + \frac{1}{2} \tilde{a}² + \frac{1}{2} \tilde{b}² + \frac{1}{2} \tilde{c}² + \frac{1}{2} \tilde{d}² + \frac{1}{2} \tilde{h}²
\]

Choosing the controllers as

\[
u₁ = -(\tilde{a} - 1)x - \tilde{a}x² - 0.15y - 0.15z - 0.08w, \quad u₂ = -0.3bxy, \quad u₃ = -(0.02z - 0.2xy), \quad u₄ = -dx
\]

and parameter update laws as

\[
\begin{align*}
\dot{\tilde{a}} &= -(x² - x³) - \tilde{a}, \quad \dot{\tilde{b}} = -(0.3xy² - 0.25y²) - \tilde{b}, \quad \dot{\tilde{c}} = z² - \tilde{c}, \quad \dot{\tilde{d}} = -wx - \tilde{d}, \quad \dot{\tilde{h}} = w² - \tilde{h}
\end{align*}

then

\[
\dot{V} = -x² - 0.25bxy - \tilde{c}² - h²w² - \tilde{a}² - \tilde{b}² - \tilde{c}² - \tilde{d}² - \tilde{h}²
\]

Since V is positive definite and \(\dot{V}\) is negative definite, it follows the equilibrium point \((x = 0, y = 0, z = 0, w = 0, \tilde{a} = 0, \tilde{b} = 0, \tilde{c} = 0, \tilde{d} = 0, \tilde{h} = 0)\) of the system (2.2) is globally asymptotically stable.

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3 Adaptive synchronization of the energy resources system

We assume that we have two energy resource system (2.1) where the drive system with the subscript 1 drives the response system having identical equations denoted by the subscript 2. However, the initial condition on the drive system is differential from that of the response system, for the actual energy drive systems and response systems, the parameters "a, b, c, d, h" in the two chaotic systems are arbitrary unknown constant, we do not know whether they are identical or not in advance. In the case, we can define drive and response systems are defined as follows:

\[
\begin{aligned}
\frac{dx_1}{dt} &= ax_1(1 - x_1) - 0.15(y_1 + z_1) - 0.08w_1 \\
\frac{dy_1}{dt} &= by_1(0.3x_1 - 0.25) \\
\frac{dz_1}{dt} &= 0.02z_1 - 0.2x_1y_1 - cz_1 \\
\frac{dw_1}{dt} &= dx_1 - hw_1
\end{aligned}
\]  

(3.5)

and

\[
\begin{aligned}
\frac{dx_2}{dt} &= ax_2(1 - x_2) - 0.15(y_2 + z_2) - 0.08w_2 \\
\frac{dy_2}{dt} &= by_2(0.3x_2 - 0.25) \\
\frac{dz_2}{dt} &= 0.02z_2 - 0.2x_2y_2 - cz_2 \\
\frac{dw_2}{dt} &= dx_2 - hw_2
\end{aligned}
\]  

(3.6)

where \(a, b, c, d, h\) and \(a', b', c', d', h'\) are unknown parameters of Eqs.(3.1) and (3.2), respectively, is controller we introduced in Eq. (3.2). The controller is to be determined for the purpose of synchronizing the two energy systems.

Let

\[
\begin{aligned}
a' &= a + \gamma_1a, b' &= b + \gamma_2b, c' &= c + \gamma_3c, d' &= d + \gamma_4d, h' &= h + \gamma_5h
\end{aligned}
\]

(3.7)

It is obvious that \(\gamma_1a, \gamma_2b, \gamma_3c, \gamma_4d, \gamma_5h\) are also unknown constant. From Eqs. (3.2) and (3.3), we have

\[
\begin{aligned}
\frac{dx_2}{dt} &= (a + \gamma_1a)x_2(1 - x_2) - 0.15(y_2 + z_2) - 0.08w_2 + u_1 \\
\frac{dy_2}{dt} &= (b + \gamma_2b)y_1(0.3x_2 - 0.25) + u_2 \\
\frac{dz_2}{dt} &= 0.02z_2 - 0.2x_2y_2 - (c + \gamma_3c)z_2 + u_3 \\
\frac{dw_2}{dt} &= (d + \gamma_4d)x_2 - (h + \gamma_5h)w_2 + u_4
\end{aligned}
\]  

(3.8)

Then, error dynamical system between Eqs. (3.1) and (3.4) can be expressed by

\[
\begin{aligned}
\dot{e}_1 &= ae_1 - a(x_1 + x_2)e_1 - 0.15(e_2 + e_3) - 0.08e_4 + \gamma_1\alpha(x_2 - x_2e_1 - x_1x_2) + u_1 \\
\dot{e}_2 &= 0.3b(x_1 + x_2)e_2 - 0.25e_2 + 0.3b_1y_1 - 0.3b_1y_2 + \gamma_2by_2(0.3x_2 - 0.25) + u_2 \\
\dot{e}_3 &= (0.02 - c)e_3 - 0.2((x_1 + x_2)e_2 - x_1y_2 + x_2y_1) - \gamma_3c + u_3 \\
\dot{e}_4 &= de_1 - he_4 + \gamma_4dx_2 - \gamma_5hw_2 + u_4
\end{aligned}
\]  

(3.9)

where \(e_1 = x_2 - x_1, e_2 = y_2 - y_1, e_3 = z_2 - z_1, e_4 = w_2 - w_1\)

The goal of control is to find a controller \(u = [u_1, u_2, u_3, u_4]^T\) and a parameter estimation update law for Eq. (3.5) such that the states of response system (3.2) and the states of drive system (3.1) are globally synchronized asymptotically, i.e.

\[
\lim_{t \to \infty} ||e(t)|| = 0, \forall a, b, c, d, h \in R^+
\]

where \(e(t) = [e_1, e_2, e_3, e_4]^T\).

We can pick a Lyapunov function for Eqs.(3.5)

\[
V(e, \hat{a}, \hat{\gamma}_1a, \hat{\gamma}_2b, \hat{\gamma}_3c, \hat{\gamma}_4d, \hat{\gamma}_5h) = \frac{1}{2}e^T e + \frac{1}{2} \hat{a}^2 + \frac{1}{2}(\hat{\gamma}_1^2a^2 + \hat{\gamma}_2^2b^2 + \hat{\gamma}_3^2c^2 + \hat{\gamma}_4^2d^2 + \hat{\gamma}_5^2h^2)
\]

(3.10)

where \(\hat{\gamma}_1a = \gamma_1a, \hat{\gamma}_2b = \gamma_2b, \hat{\gamma}_3c = \gamma_3c, \hat{\gamma}_4d = \gamma_4d, \hat{\gamma}_5h = \gamma_5h\), respectively. Choose controllers and parameters estimation update laws as follows:
\[u_1 = -[(\dot{\alpha} - 1)e_1 - a(x_1 + x_2)e_1 - 0.15(e_2 + e_3) - 0.08e_4 + \dot{\gamma}_1 a(x_2 - x_2e_1 - x_1x_2)]\]
\[u_2 = -[0.3\dot{b}(x_1 + x_2)e_2 + 0.3bx_2y_1 - 0.3bx_1y_2 + \dot{\gamma}_2 by_2(0.3x_2 - 0.025)]\]
\[u_3 = -[0.02e_3 - 0.2(x_1 + x_2)e_2 - 0.2x_1y_2 + 0.2x_2y_1 - \dot{\gamma}_3 cz]\]
\[u_4 = [-de_1 + \dot{\gamma}_4 dx_2 - \dot{\gamma}_5 hw_2]\]
\[\dot{\alpha} = -e_2^1\]
\[\dot{\gamma}_1 a = -[x_2e_1 - x_2e_1^2 - x_1x_2e_1]\]
\[\dot{\gamma}_2 b = -by_2(0.3x_2 - 0.25)\]
\[\dot{\gamma}_3 c = z_2, \dot{\gamma}_4 d = -x_2e_4, \dot{\gamma}_5 h = w_2e_4\]

From Eqs. (3.4)-(3.6), we obtain the derivative of Eq. (3.6)
\[\dot{V} = -e_2^1 - 0.25be_2^2 - ce_3^2 - he_4^2\]

It is concluded that the synchronization of two energy system is achieved under the controllers and parameter estimation update laws as above,
\[\lim_{t \to \infty} ||e(t)|| = 0, \forall a, b, c, d, h \in R^+\]

4 Conclusion
This paper investigates the control and synchronization of a new four-dimensional energy resources system of JiangSu province. Adaptive synchronization controllers and parameter update laws are introduced to make two systems with different unknown parameters asymptotically synchronized. The sufficient conditions for the synchronization have been analyzed theoretically and all results are proved using a well-known Lyapunov stability theorem.

Because the crude coal is the main energy consumption of JiangSu province, we should pay attention to the efficiency of using the crude coal, strengthen exploring technology and control the environment pollution. Besides, we should focus on exploring the renewable energy resources and saving the energy to reduce the energy pressure of JiangSu province.

References


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