Chaos Control and Chaotification for a Three-dimensional Autonomous System

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Abstract: In this paper, a periodic parametric perturbation was designed for controlling chaos and generating chaos (chaotification) from a three-dimensional autonomous system. When we added the small periodic parametric perturbations, the system became a non-autonomous system, which has abundant dynamics behavior. Periodic, chaotic and hyperchaos behaviors were identified by the Lyapunov exponents and bifurcation diagram. It was found that this method not only can control chaotic behavior, but also generate chaos and hyperchaos in non-chaotic parameter ranges.

Keywords: periodic parametric perturbations; non-autonomous system; chaos control; chaotification; hyperchaos

1 Introduction

Chaos is a very interesting non-linear phenomenon, and it has been detected in many practical applications. Chaos is generally believed to be harmful, because a nonlinear system in the chaotic state is very sensitive to its initial condition and chaos causes often irregular behavior in practical systems. For example, chaos in magnetically confined plasma may evolve into fully developed turbulence and lead to anomalous energy and particle cross-field transport. So traditional control research has mainly focused on determining ways to eliminate or reduce chaotic behavior in a system. In contrast, sometime chaos provides a system designer with a variety of special properties that are richly flexible and could be exploited. Examples include chemical reactions, liquid mixing [1], human brain and heartbeat regulation [2, 3], resonance prevention in mechanical systems [4], and certain types of secure communications [5]. Therefore, sometimes it is useful and even important to make an originally non-chaotic system chaotic, or to enhance a chaotic system to present new chaotic behavior applications. These provide a strong motivation for the current research on chaotification of the discrete-time dynamical systems and the continuous-time dynamical systems. Thus, controlling chaos can be divided into two categories: one of which is suppressing chaotic dynamical behavior and another is generating or enhancing chaos (also called chaotification) in nonlinear systems. Different methods and techniques have been proposed for controlling chaos, such as OGY method [6], active control, observer-based control, feedback and nonfeedback control, adaptive control [7-12], etc. Recently, parametric control of chaos has been studied in several papers [13-14].

In this paper, we add a simple parametric perturbation to a three-dimensional autonomous system [15] and investigate its effects on the system. By means of Lyapunov exponents and bifurcation diagrams, the dynamics of the system with parametric perturbations is investigated. This system has richer dynamical behaviors. We find out that this parametric perturbation method not only control the original chaotic behaviors, but also generate chaos and hyperchaos in non-chaotic parameter ranges.

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System description

In paper [15], Liu established a three-dimensional autonomous system, which relies on two multipliers and one quadratic term to introduce the nonlinearity necessary for folding trajectories. The chaotic attractor obtained from this system according to the detailed numerical as well as theoretical analysis is also the butterfly shaped attractor, exhibiting the abundant and complex chaotic dynamics.

This system is described by the following non-linear differential equations:

\[
\begin{align*}
\dot{x} &= -ax - ey^2 \\
\dot{y} &= by - kxz \\
\dot{z} &= -cz + mxy
\end{align*}
\]  

(1)

This system is one dissipative system and has a chaotic attractor, when \(a = 1, b = 2.5, c = 5, e = 1, k = 4, m = 4\) and initial condition \([0.2, 0, 0.5]\). The value of Lyapunov exponents of this system is obtained as \((0.4412, 0, -3.9418)\).

In this paper, suppose there exists a periodic parametric perturbations in the above system, then we can write system (1) as follows:

\[
\begin{align*}
\dot{x} &= -ax - ey^2 \\
\dot{y} &= b(1 + d_1 \sin d_2 t)y - kxz \\
\dot{z} &= -cz + mxy
\end{align*}
\]  

(2)

where \(d_1\) is the perturbation amplitude, and \(d_2\) is the perturbation angular frequency.

Dynamical analysis

In this section, we will analyse the dynamic behaviors of the system (1) without perturbations and the system (2) with parametric perturbations by means of Lyapunov exponents and bifurcation diagrams. The dynamics may be viewed globally over a range of parameter values, thereby allowing simultaneous comparisons of periodic, chaotic and hyperchaotic behaviors. The addition of the time-varying parametric perturbations \(d_1 \sin d_2 t\) changes the autonomous system (1) to the non-autonomous system (2), which is equivalent to a four-dimensional autonomous system. This new perturbed system has more rich dynamical behaviors.

In general, a hyperchaotic system is defined as a chaotic system with at least two positive exponents, implying that its dynamics are expended in several different directions simultaneously [16-17]. It means that hyperchaotic systems have more complex dynamical behaviors.

For convenience, assume that the Lyapunov exponents of the system (2) are \(\lambda_i (i = 1, 2, 3, 4)\) satisfying \(\lambda_1 > \lambda_2 > \lambda_3, \lambda_4 = 0\). Then the dynamics of the system (2) can be characterized with \(\lambda_i (i = 1, 2, 3, 4)\).

(1) For periodic orbits, \(\lambda_{1,2,3} < 0, \lambda_4 = 0\);
(2) For chaotic attractor, \(\lambda_1 > 0, \lambda_{2,3} < 0, \lambda_4 = 0\);
(3) For hyperchaotic attractor, \(\lambda_{1,2} > 0, \lambda_3 < 0, \lambda_4 = 0\).

The parameters \(a = 1, c = 5, e = 1, k = 4\) and \(m = 4\) are fixed, while \(b\) varies. Fig.1 display the Lyapunov exponents of the system (1) without perturbations versus parameter \(b\). From the Fig.1, we find that the system (1) is chaotic in \(b \in (1.45, 3.6)\), the system (1) has some typical periodic orbits in \(b \in (3.6, 4.4)\) and \(b \in (0.5, 1.44)\), there appear a steady state.

Figure 1: Lyapunov exponents of the system (1) for varying \(b\).
Throughout this paper, the system parameters $a = 1$, $c = 5$, $e = 1$, $k = 4$ and $m = 4$ are fixed.

(1) Fix $d_1 = 0.7$, $d_2 = 5$ and let $b$ vary. Lyapunov exponents of the system (2) are shown in Fig. 2, while part of the corresponding bifurcation diagram is given in Fig. 3. It can be observed that the spectrum of Lyapunov exponents with the bifurcation diagram well coincides. They reveal that periodic orbits and chaos appear alternately when $b$ is gradually increased from 0.5 to 4.5. When $b$ increases from 1.4 to 4.5, the perturbed system (2) has three periodic windows: $w_1 = [0.5, 1.25], w_2 = [2.25, 2.45]$ and $w_3 = [3.63, 3.66]$. From the second periodic window (a long periodic window $b \in [2.25, 2.45]$) of the bifurcation diagram, one can clearly see that the system (2) smoothly evolves into a chaotic region through a period-doubling bifurcation from the periodic orbit.

![Figure 2: Lyapunov exponents of the system (2) for varying $b$, with $d_1 = 0.7, d_2 = 5$.](image)

Figure 3: Bifurcation diagram of the system (2) for varying $b$, with $d_1 = 0.7, d_2 = 5$.

Comparing with the unperturbed system (1), we can see that the parametric perturbation generates chaos within originally non-chaotic parameter regions, such as $b \in (1.4, 1.42)$ and $b \in (3.8, 4.4)$, but chaos is controlled within $b \in (2.4, 2.5)$ to periodic orbit.

(2) Fix $b = 2.5$, $d_2 = 5$ and let $d_1$ vary. Lyapunov exponents of the system (2) are shown in Fig. 4a and the bifurcation diagram of the system (2) is shown in Fig. 5. The results of the four Lyapunov exponents reveal that periodic orbits, chaos and hyperchaos appear alternately with $d_1$ increasing gradually from 0.1 to 1. When only $\lambda_1$ is positive, the system (2) is chaotic, and when both $\lambda_1$ and $\lambda_2$ are positive, the system (2) is hyperchaotic. It can be seen that the system (2) is chaotic when $d_1 \in [0.1, 0.42]$ and $d_1 \in [0.58, 1]$, the perturbed the system (2) has a long periodic windows: $d_1 = [0.43, 0.57]$. In order to examine the dynamics of the system (2) carefully, the Lyapunov exponents figure of $\lambda_2$ against $d_1$ ($d_1 \in [0.1, 0.15]$) was enlarged in Fig. 4b. This indicates that the system (2) fluctuates between chaotic and hyperchaotic motions when $d_1 \in [0.1, 0.15], d_1 = 0.1$ is a typical value of parameters that generate hyperchaos.

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(3) Fix $b = 2.5$, $d_1 = 0.7$ and let $d_2$ vary. Lyapunov exponents of the system (2) are shown in Fig. 6a and the bifurcation diagram of the system (2) is shown in Fig. 7. It can be seen that the system (2) has only one periodic window: $d_2 = [5, 6.5]$, the system (2) is chaotic with other range. In order to examine the dynamics of the system (2) carefully, the Lyapunov exponents figure of $\lambda_2$ against $d_2$ ($d_2 \in [25, 30]$) was enlarged in Fig. 6b. This indicates that the system (2) fluctuates between chaotic and hyperchaotic motions when $d_2 \in [25, 30]$. 

Figure 4: Lyapunov exponents of the system (2) for varying $d_1$, with $b = 2.5$, $d_2 = 5$.

Figure 5: Bifurcation diagram of the system (2) for varying $d_1$, with $b = 2.5$, $d_2 = 5$.

Figure 6: Lyapunov exponents of the system (2) for varying $d_2$ with $b = 2.5$, $d_1 = 0.7$. 

(b) Enlarged figure for $\lambda_2$ varying $d_2$.
4 Conclusions

This paper has presented a parametric perturbation method that can change a three-dimensional autonomous system’s dynamical behaviors. The resulting perturbed system is analyzed successfully by bifurcation analysis and Lyapunov exponent spectrums. The parametric perturbation added not only suppresses the original chaotic behaviors to periodic orbits, but also generates chaos in non-chaotic parameter ranges. Furthermore, it can generate hyperchaos in some parameter ranges. Compared to OGY method, this method is simple, because it did not consider the feedback item. This also suggests an excellent methodology for the design of new generators of chaos and hyperchaos, as well as for controlling chaos.

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