New Exact Solutions of Nonlinear Variants of the RLW, the PHI-four and Boussinesq Equations Based on Modified Extended Direct Algebraic Method

A. A. Soliman\textsuperscript{a} *, H. A. Abdo\textsuperscript{b}†

\textsuperscript{a}Department of Mathematics, Faculty of Education (AL-Arish)
Suez Canal University, AL-Arish, 45111, Egypt.
\textsuperscript{b}Freie Universität Berlin, Institut für Informatik,
Takustraße 9, D-14195 Berlin, Germany,
(Received 18 April 2008, accepted 19 September 2008)

Abstract: By means of modified extended direct algebraic (MEDA) method the multiple exact complex solutions of some different kinds of nonlinear partial differential equations are presented and implemented in a computer algebraic system. New complex solutions for nonlinear equations such as the variant of the RLW equation, the variant of the PHI-four equation and the variant Boussinesq equations are obtained.

Keywords: variant of the RLW equation; variant of the PHI-four equation; variant Boussinesq equations; nonlinear partial differential equations; the MEDA method

1 Introduction

It is well known that the nonlinear physical phenomena are related to nonlinear partial differential equations which are involved in many fields such as physics, chemistry, mechanics, etc. As mathematical models of the phenomena, the investigation of exact solutions of the partial differential equations will help one to understand these phenomena better. In recent years, many powerful methods to construct exact solutions of nonlinear partial differential equations have been established and developed. Among these are variational iteration method [1-7], tanh function method [8,9], modified extended tanh function method[10-16], sine-cosine method [17,18], Exp-method [19], inverse scattering method[20], Hirota’s bilinear method [21], the homogeneous balance method[22], the Riccati expansion method with constant coefficients [23] and trial function [24]. The regularized long wave (RLW) equation is an important nonlinear wave equation. Solitary waves are wave packet or pulses, which propagate in nonlinear dispersive media. Due to dynamical balance between the non-linear and dispersive effects these waves retain a stable wave form. The regularized long wave (RLW) equation is an alternative description of non-linear dispersive waves to the more usual Kortewege-de Vries (KDV) equation [25]. Numerical solutions based on finite difference techniques [26,27]. Rung-Kutta method [28] and Galerkin’s method [29] have been given. Alexander and Morris [29] constructed a global trial function mainly from cubic splines. Gardner and Gardner [30] used the Galerkin’s method and cubic B-spline as element shape function to construct an implicit finite element solution. The least squares method using linear space-time finite elements was used to solve the RLW equation [31]. Soliman and Raslan [32] solved the RLW equation by using collocation method using quadratic B-spline at the mid point. Soliman and Hussien[33] used the collocation method with septic spline to solve the RLW equation. Soliman [34] used the finite difference method with the similarity solution of the partial differential equations to obtain the numerical scheme for RLW equation. This approach eliminates the difficulty associated to the boundary. In the end Soliman [2] used the variational iteration method to obtain the exact solutions of the generalized

\textsuperscript{* Corresponding author. Department of Mathematics, Bisha Teachers’ College, King Khalid University, P.O. Box 551, Bisha, Kingdom of Saudi Arabia. E-mail address: asoliman_99@yahoo.com.}

\textsuperscript{† E-mail: abdo@inf.fu-berlin.de}
The PHI-four equation was considered as a particular form of the Klein-Gordon equation that models phenomena in particle physics where kink and anti-kink solitary waves interact [35].

Recently, the direct algebraic method and symbolic computation have been suggested to obtain the exact complex solutions of nonlinear partial differential equations [36-38].

The aim of this paper is to extend the modified extended direct algebraic method (MEDA) method to solve three different types of nonlinear differential equations such as the variant of the RLW, the variant of the PHI-four and variant Boussinesq equations [13, 14].

2 Modified extended direct algebraic method

To illustrate the basic concepts of the modified extended direct algebraic (MEDA) method. We consider a given PDE in two independent variables given by

\[ F(u, u_x, u_t, u_{xx}, \ldots) = 0, \] (1)

We first consider its travelling solutions \( u(x, t) = u(z), z = i(x + ct) \) or \( z = i(x - ct), i = \sqrt{-1} \), then Eq. (1) becomes an ordinary differential equation

\[ H(u, iu', -icu', -u'', \ldots) = 0, \] (2)

where \( u' = \frac{du}{dz} \).

In order to seek the solutions of Eq. (1), we introduce the following ansatz

\[ u(z) = a_0 + \sum_{j=1}^{M} (a_j \phi^j + b_j \phi^{-j}), \] (3)

\[ \phi' = b + \phi^2, \] (4)

where \( b \) is a parameter to be determined, \( \phi = \phi(z), \phi' = \frac{d\phi}{dz} \). The parameter \( M \) can be found by balancing the highest-order derivative term with the nonlinear terms [39]. Substituting (3) into (2) with (4) will yield a system of algebraic equations with respect to \( a_j, b_j, b \) and \( c \) (where \( j = 1 \ldots M \)) because all the coefficients of \( \phi^j \) have to vanish. We can then determine \( a_0, a_j, b_j, b \), and \( c \). Eq. (4) has the general solutions:

(I) If \( b < 0 \),

\[ \phi = -\sqrt{-b} \tanh(\sqrt{-b}z), \text{ or } \phi = -\sqrt{-b} \coth(\sqrt{-b}z), \]

it depends on initial conditions.

(II) If \( b > 0 \),

\[ \phi = \sqrt{b} \tanh(\sqrt{b}z), \text{ or } \phi = -\sqrt{b} \coth(\sqrt{b}z). \]

it depends on initial conditions.

(III) If \( b = 0 \)

\[ \phi = \frac{-1}{z}. \] (5)

Substituting the results into (3), then we obtain the exact travelling wave solutions of Eq. (1).

To illustrate the procedure, three examples related to variant of the RLW, variant of the PHI-four and variant Boussinesq equations are given in the following.

3 Applications

3.1 Variant of the regularized long-wave (RLW) equation

Let us first consider the nonlinear variant of the regularized long wave equation which has the form [13]

\[ u_t + \alpha u_x - \lambda (u^n)_x + \beta (u^n)_{xxx} = 0, \] (6)

IIINS homepage: http://www.nonlinearscience.org.uk/
where \( \alpha \), \( \lambda \) and \( \beta \) are arbitrary constants. In order to solve Eq. (6) by the MEDA method, we use the wave transformation \( u(x, t) = U(z) \) with wave complex variable \( z = i(x - ct) \), Eq. (6) takes the form of an ordinary differential equation as

\[
(\alpha - c)U' - \lambda(U^n)' + \beta c(U^n)'' = 0. \tag{7}
\]

Integrating Eq. (7) once with respect to \( z \) and setting the constant of integration to be zero, we obtain

\[
(\alpha - c)U - \lambda U^n + \beta c(U^n)'' = 0. \tag{8}
\]

Or equivalently

\[
(\alpha - c)U - \lambda U^n + \beta cnU^{n-1}U'' + \beta cn(n-1)U^{n-2}(U')^2 = 0. \tag{9}
\]

Balancing the order of \( U^n \) with the order of \( U^{n-1}U'' \) in Eq. (9), we find \( M = -\frac{2}{n-1} \). To get a closed form analytic solution, the parameter \( M \) should be an integer. A transformation formula \( U = V^{1/(n-1)} \) should be used to obtain this analytic solution. So Eq. (9) takes the form

\[
(\alpha - c)(n-1)^2V^3 - \lambda(n-1)^2V^2 - \beta cn(n-1)VV'' + \beta cn(2n-1)(V')^2 = 0. \tag{10}
\]

Balancing the order of \( V^3 \) with the order of \( VV'' \) in Eq. (10), gives \( M = 2 \). So the solution takes the form

\[
V(z) = a_0 + a_1 \phi(z) + a_2 \phi(z)^2 + b_1 \phi(z)^{-1} + b_2 \phi(z)^{-2}. \tag{11}
\]

Inserting Eq. (11) into Eq. (10) and making use of Eq. (4), using the Maple Package, we get a system of algebraic equations, for \( a_0, a_1, a_2, b_1, b_2 \) and \( b \). We solve the obtained system of algebraic equations give the following three cases: Case (I): \( a_1, a_2, b_1 = 0, b_2 = \frac{a_0^2 \lambda(n^2-2n+1)}{2n\beta(-n\lambda-\lambda+2\alpha a_n)}, c = \frac{(-\lambda n-\lambda+2\alpha a_n)}{2n\alpha}, b = \frac{a_0 \lambda(n^2-2n+1)}{2n\beta(-n\lambda-\lambda+2\alpha a_n)}, \) with \( a_0 \) being an arbitrary constant. Then

\[
V(x, t) = a_0(1 + \cot^2 \left( \frac{1}{2}\sqrt{\frac{2a_0 \lambda(n^2-2n+1)}{n\beta(-n\lambda-\lambda+2\alpha a_n)}}(x - \frac{(-\lambda n-\lambda+2\alpha a_n)}{2n\alpha} t)) \right), \tag{12}
\]

so the travelling wave solution is given by

\[
u(x, t) = (a_0(1 + \cot^2 \left( \frac{1}{2}\sqrt{\frac{a_0 \lambda(n^2-2n+1)}{n\beta(-n\lambda-\lambda+2\alpha a_n)}}(x - \frac{(-\lambda n-\lambda+2\alpha a_n)}{2n\alpha} t)) \right))^{-\frac{1}{n-1}} \tag{13}
\]

Case (II): \( a_0 = \frac{1}{4} \lambda(n^2a_2-2na_2+2\beta n^2+2\beta n+a_2), a_1 = 0, b_1 = 0, b_2 = 0, b = \frac{1}{4} \lambda(n^2a_2-2na_2+2\beta n^2+2\beta n+a_2), c = \frac{a_0 a_0(n^2-2n+1)}{n^2a_2-2na_2+2\beta n^2+2\beta n+a_2}, \) with \( a_2 \) being an arbitrary constant, then

\[
V(x, t) = \frac{E}{4 \beta n^2} \left( 1 - \tan^2 \left( \frac{1}{2}\sqrt{\frac{E \beta}{\lambda n^2 a_2}}(x - \frac{a_0(n^2-2n+1)}{E} t)) \right) \right) \tag{14}
\]

so the travelling wave solution is given by

\[
u(x, t) = \left( \frac{1}{4} \frac{E \beta}{\lambda n^2} \left( 1 - \tan^2 \left( \frac{1}{2}\sqrt{\frac{E \beta}{\lambda n^2 a_2}}(x - \frac{a_0(n^2-2n+1)}{E} t)) \right) \right) \right)^{-\frac{1}{n-1}} \tag{15}
\]

where \( E = \frac{(n^2a_2-2na_2+2\beta n^2+2\beta n+a_2)}{\alpha} \).

IJNS email for contribution: editor@nonlinearscience.org.uk
Case (III): \( a_0 = \frac{1}{8} \frac{\lambda (n^2 a_2 - 2 n a_2 + 2 n^2 + 2 n + a_2)}{\beta n^2 a_2}, a_1 = 0, b_1 = 0, b_2 = \frac{1}{256} \frac{\lambda^2 (n^2 a_2 - 2 n a_2 + 2 n^2 + 2 n + a_2)}{a_2^2 \beta n^2 a_2}, \) with \( a_2 \) being an arbitrary constant, then
\[
V(x, t) = \frac{1}{8} \frac{E \lambda}{\beta n^2} \left( 1 - \frac{1}{2} \sec^2 \left( \frac{1}{4} \sqrt{\frac{E \lambda}{\beta n^2 a_2}} \right) (x - \frac{a_2 (n^2 - 2 n + 1)}{E} t) - \frac{1}{2} \sec^2 \left( \frac{1}{4} \sqrt{\frac{E \lambda}{\beta n^2 a_2}} \right) (x - \frac{a_2 (n^2 - 2 n + 1)}{E} t) \right),
\]
so the travelling wave solution is given by
\[
u(x, t) = \left( \frac{1}{16} \frac{E \beta}{\lambda n^2} \left( 1 - \frac{1}{2} \sec^2 \left( \frac{1}{4} \sqrt{\frac{E \lambda}{\beta n^2 a_2}} \right) (x - \frac{a_2 (n^2 - 2 n + 1)}{E} t) - \frac{1}{2} \sec^2 \left( \frac{1}{4} \sqrt{\frac{E \lambda}{\beta n^2 a_2}} \right) (x - \frac{a_2 (n^2 - 2 n + 1)}{E} t) \right) \right)^{(-\frac{1}{4})}.
\]

### 3.2 Variant of the PHI-four equation

A second important example is the Variant of the PHI-four equation [13], which can be written as
\[
u_{tt} - \alpha \nu_{xx} - \lambda \nu + \beta \nu^n = 0, \quad n > 0
\]
where \( \alpha, \lambda \) and \( \beta \) are arbitrary constants. In order to solve Eq. (18) by the MEDA method, we use the wave transformation \( u(x, t) = \nu(z) \) with wave variable \( z = i(x - ct) \). Eq. (18) takes the form of an ordinary differential equation
\[
c^2 \nu'' - \alpha \nu'' - \lambda \nu + \beta \nu^n = 0.
\]
Or equivalently
\[
-\lambda \nu + \beta \nu^n - (c^2 - \alpha) \nu'' = 0.
\]
Balancing the order of \( \nu^n \) with the order of \( \nu'' \) in Eq. (20), we find \( M = \frac{2}{n-1} \). To obtain a closed form analytic solution we use a transformation formula \( U = V \frac{n}{1-x} \), that transforms Eq. (20) to
\[
-\lambda (n - 1)^2 V^2 + \beta (n - 1)^2 V^3 + (\alpha - c^2)(n - 1)VV'' + (\alpha - c^2)(2 - n)(V')^2 = 0.
\]
Balancing the order of \( V^3 \) with the order of \( VV'' \) in Eq. (21), we find \( M = 2 \). So the solution takes the form
\[
V(z) = a_0 + a_1 \phi(z) + a_2 \phi(z)^2 + b_1 \phi(z)^{-1} + b_2 \phi(z)^{-2}.
\]
Substituting Eq. (22) into Eq. (21) and making use of Eq. (4), we obtain a system of algebraic equations, for \( a_0, a_1, a_2, b_1, b_2 \) and \( b \). Solving the obtained system of algebraic equations gives the following three cases:

**Case (I):** \( a_0 = \frac{1}{2} \frac{\lambda (n + 1)}{\beta}, a_1 = 0, a_2 = 0, b_1 = 0, b_2 = \frac{1}{8} \frac{(n+1)(n^2 - 2 n + 1)}{\beta (c^2 - \alpha)} \), \( b = \frac{1}{4} \frac{\lambda (n^2 - 2 n + 1)}{c^2 - \alpha} \), with \( c \) being an arbitrary constant. Then
\[
V(x, t) = \frac{1}{2} \frac{\lambda (n + 1)}{\beta} + \frac{\lambda (n + 1)}{2 \beta} \cot^2 \left( \frac{i}{2} \sqrt{\frac{\lambda (n^2 - 2 n + 1)}{c^2 - \alpha}} (x - ct) \right),
\]
so the travelling wave solution is given by
\[
u(x, t) = \left( \frac{1}{2} \frac{\lambda (n + 1)}{\beta} + \frac{\lambda (n + 1)}{2 \beta} \cot^2 \left( \frac{i}{2} \sqrt{\frac{\lambda (n^2 - 2 n + 1)}{c^2 - \alpha}} (x - ct) \right) \right)^{\frac{1}{2}}.
\]

**Case (II):** \( a_0 = \frac{1}{2} \frac{\lambda (n + 1)}{\beta}, a_1 = 0, a_2 = \frac{2 (\alpha + c^2 n - \alpha + c^2)}{\beta (n^2 - 2 n + 1)}, b_1 = 0, b_2 = 0, b = \frac{1}{4} \frac{\lambda (n^2 - 2 n + 1)}{c^2 - \alpha} \), with \( c \) being an arbitrary constant, then
\[
V(x, t) = \frac{1}{2} \frac{\lambda (n + 1)}{\beta} + \frac{\lambda (\alpha + c^2 n - \alpha + c^2)}{\beta (c^2 - \alpha)} \tan^2 \left( \frac{i}{2} \sqrt{\frac{\lambda (n^2 - 2 n + 1)}{c^2 - \alpha}} (x - ct) \right)
\]

IINS homepage: http://www.nonlinearscience.org.uk/
Therefore, system (31,32) is reduced to the ordinary differential equations in the form

\[
\begin{align*}
\frac{d}{dt}(1) + \frac{1}{2} \frac{\lambda(n+1)}{\beta} + \frac{1}{2} \frac{\lambda(-\alpha n + c^2 n - \alpha + c^2)}{\beta(c^2 - \alpha)} \tan^2 \left( i \frac{\lambda(n^2 - 2n + 1)}{c^2 - \alpha} (x - ct) \right) &= 0, \\
\frac{d}{dt}(2) + \frac{1}{2} \frac{\lambda(n+1)}{\beta} + \frac{1}{2} \frac{\lambda(-\alpha n + c^2 n - \alpha + c^2)}{\beta(c^2 - \alpha)} \tan^2 \left( i \frac{\lambda(n^2 - 2n + 1)}{c^2 - \alpha} (x - ct) \right) &= 0,
\end{align*}
\]

so the travelling wave solution is given by

\[
\begin{align*}
\frac{1}{2} \frac{\lambda(n+1)}{\beta} + \frac{1}{2} \frac{\lambda(-\alpha n + c^2 n - \alpha + c^2)}{\beta(c^2 - \alpha)} \tan^2 \left( i \frac{\lambda(n^2 - 2n + 1)}{c^2 - \alpha} (x - ct) \right) &= 0, \\
\end{align*}
\]

Case (III): \(a_0 = \frac{1}{4} \frac{(n+1)}{\beta}, a_1 = 0, a_2 = \frac{1}{8} \frac{(n+1)}{\beta}, b_1 = 0, b_2 = \frac{1}{8} \frac{(n+1)}{b}, c = i \sqrt{2\lambda n - \lambda^n - \lambda - 16ab},
\]

with \(b\) being an arbitrary constant. Then,

\[
\begin{align*}
\frac{\lambda(n+1)}{4 \beta} + \frac{\lambda(n+1)}{8 \beta} \tan^2 \left( \sqrt{-b} \left( \frac{2\lambda n - \lambda^n - \lambda - 16ab}{b} t \right) \right) + \frac{\lambda(n+1)}{8 \beta} \cot^2 \left( \sqrt{-b} \left( \frac{2\lambda n - \lambda^n - \lambda - 16ab}{b} t \right) \right) &= 0,
\end{align*}
\]

so the travelling wave solution is given by

\[
\begin{align*}
\frac{1}{2} \frac{\lambda(n+1)}{\beta} + \frac{1}{2} \frac{\lambda(-\alpha n + c^2 n - \alpha + c^2)}{\beta(c^2 - \alpha)} \tan^2 \left( i \frac{\lambda(n^2 - 2n + 1)}{c^2 - \alpha} (x - ct) \right) &= 0, \\
\end{align*}
\]

Case (IV): \(a_0 = \frac{1}{4} \frac{(n+1)}{\beta}, a_1 = 0, a_2 = \frac{1}{8} \frac{(n+1)}{\beta}, b_1 = 0, b_2 = \frac{1}{8} \frac{(n+1)}{b}, c = -i \sqrt{2\lambda n - \lambda^n - \lambda - 16ab},
\]

with \(b\) being an arbitrary constant. Then,

\[
\begin{align*}
\frac{\lambda(n+1)}{4 \beta} + \frac{\lambda(n+1)}{8 \beta} \tan^2 \left( \sqrt{-b} \left( \frac{2\lambda n - \lambda^n - \lambda - 16ab}{b} t \right) \right) + \frac{\lambda(n+1)}{8 \beta} \cot^2 \left( \sqrt{-b} \left( \frac{2\lambda n - \lambda^n - \lambda - 16ab}{b} t \right) \right) &= 0,
\end{align*}
\]

so the travelling wave solution is given by

\[
\begin{align*}
\frac{1}{2} \frac{\lambda(n+1)}{\beta} + \frac{1}{2} \frac{\lambda(-\alpha n + c^2 n - \alpha + c^2)}{\beta(c^2 - \alpha)} \tan^2 \left( i \frac{\lambda(n^2 - 2n + 1)}{c^2 - \alpha} (x - ct) \right) &= 0, \\
\end{align*}
\]

All the solutions of the equations are new.

### 3.3 The variant Boussinesq equations

Finally, we consider a very important example as an illustration of the modified extended direct algebraic method for solving the variant Boussinesq equations [14], we will consider the following system of equations

\[
\begin{align*}
\frac{du}{dt} + v_x + u u_x &= 0, \\
\frac{dv}{dt} + (uv)_x + u_{xxx} &= 0.
\end{align*}
\]

To solve the system of Eqs. (31,32) by means of the modified extended direct algebraic method, we use the wave transformation \(u(x,t) = U(z)\) and \(v(x,t) = V(z)\) with complex wave variable \(z = i(x + \lambda t)\). Therefore, system (31,32) is reduced to the ordinary differential equations in the form

\[
\begin{align*}
\lambda U' + V' + \frac{1}{2} (U')^2 &= 0, \\
\lambda V' + (UV)' - U'' &= 0,
\end{align*}
\]
wave solutions are:

\[ C_1 - \lambda U - \frac{1}{2}U^2 = V, \]  
\[ \lambda V + UV - U'' = C_2, \]  

where \( C_1 \) and \( C_2 \) are integrating constants, so as to we find the special forms of the exact solutions, for simplicity purpose, we take \( C_1 = C_2 = 0 \). Substituting (35) into (36) gives

\[ U'' + \frac{1}{2}U^3 + \frac{3\lambda}{2}U^2 + \lambda^2 U = 0, \]  

By balancing \( U'' \) with \( U^3 \) in Eq. (37), we find \( M = 1 \). So the solutions take the form

\[ U(z) = a_0 + a_1\phi(z) + b_1\phi(z)^{-1}, \]  
\[ V(z) = -\lambda[a_0 + a_1\phi(z) + b_1\phi(z)^{-1}] - \frac{1}{2}[a_0 + a_1\phi(z) + b_1\phi(z)^{-1}]^2. \]

Inserting Eqs. (38,39) into Eq. (36), making use of Eq. (4), and by using Maple Package, we get a system of algebraic equations, for \( a_0, a_1, b_1, \lambda \) and \( b \). We solve the obtained algebraic system of equations by Mable Package and select four cases of solutions as:

Case (1) : \( a_1 = 2i, b_1 = 0, b = -\frac{1}{4}a_0^2, \lambda = -a_0 \), with \( a_0 \) being an arbitrary constant, the complex wave solutions are:

\[ u(x,t) = a_0(1 + i\tan\left(\frac{ia}{2}(x - a_0t)\right)), \]  
\[ v(x,t) = a_0\left(a_0(1 + i\tan\left(\frac{ia}{2}(x - a_0t)\right)) - \frac{1}{2}(a_0 + i\tan\left(\frac{ia}{2}(x - a_0t)\right))^2\right). \]

Case (2) : \( a_1 = -2i, b_1 = 0, b = -\frac{1}{4}a_0^2, \lambda = -a_0 \), with \( a_0 \) being an arbitrary constant, the complex wave solutions are:

\[ u(x,t) = a_0(1 - i\tan\left(\frac{ia}{2}(x - a_0t)\right)), \]  
\[ v(x,t) = a_0\left(a_0(1 - i\tan\left(\frac{ia}{2}(x - a_0t)\right)) - \frac{1}{2}(a_0 - i\tan\left(\frac{ia}{2}(x - a_0t)\right))^2\right). \]

Case (3) : \( a_1 = 2i, b_1 = \frac{1}{4}a_0^2i, b = -\frac{1}{8}a_0^2, \lambda = -a_0 \), with \( a_0 \) being an arbitrary constant, the complex wave solutions are:

\[ u(x,t) = a_0 + \frac{\sqrt{2}a_0}{2}\left(\tan\left(\frac{\sqrt{2}a_0}{4}(x - a_0t)\right) + \cot\left(\frac{\sqrt{2}a_0}{4}(x - a_0t)\right)\right), \]  
\[ v(x,t) = a_0\left(a_0 + \frac{\sqrt{2}a_0}{2}\left(\tan\left(\frac{\sqrt{2}a_0}{4}(x - a_0t)\right) + \cot\left(\frac{\sqrt{2}a_0}{4}(x - a_0t)\right)\right)\right) \]  
\[ - \frac{1}{2}(a_0 + \frac{\sqrt{2}a_0}{2}\left(\tan\left(\frac{\sqrt{2}a_0}{4}(x - a_0t)\right) + \cot\left(\frac{\sqrt{2}a_0}{4}(x - a_0t)\right)\right))^2. \]

Case (4) : \( a_1 = -2i, b_1 = \frac{1}{4}a_0^2i, b = \frac{1}{8}a_0^2, \lambda = -a_0 \), with \( a_0 \) being an arbitrary constant, the complex wave solution are:

\[ u(x,t) = a_0 - \frac{\sqrt{2}a_0}{2}\left(\tan\left(\frac{\sqrt{2}a_0}{4}(x - a_0t)\right) + \cot\left(\frac{\sqrt{2}a_0}{4}(x - a_0t)\right)\right), \]  
\[ v(x,t) = a_0\left(a_0 - \frac{\sqrt{2}a_0}{2}\left(\tan\left(\frac{\sqrt{2}a_0}{4}(x - a_0t)\right) + \cot\left(\frac{\sqrt{2}a_0}{4}(x - a_0t)\right)\right)\right) \]  
\[ - \frac{1}{2}(a_0 - \frac{\sqrt{2}a_0}{2}\left(\tan\left(\frac{\sqrt{2}a_0}{4}(x - a_0t)\right) + \cot\left(\frac{\sqrt{2}a_0}{4}(x - a_0t)\right)\right))^2. \]

All the solutions of the variant Boussinesq equations are new.
4 Conclusions

In this paper, the MEDA method has been successfully applied to find the solution for three nonlinear partial differential equations such as the variant of the RLW equation, the variant of the PHI-four equation, and the variant Boussinesq equations. The modified extended direct algebraic method is used to find a new complex travelling wave solutions. The results show that the modified extended direct algebraic method is a powerful mathematical tool to solve the variant of the RLW equation, the variant of the PHI-four equation, and the variant Boussinesq equations. It is also a promising method to solve other nonlinear equations.

References


IINS homepage:http://www.nonlinearscience.org.uk/