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Internet Traffic Modelling -Variance Based Markovian Fitting of Fractal Point Process from Self-Similarity Perspective

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Abstract. Most of the proposed self-similar traffic models could not address fractal onset time at which self-similar behavior actually begins. This parameter has considerable impact on network performance. Fractal point process (FPP) emulates self-similar traffic and involves fractal onset time (FOT). However, this process is asymptotic in nature and has less effective in queueing based performance. In this paper, we propose a model of variance based Markovian fitting. The proposed method is to match the variance of FPP and superposed Markov modulated Poisson Process (MMPP) while taking FOT into consideration. Superposition consists of several interrupted Poisson processes (IPPs) and Poisson process. We present how well resultant MMPP could approximate FPP which emulates self-similar traffic. We investigate queueing behavior of resultant queueing system in terms of a packet loss probability. We demonstrate how FOT affects the fitting model and queueing behavior. We conclude from the numerical example that network nodes with a self-similar input traffic can be well represented by a queueing system with MMPP input

Keywords: *Self-similarity; Fractal point process; fractal onset time; Markov modulated Poisson process, Variance; Packet loss probability.*

1. Introduction

Seminal studies revealed the presence of self-similarity or long range dependence (LRD) in LAN, WAN, the variable bit rate (VBR) video traffic, and its impact on the network traffic [1-3]. This type of traffic exhibits statistical similarity over different time scales and is highly correlated. Characterizing the statistical behavior of traffic is crucial to proper design of routers to provide the quality of service (QoS). If the traffic models do not accurately represent the real traffic, then the network performance may be estimated over or underestimated [4]. Traffic models such as Fractional Brownian Motion (FBM), Fractional Auto Regressive Integrated Moving Average (FARIMA), Chaotic maps are proposed to characterize the self-similarity. Although, these processes are parsimonious, but are less effective in the case of queueing based performance evaluation when buffer sizes are small. In [5-8], Markovian arrival process (MAP) is employed to model self-similar behavior over the different time scales. These fitting models equate the second order statistics of self-similar traffic and that of superposition of several 2-state Markov modulated Poisson Process (MMPP) over desired time-scales. However, in the paper [5], covariance function of resultant MMPP is approximated by suppressing the higher order terms in Taylor's expansion. In the paper [6], MMPP emulating the self-similar traffic is fitted by matching variance over the desired time-scales. Resultant MMPP here is superposition of several Interrupted Poisson Process (IPPs) wherein two modulating parameters of each IPP are equal. The fitting method [6, 7] is generalized in the paper [8] by taking distinct modulating parameters in each IPP. Paulo Salvador et.al [9] proposed a model to fit discrete time MMPP that matches both autocovariance and marginal distribution of the counting process in such a way that model can capture self-similar behavior up to the time-scales of interest. Fractal on set time (FOT) defines the time scale from which self-similar behavior begins and is denoted by T_0 [11]. In the paper [12], the impact of FOT is realized besides the impact of another important characteristic Hurst parameter H of the self-similar traffic. According to the measurement studies, FOTs of the network traffic are at scales in the order of a few hundreds of milliseconds. The FOT plays an important role in characterizing the burstiness of the network traffic. In the said papers, the Markov-modulated Poisson process (MMPP) emulating the self-similar traffic over the different time scale is fitted, however, the time scale where self-similar nature actually begins is not considered.

Fractal point processes (FPPs) are proved to be self-similar [12], and they provide network traffic models [13]. The second order statistics of FPP involve not only the Hurst parameter but also FOT [12-13].

However, these processes are asymptotic in nature and has less effective in queueing based performance when the buffer sizes are small. That is, FPP can be used as a self-similar traffic generator, but it is not so useful in the context of the queueing theory. Hence, in this paper, first, we fit the MMPP for FPP by equating the variance while taking FOT into consideration and then we model network nodes such as routers by the $MMPP/D/1/K$ queueing system to investigate the queueing behavior. It is found from the numerical results that the MMPP model could emulate both the FPP traffic and the exact self-similar traffic. There are two objectives with the fitting described in this paper. First one is that, the resulting MMPP which works well for the queueing theory will have same statistical characteristics as that of FPP and self-similar process. The second one is to investigate queueing behavior under any traffic conditions. For the first objective, variance-time results of the self-similar traffic, FPP, and resultant MMPP are presented. In the context of the second objective, packet loss probability (PLP) against traffic intensity is presented.

The rest of the paper is organized as follows. In section 2, we first overview the definitions of self-similar process and FPP. In section 3, we present the fitting procedure. We then present the analytical results of $MMPP/D/1/K$, in section 4. In section 5, we demonstrate accuracy of the proposed model by means of numerical results. Finally, some conclusions are made in section 6.

2. Self-Similar Process and Fractal Point Process (FPP)

The second order statistics, namely variance, index of dispersion of counts (IDC), and auto covariance function (ACF) are relatively straightforward to fit the parameters of a model emulating self-similar traffic and gives much information [11]. As a result, these statistics are exploited by several authors. In this section, first we overview the definition of the self-similar process and the fractal point processes in terms of the second order statistics.

2.1 Self-Similar Process

Consider X to be a second -order stationary process with variance σ^2, and divide time axis into disjoint intervals of unit length, we could define $X = \{X_t / t = 1,2,3........\}$ to be a number of points (packet arrivals) in the t^{th} interval. A new sequence $X^{(m)} = X_t^{(m)}$, where $X_t^{(m)} = \frac{1}{m}\sum_{i=1}^{m} X_{(t-1)m+i}$, t = 1, 2, 3……. is the average of the original sequence in m non-overlapping blocks. Then the definitions of the exact second-order self-similar process and the asymptotically second-order self-similar process are defined as follows:

Definition 1. The second-order stationary process X is defined as an exact second-order self-similar process with the Hurst parameter, $H = 1 - \beta/2$ if

$$Var(X^{(m)}) = \sigma^2 m^{-\beta}, \forall m \geq 1.$$

Definition 2. The second-order stationary process X is defined as an exact second-order self-similar process with the Hurst parameter, $H = 1 - \beta/2$ if

$$r(k) = \frac{1}{2}\nabla^2(k^{2-\beta})$$

where $r(k)$ is an auto covariance function, $\nabla^2(.)$ is the second central difference operator and it is defined as

$$\nabla^2(f(k)) = f(k+1) - 2f(k) + f(k-1).$$

Definition 3. The process X is called asymptotically second-order self-similar process with the Hurst parameter, $H = 1 - \beta/2$ if

$$r^{(m)}(k) \to \frac{1}{2}\nabla^2(k^{2-\beta}) \quad \text{as } m \to \infty.$$

2.2 Fractal Point Process

Let $N(t)$ be the number of arrivals up to the time t and define X_n as the number of packets that are arrived during the n^{th} time interval of size T sec, i.e., $X_n \equiv N(nT) - N((n-1)T)$ then $c(n,T) = \mathrm{cov}(X_n, X_{n+k})$, is defined as the covariance between the number of arrivals in two counting windows of counting time T and separation kT. Then the index of dispersion for counts (IDC) in a specified window of width T is given by [13],

$$IDC(T) = \frac{Var(N(T))}{E(N(T))}$$

$$= 1 + \left(\frac{T}{T_0}\right)^{\alpha}, \quad \dots (1)$$

where $\alpha = 2H - 1$, and $0 < \alpha < 1$, $\frac{1}{2} < H < 1$ in the case of self-similar process. The autocorrelation function is given by [13]

$$C(k,T) = \lambda T . \begin{cases} 1 + (T/T_0)^{\alpha} & k = 0 \\ (T/T_0)^{\alpha} \frac{1}{2} \nabla^2 (k^{\alpha+1}) & k > 0. \end{cases} \quad \dots (2)$$

The auto covariance function (ACF) denoted by $r(k,T)$ is given by

$$r(k,T) = \frac{C(k,T)}{C(0,T)}$$

$$= \frac{T^{\alpha}}{T^{\alpha} + T_0^{\alpha}} \frac{1}{2} \nabla^2 (k^{\alpha+1}) \qquad (k > 0) \cdot \quad \dots (3)$$

Recall that X_n represents the number of packets during the n^{th} time interval of size T and $X_n^{(m)} = \frac{1}{m} \sum_{i=1}^{m} X_{(n-1)m+i}$, $n = 1,2,3,\dots$, is the average of the original sequence in m non-overlapping blocks, then the covariance $C^{(m)}(k,T)$, ACF $r^{(m)}(k,T)$, and variance $Var(X^{(m)})$ of this aggregated process are, respectively, given by [13]

$$C^{(m)}(k,T) = m^{-2} C(k, mT), \quad \dots (4)$$

$$r^{(m)}(k,T) = \frac{(mT)^{\alpha}}{(mT)^{\alpha} + T_0^{\alpha}} \frac{1}{2} \nabla^2 (k^{\alpha+1}) \qquad (k > 0), \quad \dots (5)$$

$$Var\,(X^{(m)}) = \lambda T\, [m^{-1} + (T/T_0)^{\alpha} m^{-(1-\alpha)}]. \quad \dots (6)$$

We shall make use of the Eq. (6) to fit the MMPP in the next section.

3. Fitting Procedure

In this section, we fit the model for FPP that emulates the self-similar traffic using the Markovian approach. This procedure is based on variance while taking FOT into consideration. This model is similar to that of the paper [8] involving superposition of d two state interrupted Poisson process (IPP) and Poisson process. IPP is a particular case of MMPP. We can describe i^{th} IPP as follows:

$$Q_i = \begin{bmatrix} -c_{1i} & c_{1i} \\ c_{2i} & c_{2i} \end{bmatrix}, \; \Lambda_i = \begin{bmatrix} \lambda_i & 0 \\ 0 & 0 \end{bmatrix}, \; 1 \le i \le d. \quad \dots (7)$$

Superposition of above d IPPs and a Poisson process is again an MMPP, and is characterized by

$$Q = Q_1 \oplus Q_2 \oplus ...... \oplus Q_d ,$$

$$\Lambda = \Lambda_1 \oplus \Lambda_2 \oplus .... \oplus \Lambda_d \oplus \lambda_p , \qquad \text{… (8)}$$

where $\oplus$ denotes the Kronecker sum, and λ_p is the arrival rate of the Poisson process to be superposed. Then the whole arrival rate λ is given by

$$\lambda = \lambda_p + \sum_{i=1}^{d} \frac{c_{2i}}{c_{1i} + c_{2i}} \lambda_i . \qquad \text{… (9)}$$

Let $N_{t,i}$ be the number of arrivals from the i^{th} IPP during the t^{th} timeslot and $N_{t,p}$ be the corresponding number of arrivals from the Poisson process, and let $N_{t,i}^{(m)}$ and $N_{t,p}^{(m)}$ be the number of arrivals from the averaged process of i^{th} IPP and Poisson process, respectively. The variance of this averaged process is given by [7, 8],

$$Var\ [N_{t,i}^{(m)}] = \frac{c_{2i}\lambda_i}{m(c_{1i} + c_{2i})} + \frac{c_{1i}c_{2i}}{m(c_{1i} + c_{2i})^3}[1 - \frac{1 - e^{-m(c_{1i}+c_{2i})}}{m(c_{1i} + c_{2i})}]\lambda_i^2 \qquad 1 \le i \le d \qquad \text{… (10)}$$

$$\text{and } Var\ [N_{t,p}^{(m)}] = \frac{\lambda_p}{m} . \qquad \text{… (11)}$$

Hence, the variance of the whole process is

$$Var[X_t^{(m)}] = \frac{\lambda}{m} + \sum_{i=1}^{d} \eta_i \lambda_i^2 , \qquad \text{… (12)}$$

$$\text{where } \eta_i = \frac{2c_{1i}c_{2i}}{m(c_{1i} + c_{2i})^3}[1 - \frac{1 - e^{-m(c_{1i}+c_{2i})}}{m(c_{1i} + c_{2i})}]$$

For the given traffic parameters H, T_0, and λ we match the variance at d different points m_i , $i = 1, 2, \cdots, d$. Let $[m_{\min}, m_{\max}]$ (that is $m_{\min} \le m \le m_{\max}$) be the time interval over which we want the process to emulate self-similarity of the original process, then m_i is given by

$$m_i = m_{\min} a^{i-1}, \qquad i = 1, 2, \cdots, d ,$$

where

$$a = \left(\frac{m_{\max}}{m_{\min}} \right)^{\frac{1}{d-1}} , \qquad d > 1 . \qquad \text{… (13)}$$

Now, we assume the following relations

$$m_i c_{1i} = c_1 \text{ (say)}, \quad m_i c_{2i} = c_2 \text{ (say)} \quad i = 1, 2, \cdots, d$$

$$\text{that is } \quad c_{1i} = \frac{m_1}{m_i} c_{11} \qquad \text{… (14)}$$

These are due to the fact that a self-similar process looks the same in any time scale. Because of these assumptions, the number of parameters to be determined is reduced. That is, if we determine c_{11}, c_{21}, we can obtain the values of c_{1i}, c_{2i} $(i = 1, 2, \cdots, d)$. Having c_{1i}, c_{2i} $(i = 1, 2, \cdots, d)$, and λ_i we can obtain the λ_p from the Eqn. (12). Now the parameters need to find are c_{11}, c_{21} and λ_i $(i = 1, 2, \cdots, d)$. Step by step method to determine these parameters is as follows [8]:

Step 1. [Assign appropriate initial approximations for both c_{11}, c_{21}].

Step 2. [Determine λ_i as the function of c_{11}, c_{21}].

From (6) and (12), we have

$$\lambda \begin{bmatrix} 1+\frac{m_1^{2\alpha}}{T_0^{\alpha}} \\ 1+\frac{m_2^{2\alpha}}{T_0^{\alpha}} \\ \cdot \\ \cdot \\ \cdot \\ 1+\frac{m_d^{2\alpha}}{T_0^{\alpha}} \end{bmatrix} = \lambda \begin{bmatrix} m_1^{-1} \\ m_2^{-1} \\ \cdot \\ \cdot \\ \cdot \\ m_d^{-1} \end{bmatrix} + B \begin{bmatrix} \lambda_1^2 \\ \lambda_2^2 \\ \cdot \\ \cdot \\ \cdot \\ \lambda_d^2 \end{bmatrix} \quad \ldots (15)$$

which gives

$$\lambda \begin{bmatrix} 1+\frac{m_1^{2\alpha}}{T_0^{\alpha}} - m_1^{-1} \\ 1+\frac{m_2^{2\alpha}}{T_0^{\alpha}} - m_2^{-1} \\ \cdot \\ \cdot \\ \cdot \\ 1+\frac{m_d^{2\alpha}}{T_0^{\alpha}} - m_d^{-1} \end{bmatrix} = B \begin{bmatrix} \lambda_1^2 \\ \lambda_2^2 \\ \cdot \\ \cdot \\ \lambda_d^2 \end{bmatrix} \quad \ldots (16)$$

where B is the '$d \times d$' matrix whose (i, j) element is

$$B_{ij} = \frac{2c_{1j}c_{2j}}{m_i(c_{1j}+c_{2j})^3}\left[1-\frac{1-e^{-m_i(c_{1j}+c_{2j})}}{m_i(c_{1j}+c_{2j})}\right].$$

Eqn. (16) is a non-homogeneous system of equations in $\lambda_1^2, \lambda_2^2, \ldots\ldots \lambda_d^2$. Solving this system by any Matrix method, Cramer's rule (say) and using (14) and (16), we can express $\lambda_i \;\; (i=1,2,\cdots,d)$ as the function of c_{11}, c_{21}.

Step 3. [Determine the values of c_{11}, c_{21}].

Using (16) and the expressions for λ_i obtained in step 2, consider the integral

$$\int_{m_{\min}}^{m_{\max}} [R.H.S\, of\, (6) - R.H.S\, of\, (12)]\, dm \;. \quad \ldots (17)$$

This integral is the function of two parameters c_{11}, c_{21}. Determine the values of c_{11}, c_{21} so that the value of the integral is minimum.

Step 4. [Compute the values of λ_i from the equations obtained in step 2].

From step 2, it is clear that matrix B must be non-singular. Sufficient condition under which B is non-singular is given in the papers [6, 8].

3.1 Analytical Results of Index of Dispersion for Counts (IDC)

The Index of Dispersion for Counts (IDC) of an aggregated process of the Second order Self-similar process and fractal point process is given as follows.
The mean of the aggregated process is

$$E(X_t^m) = \frac{\lambda}{m}, \qquad \dots (18)$$

From Eqs. (12), (18), The Index of dispersion for counts of an aggregated process is

$$IDC(X_t^m) = \frac{Var(X_t^m)}{E(X_t^m)},$$

$$IDC(X_t^m) = \frac{\frac{\lambda}{m} + \sum_{i=1}^{d} \eta_i \lambda_i^2}{\frac{\lambda}{m}}. \qquad \dots (19)$$

The Index of dispersion counts of an exact self-similar process is

$$IDC(X_t^m) = \frac{Var(X_t^m)}{E(X_t^m)} = \frac{\sigma^2 m^{-\beta}}{\frac{\lambda}{m}} = \frac{\sigma^2 m^{-\beta+1}}{\lambda}. \qquad \dots (20)$$

and from Eqs. (6), (18) Index of dispersion counts of fractal point process

$$IDC(X_t^m) = mT\,[m^{-1} + (T/T_0)^{\alpha} m^{-(1-\alpha)}]. \qquad \dots (21)$$

4. Loss Behavior of Resultant Queueing System

In this section, having the MMPP emulating self-similar input traffic, network nodes such as router or switch is modeled as $MMPP/D/1/K$ queueing system to analyze the loss behavior. In the $MMPP/D/1/K$ queue, the packets arrive according to the MMPP of m states and is characterized by a matrices Q, R, where Q, R are $m \times m$ matrices. The service time is deterministic with mean service rate h. That is, we are considering synchronous input traffic of fixed length h (in time units). Let D_k, $k \geq 0$ denote the matrices of order $m \times m$ whose (*i*,*j*) element is the probability that given departure at time 0, which left at least one packet in the system and the process is in state *i*, the next departure occurs when the arrival process in *j*, and during that service time there were k arrivals. Then D_k in the case of general service time distribution $H(t)$ satisfies the following equation

$$\sum_{k=0}^{\infty} D_k z^k = \int_0^{\infty} e^{[Q-R+Rz]t} dH(t). \qquad \dots (22)$$

Since the service time is deterministic with h time units, then the above equation can be reduced to

$$\sum_{k=0}^{\infty} D_k z^k = e^{[Q-R+Rz]h}. \qquad \dots (23)$$

Now we compute the D_k s using the recurrence formulae [14]. Consider the embedded Markov chain $\{L(n), J(n) / n \geq 0\}$ at the departure epochs of the queueing system on the state space $S = \{(b,i) / 0 \leq b \leq K-1, 1 \leq i \leq m\}$, where *L(n)* denotes buffer occupancy and *J(n)* denotes the state of MAP. Then the pertinent embedded Markov chain has the following transition probability matrix :

$$P = \begin{bmatrix} GD_0 & GD_1 & \dots & GD_{K-2} & GE_{K-1} \\ D_0 & D_1 & \dots & D_{K-2} & E_{K-1} \\ 0 & D_0 & \dots & D_{K-3} & E_{K-2} \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & D_1 & E_2 \\ 0 & 0 & \dots & D_0 & E_1 \end{bmatrix} \qquad \dots (24)$$

where $G = (R - Q)^{-1} R,$ and its (i, j) element is conditional probability that the system is not busy when the system is in j state given that system was in i state, and $E_i = \sum_{k=i}^{\infty} D_k$. Let $\overrightarrow{y_k}$, $(0 \le k \le K - 1)$ be an $1 \times m$ vector whose i^{th} element is the stationary conditional probability that the number of packets in the system is k and the state of underlying arrival process is in i at an arbitrary time. The packet loss probability (PLP) is given by [7]

$$PLP = 1 - \frac{\left(1 - \overrightarrow{y_0} e\right)}{\rho}. \qquad \dots (25)$$

In the above equation, e is column vector consisting of 1 and ρ is the traffic intensity.

5. Numerical examples

In this section, we investigate the accuracy of the proposed fitting model in terms of the second order statistics of counts and queueing-based performance measure, namely packet loss probability. We demonstrate how the proposed fitting model and the queueing behavior are affected by T_0. We have fitted MMPP by the equating its variance to that of real time traffic measured at AT&T Bell Labs [13], over the time scale [10^2,10^8]. The values of traffic parameters are $H = 0.92$, $\lambda = 8.1, \sigma^2 = 133.5$, and $T_0 = 0.033$, The variance time curves of the resultant MMPP, that of FPP, and that of the above real-time traces are shown in Fig.1. Also, for the said case IDC curves are presented in Fig. 2. From the figures, we conclude that our results exhibit good concord with that of the self-similar traffic and that of FPP. Variance-time and IDC-time results pertaining to the values $H = 0.6$, $\lambda = 1, \sigma^2 = 0.6$ (Sample1), $H = 0.7$, $\lambda = 1, \sigma^2 = 0.6$ (Sample 2), and $H = 0.8$, $\lambda = 1, \sigma^2 = 0.6$ (Sample 3), $T = 1$, and for arbitrary values of FOT are presented in Figs. 3-8. The pertinent self-similar traffic of these samples are generated in the paper [6] by the random midpoint displacement algorithm [10]. In all the fitting cases, the number of two state MMPPs, d , is equal to 4. From these figures it is observed that fitted MMPP could emulate the FPP and the self-similar traffic. Next, we investigate queueing behavior in terms of a performance measure namely packet loss probability in the resultant MMPP/D/1/K queue for arbitrary values of FOT. Following [8], and [15], we use matrix analytic methods to compute steady state probability distribution of the transition probability matrix P of buffer occupancy that in turn gives the packet loss probability. The packet loss probability is computed using Eqn. (25). Numerical calculations are performed using MATLAB and results are shown in figures 9-16. The buffer depth K is taken to be 10. Figures 9 and 10 illustrate the results for the case of Hurst parameter H=0.7 for different values of T_0 over the time scales [10^2, 10^7] and [10^2, 10^6]. Figures .11 and 12 depict the results for the case of Hurst parameter H=0.6 for different values of T_0 over the time scales [10^2, 10^8] and [10^2, 10^6] , respectively. From the figures we conclude that PLP is affected with T_0 for the same value of Hurst parameter. i.e., packet loss probability decreases as T_0 increases for $T_0 \le T$. Figure 13 depicts the results for the case of T_0=0.95 and Hurst parameter H=0.6 over the different time scales [10^2, 10^6], [10^2, 10^7], and [10^2, 10^8]. From this figure it is clear that PLP increases with the time-scale as in the paper [8]. Figures 14, 15, and 16 illustrate the results for the case of T_0=0.95 for different Hurst parameter values $H = 0.6$,

$H = 0.7$, and $H = 0.8$ over the different time scales [10^2, 10^6], [10^2, 10^7], and [10^2, 10^8]. From the figures, we conclude that packet loss probability increases as H increases. From these observations, we conclude that fractal onset time T_0 does have significant effect on the queueing behavior of network nodes.

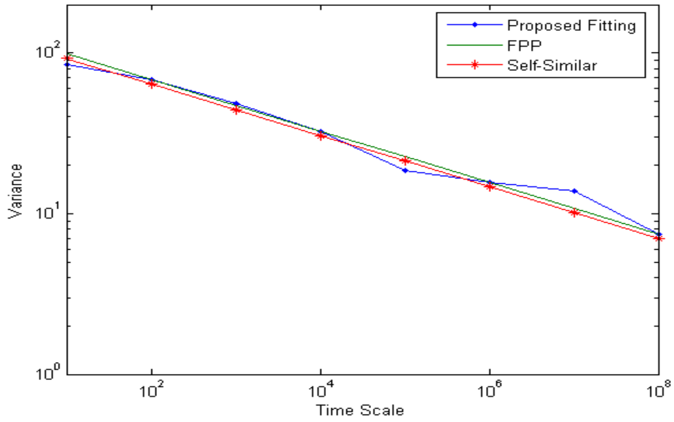

Fig.1 Variance-time curves of the FPP, resultant MMPP and real time self-similar traffic with the values $H = 0.92$, $\lambda = 8.1$, $\sigma^2 = 133.5$, and $T = 1$, $T_0 = 0.033$ over the time scale range [10^2, 10^8].

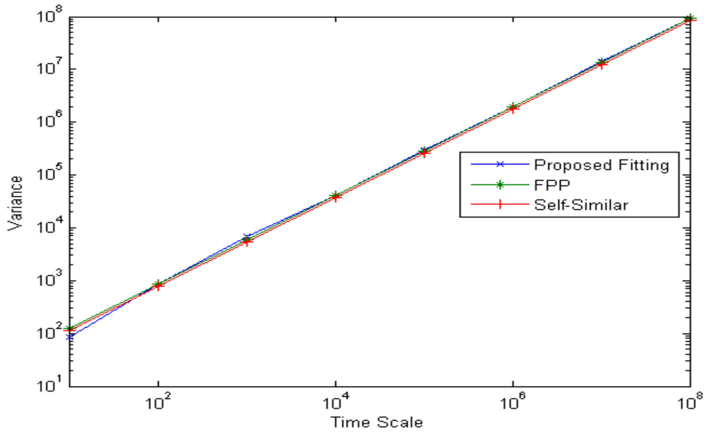

Fig.2. log (IDC)-log (Time) curves of the resultant MMPPs and FPPs over the time scale [10^2, 10^8] at $H = 0.92$, $\lambda = 8.1$, $\sigma^2 = 133.5$, and $T = 1$, $T_0 = 0.033$.

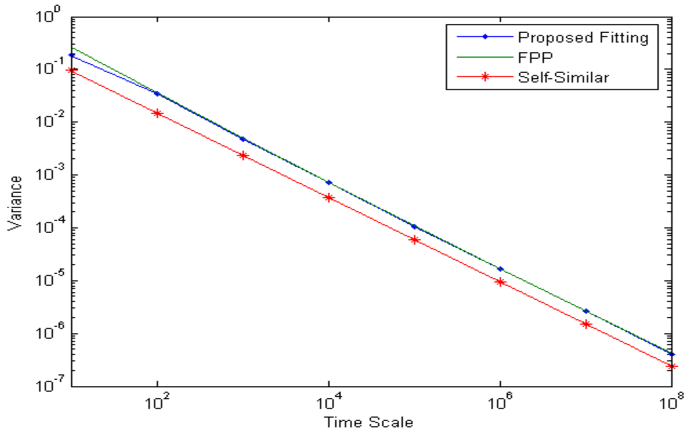

Fig.3. Variance-time curves of the FPP, resultant MMPP and self-similar traffic over the time scale range [10^2, 10^8] when $H = 0.6$, $\lambda = 1$, K=10 and $T = 1$, $T_0 = 0.95$.

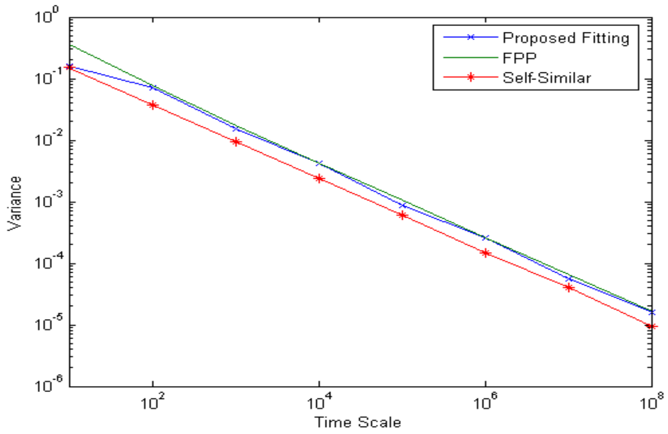

Fig.4. Variance-time curves of the FPP , resultant MMPP, and self-similar traffic over the time scale [10^2, 10^8] when $H=0.7$, $\lambda=1$, K=10 and $T=1$, $T_0=0.95$.

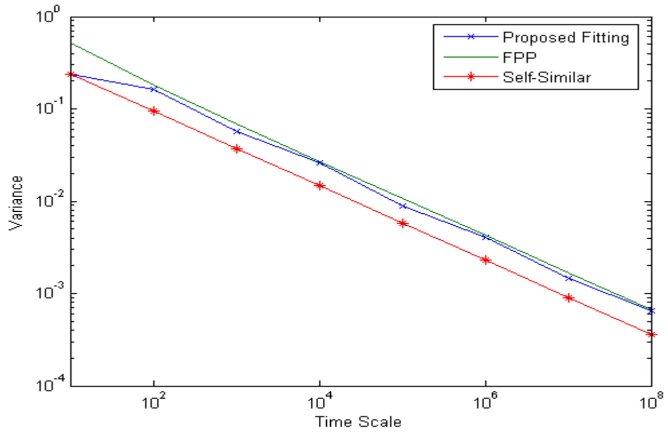

Fig.5. Variance-time curves of the FPP, resultant MMPP, and self-similar traffic over the time scale [10^2, 10^8] at $H=0.8$, $\lambda=1$, K=10 and $T=1$, $T_0=0.95$.

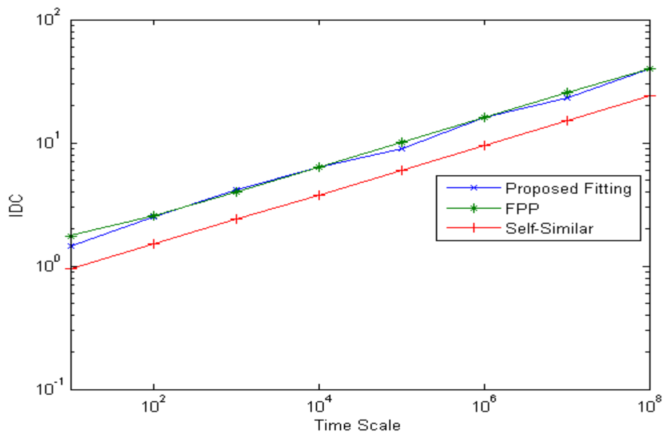

Fig.6. log (IDC)-log (Time) curves of the resultant MMPPs and FPPs over the time scale [10^2, 10^8] at $H=0.6$, $\lambda=1$, $\sigma^2=0.6$, and $T=1$, $T_0=0.95$.

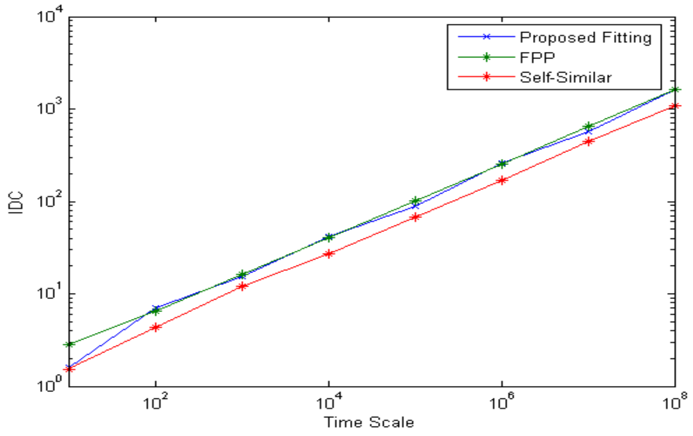

Fig.7. log (IDC)-log (Time) curves of the resultant MMPPs and FPPs over the time scale [10^2, 10^8] at $H=0.7$, $\lambda=1$, $\sigma^2=0.6$, and $T=1$, $T_0=0.95$.

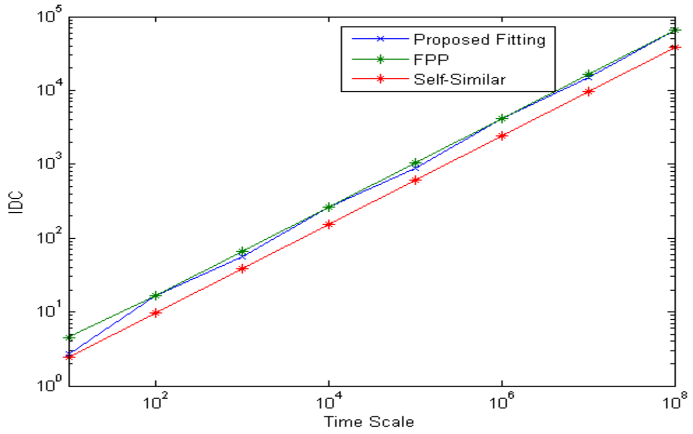

Fig.8. log (IDC)-log (Time) curves of the resultant MMPPs and FPPs over the time scale [10^2, 10^8] at $H=0.8$, $\lambda=1$, $\sigma^2=0.6$, and $T=1$, $T_0=0.95$.

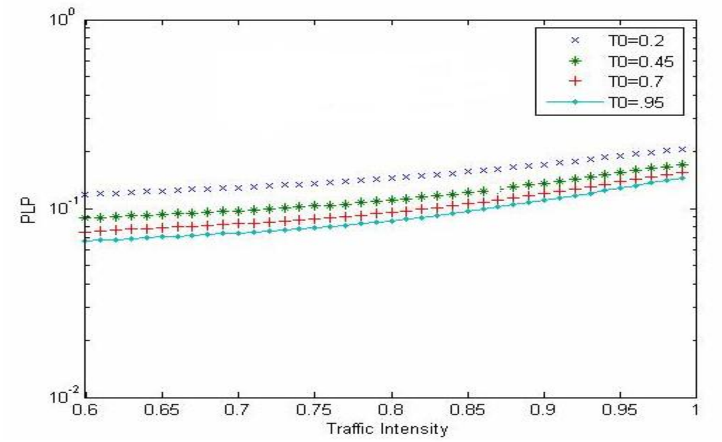

Fig.9. Loss probability of the resultant MMPP/D/1/K queues with d=4, H=0.7 $\lambda=1$, K=10 and T=1 over the time scale [10^2, 10^6].

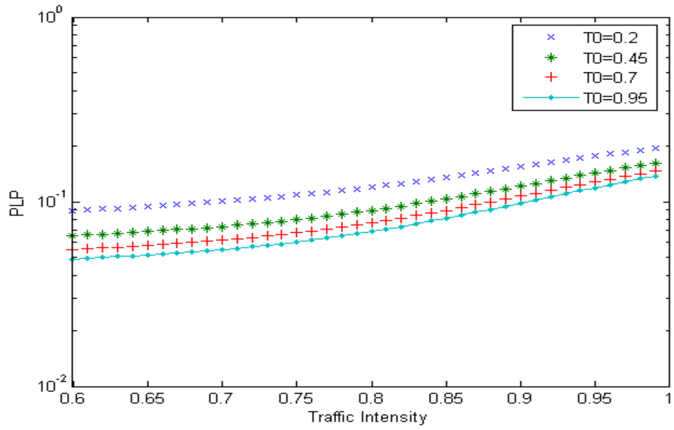

Fig.10. Loss probability of the resultant MMPP/D/1/K queues with d=4, $\lambda = 1$, H=0.7, K=10 and T=1 over $[10^2, 10^7]$.

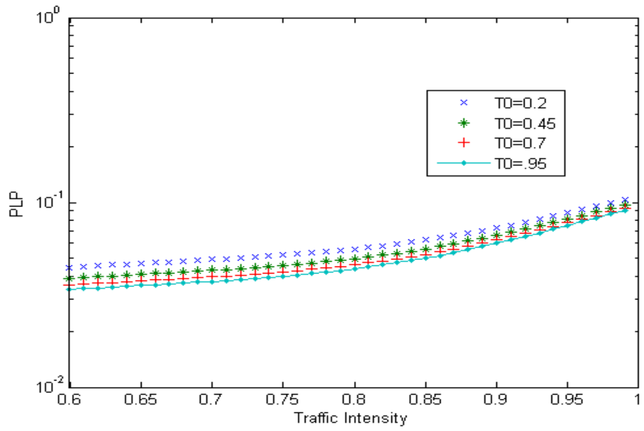

Fig.11. Loss probability of the resultant MMPP/D/1/K queues with d=4, $\lambda = 1$, H=0.6, K=10 and T=1 over $[10^2, 10^8]$.

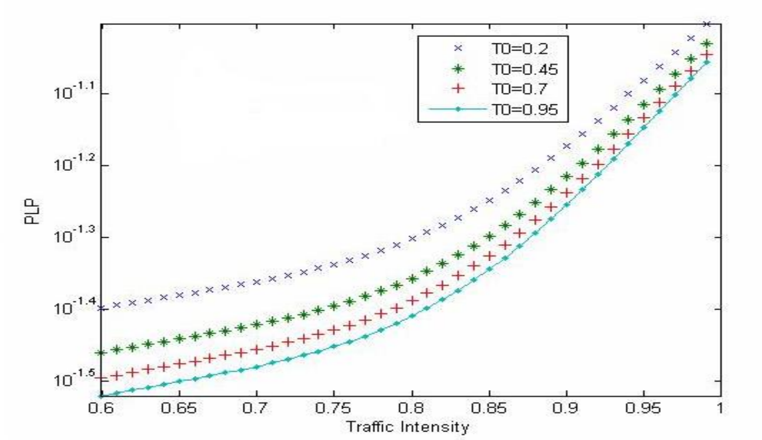

Fig.12. Loss probability of the resultant MMPP/D/1/K queues with d=4, H=0.6, $\lambda = 1$, K=10, and T=1 over the time scale $[10^2, 10^6]$.

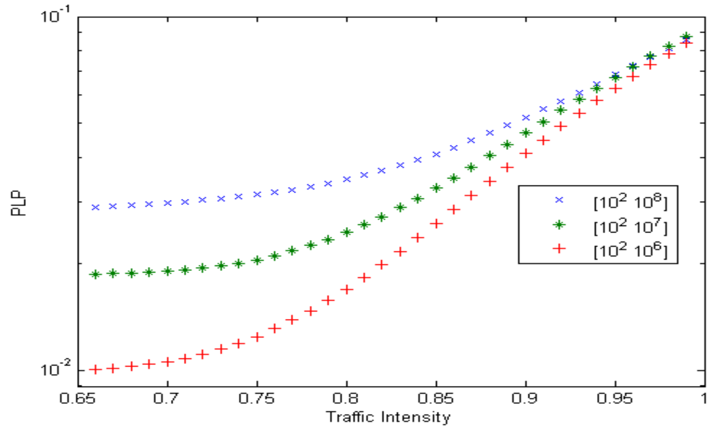

Fig.13. Loss probability of the resultant MMPP/D/1/K queues with d= 4, $\lambda = 1$, H=0.6, K=10, $T_0 = 0.95$ and T=1 over the different time scale .

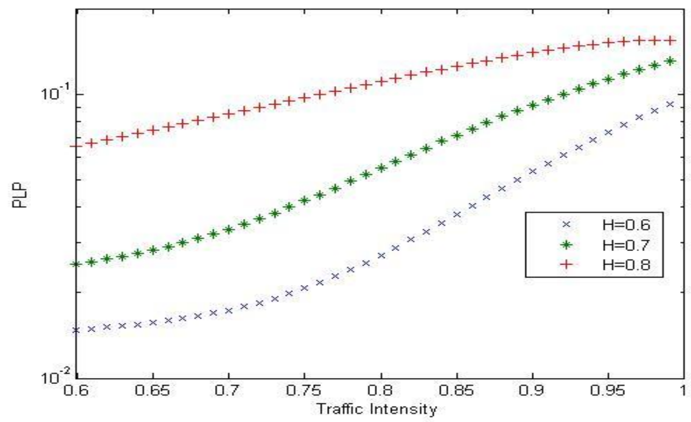

Fig.14. Loss probability of the resultant MMPP/D/1/K queues with d= 4, $\lambda = 1$, K=10, $T_0 = 0.95$ and T=1 over the time scale [10^2, 10^6].

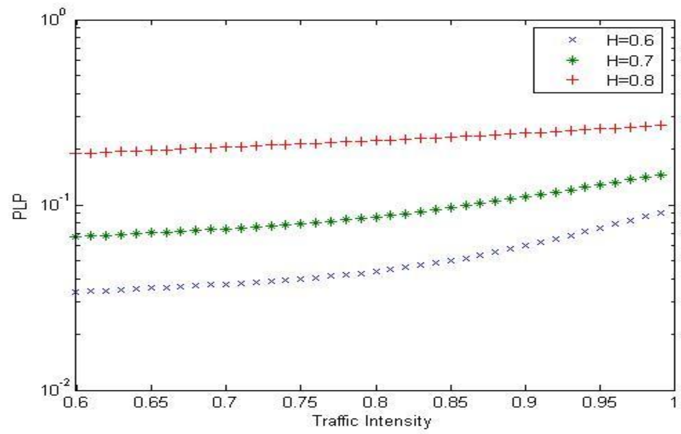

Fig.15. Loss probability of the resultant MMPP/D/1/K queues with d= 4, $\lambda = 1$, K=10, $T_0 = 0.95$ and T=1 over the time scale [10^2, 10^7].

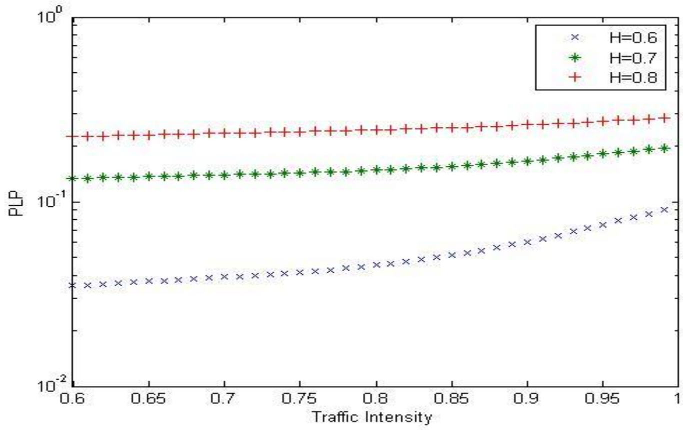

Fig.16. Loss probability of the resultant MMPP/D/1/K queues with d= 4, $\lambda = 1$, K=10, $T_0 = 0.95$ and T=1 over the time scale [10^2, 10^8].

6. Conclusion

The earlier self-similar traffic models are parsimonious, but asymptotic in nature. They are less effective in queueing based performance evaluation when the buffer sizes are small. Therefore, Markovian models are well recognized as the appropriate self-similar traffic models. However, these models do not involve fractal onset time. Fractal point process (FPP) involves fractal onset time, but is not suitable for queueing based performance evaluation. In order to have suitable model emulating FPP, we use variance based Markovian fitting procedure to match the variance of FPP over several time scales. We present how well the resultant MMPP could emulate the variance and IDC of FPP and the original self-similar traffic.

On the other hand, in order to dimension the internet router or switch accurately, we model it as a MMPP/D/1/K queueing system. We investigate queueing behaviour in terms of packet loss probability against traffic intensity, Hurst parameter, time-scale, and FOT. From the results, it is clear that loss probability increases as FOT and Hurst parameter increases. Numerical results show that self-similar network traffic can be investigated by means of FPP and MMPP. Proposed model is not parsimonious, in the sense, there is an extra parameter. However, there is no enhancement in the computational complexity because of this extra parameter.

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