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A Fliter Method for Sloving LCP Based on Nonmonotone Line Search

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Abstract. In this paper, we propose a filter method to solve the linear complementarity problem(LCP). By using the Fisher-Burmeister function, we convert the LCP to an equivalent optimization problem with linear equality constraints. A filter approach is employed to tackle the optimization problem and the proposed mechanism for accepting the trial step is obtained by a nonmonotone filter technique. Under some conditions, we establish the global convergence of the algorithm.

Keywords: linear complementarity, Fisher-Burmeister function, constrained optimization, filter method, nonmontone technique, global convergence.

1. Introduction

In this paper, we consider the following linear complementarity problem (LCP)

$$\begin{gathered} y = Mx + q \\ x \geq 0, y \geq 0, x^T y = 0, \end{gathered} \tag{1}$$

where $M \in R^{n\times n}, x, y \in R^n$ and $x \geq 0 (y \geq 0)$ means that $x_i \geq 0 (y_i \geq 0)$ $i = 1, 2, \cdots, n$. In this paper, we assume that the solution set of (1) is nonempty, and let X denote the solution set of (1) . For convenience, we use $w = (x^T, y^T)^T$.

LCP problem, arising in transportation, economy, engineering and many fields in the society, see [1,2] for survey. Optimization reformulation method is one of the most popular method for solving the LCP, one is the equivalent unconstrained optimization reformulation[3], and the other the equivalent constrained optimization reformulation[4]. In the last few years, a great deal of numerical methods had been proposed to deal with the responding optimization reformulation problems, such as nonsmooth Newton methods(see[4,5,6,7,8]), interior method(see[9])and smoothing method (see[10,11,12,13]and[14]for survey).

This paper will focus on the equivalent constrained optimization reformulation and the filter method to deal with linear equality constrained optimization reformulation of the LCP. The filter methods was proposed first by Fletcher and Leyffer[15], in which the use of a penalty function, a common feature of the large majority of the algorithms for constrained optimization, is replaced by the technique so-called "filter", and filter method has been actually applied in many optimization techniques, for instance, the pattern search method [16], the SLP method [17], the interior point method [18], the bundle approaches [19], the system of nonlinear equations and nonlinear least squares [20], multidimensional filter method[21], and so on.

In fact, filter method exhibits a certain degree of nonmonotonicity. The idea of nonmonotone technique can be traced back to Grippo et al.[22] in 1986. Due to its excellent numerical exhibition, over the last decades, the nonmonotone technique has been used in trust region method to deal with unconstrained and constrained optimization problems. Motivated by above ideas and methods, in this paper we use a filter algorithm that combines the nonmonotone technique for solving LCP.

The rest of paper is organized as follows: In the section 2, we state the knowledge summary and algorithm model. In the section 3 we analyze the convergence property of the algorithm. In the section 4, some discussions and remarks are given.

2. Knowledge summary and algorithm Model

It has been well known that by means of a suitable function: $\phi\ R^2 \to R$ the system

$$a \geq 0, b \geq 0, ab = 0, \tag{2}$$

can be transformed into an equivalent nonlinear equation

$$\phi(a,b) = 0, \tag{3}$$

In this situation, function ϕ is called as NCP-function. Then (1) can be reformulated as the following equivalent nonlinear equation system:

$$\Psi(x) = \begin{pmatrix} \phi(x_1, Mx+q)_1 \\ \vdots \\ \phi(x_n, Mx+q)_n \end{pmatrix}, \tag{4}$$

$$H(x,y) = \begin{pmatrix} \phi(x_1, y_1) \\ \vdots \\ \phi(x_n, y_n) \\ y - Mx - q \end{pmatrix}, \tag{5}$$

A lot of methods have been proposed to solve (4) or (5) to minimize their natural residual

$$\Phi_1(x) = \frac{1}{2}\|\Psi(x)\|^2 \quad or \quad \Phi_2(x) = \frac{1}{2}\|H(x,y)\|^2 \tag{6}$$

In general, (5) is nonsmooth and nonlinear, hence it is not easy to solve. However, in (5), the first *n* components are nonsmooth and nonlinear which is difficult to solve, contrarily to the first part, the last *n* components are easy to handle. Therefore, it is reasonable to handle the first part which consists of the *n* nonsmooth components and the second part which consists of the n linear equations separately. Based on this idea, we transform further (5) into the following equivalent minimization problem.

$$\min_{(x,y)\in R^{2n}} \Phi(x,y) = \frac{1}{2}\sum_{i=1}^{n} \phi\left(x_i, y_i\right)^2$$
$$s.t \qquad y - Mx - q = 0$$

Throughout the paper, we shall use the famous Fisher-Burmeister function defined by $\phi(a,b) = \sqrt{a^2 + b^2} - a - b, (a, b \in R)$, which has many promised properties and attracted the attention of many researchers.

As mentioned in the former, we exploit the famous Fisher-Burmeister function. Then (1) can be converted to the equivalent nonlinear equation system (5).

$$\min_{(x,y)\in R^{2n}} \Phi(x,y) = f(w) = \frac{1}{2}\sum_{i=1}^{n} \phi\left(x_i, y_i\right)^2 \tag{7}$$
$$s.t \qquad y - Mx - q = 0,$$

Since the algorithm we proposed in this paper converges to KKT point of (7). The first question needed to be answered is what condition guarantee that a KKT point of (1) is global solution of (7). Then, we easily know w^* solves (1) if and only if w^* solves (7). We have the following properties.

Lemma 2. 1[23] Function ϕ has the following properties:

(1) $\phi(a,b) = 0 \Leftrightarrow a \geq 0, b \geq 0, ab = 0$;

(2) ϕ is Lipschitz continuous with modulus $L = 1+\sqrt{2},\ i.e, \left|\phi(w)-\phi(w^{'})\right| \le L\left|w-w^{'}\right|$ for all $w, w^{'} \in R^2$;

(3) ϕ is directionally differentiable;

(4) The generalized gradient $\partial\phi(a,b)$ of ϕ at $(a,b)\in R^2$ equals to

$$\partial\phi(a,b) = \begin{cases} \{(\frac{a}{\sqrt{a^2+b^2}}-1, \frac{b}{\sqrt{a^2+b^2}}-1)\} & if\ (a,b)\neq(0,0) \\ \{(\xi-1,\zeta-1)\} & if\ (a,b)=(0,0) \end{cases},$$

where (ξ,ζ) is any vector satisfying $\sqrt{\xi^2+\zeta^2} \le 1$

Definition: If matrix M satisfy for any $x \in R^n$, $x \neq 0$, there exists a component $x_k \neq 0$, such that $x_k\left(Mx\right)_k \ge 0$, then we call M is p_0 matrix.

Lemma 2. 1[24] Let M be p_0 matrix. Furthermore, let vector $v, u \in R^n$ such that $u_i v_i \ge 0$ for all $i = 1,\cdots,n$. Then

$$u + Mv = 0 \quad if\ and\ only\ if \quad u = v = 0.$$

Lemma 2.3 If M is p_0 matrix, then

$$w^* = (x^*, y^*)\ solve\ (1) \Leftrightarrow w^*\ is\ a\ K-T\ point\ of\ (7).$$

Proof: see lemma2.5 in [23].

It is well known that the traditional SQP method generates a sequence { w_k } converging to the desired solution by solving quadratic programming problem

$$QP\left(w_k, G_k\right)\min\frac{1}{2}\|V_k d + \phi(w_k)\|^2$$

$$\min g(w_k)^T d + \frac{1}{2}d^T G_k d \tag{8}$$

$$s.t \quad C(w_k) + A(w_k)^T d = 0,$$

where we denote $g(w_k) = \nabla\Phi(x_k, y_k), C(w) = \left[-M, I\right]\begin{bmatrix} x_k \\ y_k \end{bmatrix} - q\ and\ A(w) = \nabla C\left(w\right) = \left[-M, I\right]$,

for a given $w_k = (x_k, y_k)$, we write $f(w_k), C(w_k), g(w_k)\ and\ A(w_k)\ as\ f_k, C_k, g_k, A_k$ respectively, $V_k^T \in \partial\phi\left(x^k, y^k\right)$ is a generalized Jacobian of $\phi\left(w\right)$, and $G_k = V_k^T V_k$.

As to the idea of filter method, solving problem (1) equivalents to minimize the objective function (8) and to satisfy the constraints. To inspect if the constraints are satisfied or not, we denote the violation function h as follows:

$$h(w) = \|C(w)\|. \tag{9}$$

It is easy to see that $h\left(w\right) = 0$ if and only if x is a feasible point ($h\left(w\right) > 0$ if and only if x is infeasible). To decide whether each point is better than the former one, we adopt nonmonotone technique to control $h(w)$ decreased nonmonotonically and to minimize function $f(w)$.

In our Algorithm, the nonmonotone parameter $m\left(k\right)$ satisfies

$$m\left(0\right) = 0, 0 \le m\left(k\right) \le min\left\{m\left(k-1\right)+1, N\right\},\ for\ k \ge 1, N > 0,$$

and N is positive integer, for the convenience, we denote

$$f\left(w_{l(k)}\right) = \max_{0\le j\le m(k)}\left[f\left(w_{k-j}\right)\right] \quad h\left(w_{l(k)}\right) = \max_{0\le j\le m(k)}\left[h\left(w_{k-j}\right)\right],$$

where $k-m(k)\le l(k)\le k$.

In the coming algorithm, we aim to decrease the value of $h(w)$, more precisely, we will use a trust region type method to obtain $C\left(w_k^i\right)=0$ by the help of the nonmonotone technique. Let

$$M_k^i = h\left(w_k^i\right)-h\left(w_k^i+d\right)$$
$$\Omega_k^i = h\left(w_k^i\right)-\left\|\left(C\left(w_k^i\right)+A\left(w_k^i\right)d\right)\right\|, \quad (10)$$

and $$r_k^i=\frac{M_k^i(d)}{\Omega_k^i(d)}.$$

Definition: A vector $w\in R^{2n}$ is called

(1) a stationary point of problem (7) if w it is feasible and there exists a vector $\rho=\left(\rho_1,\rho_2,\cdots,\rho_n\right)^T\in R^n$ such that

$$g(w)+A(w)\rho=0,$$
$$c_i(w)=0, i=1,2,\cdots,n$$

(2) an infeasible stationary point defined above is precisely a KKT point of problem(7) if w it is infeasible and

$$\gamma(w)=\min_{d\in R^{2n}}\max_{i\in 1,\cdots,n}\left\{c_i(w)+c_i(w)^T d\right\},$$

where $\gamma(w)=\max_{i\in 1,\cdots,n}\left\{c_i(w)\right\}$.

The definition about the stationary point can be found in [25].

(3) The function $\sigma:[0,+\infty]\to[0,+\infty]$ is a forcing function (F-function) if for any sequence $\{t_i\}\subset[0,+\infty]$ $\lim_{i\to 0}\sigma\left(t_i\right)=0$ implies $\lim_{i\to 0}t_i=0$.

(4) Let $\eta=\sup\left\{\left\|g\left(w_1\right)-g\left(w_2\right)\right\| w_1,w_2\in R^{2n}\right\}>0$. Then the mapping $\sigma:[0,+\infty]\to[0,+\infty]$ defined by

$$\theta(t)=\begin{cases}\left\{\inf \|w_1-w_2\| \big/ \left\|g\left(w_1\right)-g\left(w_2\right)\right\|\right\}\ge t & t\in[0,\eta)\\ \left\{\lim_{s\to\eta^-}\sigma(s)\right\} & t\in[\eta,+\infty)\end{cases}$$

is the reverse modulus of continuity of gradient $g(w)$.

The algorithm is given as follows:

Algorithm 2.1

Step 0: Chosen $x_0,y_0\in R^n$, we computer $G_0\in R^{2n\times 2n}$ and $\frac{1}{2}<\beta<1,\alpha_k\ge 0,0<\lambda<1,\lambda_{kr}\ge\lambda$ $r=1,2,\cdots m(k)-1,$Positive integer $N\ge 1$, set $k=0,m(k)=0$;

Step 1: Solve $QP\left(w_k,G_k\right)$ to get d_k,if $d_k=0$,stop;

Step 2: If

, (11)

set $\alpha_k=1$. Otherwise go to step 3;

Step 3: Let $\alpha_k \geq 0$ be bound above and satisfy.

$$f\left(w_k+\alpha_k d_k\right) \leq max\left[f\left(w_k\right), \sum_{r=0}^{m(k)-1} \lambda_{kr} f\left(w_{k-r}\right)\right]-\sigma\left(t_k\right) \tag{12}$$

Where $\sigma\left(t_k\right)$ is a forcing function and $t_k=\frac{-g_k^T d_k}{\left\|d_k\right\|}$, See [25].

Step4: Set $w_{k+1}=w_k+\alpha_k d_k$. Let $\bar{h}=\max_{0 \leq j \leq m(k)} h\left(w_{k-j}\right)$ if $h\left(w_{k+1}\right) \leq \beta \bar{h}$, that set $m(k+1)=$ $min\{m(k)+1, N\}$. Otherwise, let $k=k+1$, call restoration algorithm 2.2 to obtain w_k^r, let $w_k=w_k^r$. $m(k)=m(i)$ and go to step 1.

We describe the Restoration Algorithm as follows;

Algorithm 2.2

Step 0: $w_k^0=w_k, \Delta^0>0, i=0,0<\eta<1, m(i)=0$;

Step 1: If $h\left(x_{k+1}\right) \leq \beta \bar{h}$, then $x_k^r=x_k^i$, stop;

Step 2: compute

$$\begin{aligned} & max\ \Omega_k^i(d) \\ & s.t\ \ \|d\| \leq \Delta^i \end{aligned} \tag{13}$$

to get d_k^i. Calculate r_k^i;

Step 3: If $r_k^i \leq \eta$, then let $w_k^{i+1}=w_k^i, \Delta^{i+1}=\frac{\Delta^i}{2}, i=i+1, m(i)=min\{m(i-1)+1, N\}$ and go to step 2;

Step 4: If $r_k^i \geq \eta$, then $w_k^{i+1}=w_k^i+d_k^i$, $\Delta^{i+1}=2\Delta^i$, $i=i+1, m(i)=min\{m(i-1)+1, N\}$ and go to step1;

3. The convergence properties

In this section, we discuss the global convergence property, we make the following assumptions;

Assumptions:

(1)The iterate $\{w_k\}$ remains in compacted subset $K \subset R^{2n}$;

(2) There exist two constants $0<a \leq b$ such that $a\|d\|^2 \leq d^T G d \leq b\|d\|^2$, for all iteration k and $d \in R^{2n}$;

(3) The solution of problem (11) satisfies

$\Omega_k^i(d)=h\left(w_k^i\right)-\left\|\left(C\left(w_k^i\right)+A\left(w_k^i\right) d\right)^{+}\right\| \geq v \Delta^i min\left\{h\left(w_k^i\right), \Delta^i\right\}$, where $v>0$ is constant.

Assumptions (1) are the standard assumptions. (2) plays an important role in obtaining the convergence results. (3) is the sufficient reduction which guarantees the global convergence in a trust region method.

Since the point we obtain by the filter method may not satisfy the constraints, the cluster point of the sequence generated by Algorithm 2.1 can be either of the two different types of stationary points. Clearly, a strong stationary point defined above is precisely a KKT point of problem (7). The following lemma describes the properties of infeasible stationary point, see [25].

Lemma 3. 1[25] If $x, y \in R^n$ is an infeasible stationary point, there exists $\rho_0 \geq 0$ and $\rho \in R^{2n}$ such that the following first-order necessary condition

$$\rho_0 g(w)+\sum_{i=1}^n \rho_i \nabla c_i(w)=0,$$

holds.

Lemma 3.2 If $\{w_k\}$ is generated by (12). Then we have

$$f(w_k) \le f(w_0) - \lambda \sum_{r=1}^{k-2} \sigma(t_r) - \sigma(t_{k-1}) \le f(w_0) - \lambda \sum_{r=1}^{k-1} \sigma(t_r).$$

Proof. The proof see [26].

Lemma 3.3 Let $\{w_k\}$ be an infinite sequence generated by Algorithm2.1, then $h(w_k) \to 0$ $(k \to \infty)$.

Proof. According to the definition of the $m(k)$, we have $m(k+1) \le m(k) + 1$.so

$$\begin{aligned} h\left(w_{l(k+1)}\right) &\le \max_{0\le j \le m(k+1)} \left[h\left(w_{k+1-j}\right)\right] \\ &\le \max_{0\le j\le m(k)+1} \left[h\left(w_{k+1-j}\right)\right] \\ &= \max\left\{ h\left(w_{l(k)}\right), h\left(w_{k+1}\right)\right\} \\ &= h\left(w_{l(k)}\right) \end{aligned}$$

This implies that $h\left(w_{l(k)}\right)$ converges. Then by $h(w_{k+1}) \le \beta \max_{0\le j\le m(k)} \left[h\left(w_{k-j}\right)\right]$.

We get $h\left(w_{l(k)}\right) \le \beta h\left(w_{l(k)-1}\right)$.

Since $\beta \in (0,1)$, we deduce that $h(w_k) \to 0$ $(k \to \infty)$.

Therefore $h(w_{k+1}) \le \beta h\left(w_{l(k)}\right) \to 0$, hold by Algorithm 2.1. So we have $\lim_{k\to\infty} h(w_k) = 0$.

Lemma 3.4 If Algorithm 2.1 terminates at $\{w_k\}$, then $\{w_k\}$ is the either an infeasible stationary or a stationary point.

Proof. (i) If the algorithm terminates at step 1. Then we have $d_k = 0$. Obviously, we compute $h(w_k)$. If $h(w_k) = 0$, then we get the solution of the subproblem (7). Which indicate $d_k = 0$ is a feasible point. Otherwise $h(w_k) \ne 0$, which means $d_k = 0$ is an infeasible point.

(ii) If the algorithm terminates at step 2. Then $d_k = 0$ is the solution of subproblem (7). In this case, $d_k = 0$ satisfies the following K-T condition.

$$g_k + G_k d + A_k \rho = 0 \tag{14}$$

$$c_i\left(w_k\right) + \nabla c_i\left(w_k\right)^T d = 0, i \in 1, 2, \cdots, n \tag{15}$$

Form lemma 3.3, we have

$$h\left(w_k\right) \to 0, k \to \infty. \tag{16}$$

Namely w_k is a stationary point.

Lemma 3.5 In step 3, the line search procedure is well defined.

Proof. By the algorithm 2.1. We have $g_k^T d_k \le -\frac{1}{2} d_k^T G_k d_k$.

By the definition of $f\left(w_{l(k)}\right)$, we have

$$f\left(w_{l(k)}\right) = \max_{0\le j\le m(k)} \left[f\left(w_{k-j}\right)\right].$$

And $\lim_{k\to\infty}\sigma\left(t_k\right)=0$ implies $\lim_{k\to\infty} t_k=0$. From lemma 2, we get

$$\lambda\sum_{r=0}^{k}\sigma\left(t_r\right)<\infty$$

Under assumption (1), we know $f\left(w_k\right)$ is bounded below.

$$\lambda\sum_{r=0}^{k}\sigma\left(t_r\right)\le f\left(w_0\right)-f\left(w_{k+1}\right)$$

Let $k\to\infty$, we have $\lambda\sum_{r=0}^{k}\sigma\left(t_r\right)<\infty$. Hence we have $\lim_{k\to\infty}\sigma\left(t_k\right)=0$.

Case 1: assume that $f\left(w_k+\alpha d_k\right)-f\left(w_k\right)<-\sigma\left(t_k\right)$,

Now we prove that there exist a $\alpha\in\left(0,1\right)$, such that

$$f\left(w_k+\alpha d_k\right)-f\left(w_k\right)<-\sigma\left(t_k\right)$$

Assume by contradiction that for $\alpha\in\left(0,1\right)$ it holds.

$$f\left(w_k+\alpha d_k\right)-f\left(w_k\right)>-\sigma\left(t_k\right)=-\sigma\left(\frac{-g_k^T d_k}{\left\|d_k\right\|}\right)=\frac{g_k^T d_k}{\left\|d_k\right\|}. \tag{17}$$

Divided by α on the both sides of (16),

$$\frac{f\left(w_k+\alpha d_k\right)-f\left(w_k\right)}{\alpha}>\frac{g_k^T d_k}{\alpha\left\|d_k\right\|}\quad \alpha\to 0,\ g_k^T d_k>\frac{g_k^T d_k}{\alpha\left\|d_k\right\|}$$

Which implies that $g_k^T d_k>0$. This is contradicts (11).

Case2: when $max\left[f\left(w_k\right),\sum_{r=0}^{m(k)-1}\lambda_{kr}f\left(w_{k-r}\right)\right]=\sum_{r=0}^{m(k)-1}\lambda_{kr}f\left(w_{k-r}\right)$.

This means $\sum_{r=0}^{m(k)-1}\lambda_{kr}f\left(w_{k-r}\right)\ge f\left(w_k\right)$

On the same way we assume that $f\left(w_k+\alpha d_k\right)-\sum_{r=0}^{m(k)-1}\lambda_{kr}f\left(w_{k-r}\right)<-\sigma\left(t_k\right)$.

So use scaling law, we have $f\left(w_k+\alpha d_k\right)-f\left(w_k\right)<-\sigma\left(t_k\right)$, which is the same with the case 1. This completes the proof.

Lemma 3.6 Under assumption (3), the Restoration Algorithm terminates finitely.

Proof. The proof see [25].

Theorem 3.1 Suppose $\left\{w_k\right\}$ is an infinite sequence generated by Algorithm 2.1, d_k is the solution of $QP\left(w_k,G_k\right)$. If the multiplier according to the subproblem (7) is uniform bounded, then $\lim_{k\to\infty}\left\|d_k\right\|=0$.

Proof: By assumption (1), there exists point w^* such that $w_k\to w^*$ *for* $k\in O$, where O is a infinite index set, by Algorithm 2.1 and lemma 3. We talk about the following two possible cases.

Case 1: $O_0=\left\{k\in O\left|g_k{}^T d_k\le -\frac{1}{2}d_k{}^T G_k d_k\right.\right\}$ is infinite index set. In this case, we have

$$f\left(w_k+\alpha_k d_k\right)\le max\left[f\left(w_k\right),\sum_{r=0}^{m(k)-1}\lambda_{kr}f\left(w_{k-r}\right)\right]-\sigma\left(t_k\right).$$

No matter $max\left[f\left(w_k\right),\sum_{r=0}^{m(k)-1}\lambda_{kr}f\left(w_{k-r}\right)\right]=f\left(w_k\right) or \sum_{r=0}^{m(k)-1}\lambda_{kr}f\left(w_{k-r}\right)$.

Since f is bounded below. Since $\lambda\sum_{r=0}^{k}\sigma(t_r)\le f\left(w_0\right)-f\left(w_{k+1}\right)$.

We can get

$$\lim_{k\to\infty}\sigma\left(t_k\right)=0 \qquad \lim_{k\to\infty}t_k=0$$

So
$$\lim_{k\to\infty}-\frac{g_k^T d_k}{\|d_k\|}=0.$$

Then we have
$$\lim_{k\to\infty}g_k^T d_k=0.$$

Since
$$g_k{}^T d_k\le -\frac{1}{2}d_k{}^T G_k d_k\le 0.$$

According to the squeeze theorem, we obtain

$$\lim_{k\to 0}d_k{}^T G_k d_k=0$$

Since G_k is a positive matrix. So we get $\lim_{k\to\infty}\left\|d_k\right\|=0$. (18)

Case 2: O_0 is a finite index set, which implies $O_1=\left\{k\in O\left|g_k{}^T d_k>-\frac{1}{2}d_k{}^T G_k d_k\right.\right\}$ is an infinite index set. If (18) does not hold, then there exist a positive number c and an infinite index set O_2, such that $\|d_k\|>c \ for\ k\in O_2\subset O_1$. Since d_k is the solution of $QP\left(w_k,G_k\right)$, we have

$$g_k+G_k d+A_k\rho_k=0,\quad \rho_k^T\left(C_k+A_k d\right)=0$$

Where $\rho_k\in R^{2n}$ are the multipliers. Then we can assume that exist $\overline{N}>0$ such that $\|\rho_k\|\le\overline{N}$. By lemma 3, we know $h\left(w_k\right)\to 0$, hence there exist $k_0>0$, such that for $\forall k>k_0, k\in O_2$, it holds

$$h\left(w_k\right)\le\frac{ac^2}{2\overline{N}}\le\frac{a\left\|d_k\right\|^2}{2\overline{N}}\le\frac{d_k^T G_k d_k}{2\overline{N}}$$

Consequently, we deduce

$$\begin{aligned} g_k^T d_k &= -d_k^T G_k d_k - d_k^T A_k\rho_k \\ &= -d_k^T G_k d_k+\rho_k^T C_k \\ &\le \overline{N}h\left(w_k\right)-d_k^T G_k d_k \\ &\le -\frac{1}{2}d_k^T G_k d_k \end{aligned}$$

This contradicts the definition of O_1. The proof is complete.

Theorem 3.2. Suppose $\{w_k\}$ is an infinite sequence generated by Algorithm 2.1 and the assumption in Theorem 1 hold. Then cluster point of $\{w_k\}$ is a stationary point (KKT point) of (7).

Proof. Since $\{w_k\}$ lies in a compacted set, there exist $w* \in R^{2n}$, such that $w_k \to w^*, k \in O$. By the lemma 3.3, we have $h(w_k) \to 0,\ k \in O$, which means that w^* is a feasible point. Form Theo rem1, we have $\|d_k\| \to 0$. By the lemma 3 and Theorem3.1, we get $d^* = 0$ is the solution of subproblem $QP(w^*, G^*)$. Then by the KKT condition, we obtain

$$g^* + \rho^* A^* = 0$$
$$c_i^* = 0, i \in 1, 2, \cdots, n$$

Therefore w^* is a KKT point of problem (7), so we obtain the solution of (1) form lemma 2.3.

4. Conclusions

In this paper, we propose a new filter method with the nonmonotonne line search technique for LCP, and the global convergence can be established under the weaker assumption than those of existed nonmonotone line serach.

5. References

[1] Cottle, R.W, Pang, J.-S, Stone, R.W, The Linear Complementarity Problem, Academic, NewYork, NY,1992

[2] Ferris, M.C, Pang, J.-S, Engineering and economic application of complementarity problems, *SIAM J. Review,* 39 (1997), pp, 669-713.

[3] J.S.Pang, Global method of foe monotone variational problems with inequality, Optimization and application. 95(1997), pp, 419-430.

[4] De Luca, T, Fancchinei, F, Kanzow, C, A semismooth equation approach to the solution of nonlinear complementarity problems. *Math. Prog.* 75 (1996), pp, 407-439.

[5] Facchinei, F, Kanzow, C, A nonsmooth inexact Newton method for the solution of large-scale nonlinear complementarity problems. *Math. Prog.*76 (1997), pp, 493-512.

[6] Fischer, A, Solution of monotone complementarity problems with locally lipschitzian functions. *Math. Prog.*76 (1997), pp, 513-532.

[7] Harker, P.T, Pang, J.S, Finite-dimensional variational inequality and nonlinear complementarityproblem: A survey of theory, algorithms and applications. *Math. Prog.* 48 (1990), pp, 161-220.

[8] Pang, J.S, Gabriel, S.A, NE/SQP: A robust algorithm for the nonlinear complementarity problem.*Math. Prog.* 60 (1993), pp, 295-337.

[9] Yoshise, A, Gomplementarity problems. In: Terlaky, T, eds. Interior point methods of mathematical programming, Kluwer Academic Publishers, 1996.

[10] Chen, B, Chen, X, A globally and locally superlinear comtinuation-smoothing method for *Po+Ro* and monotone NCP, *SIAM J. Optim.*, 9 (1999), pp, 624-645.

[11] Chen, B, Harker, P.T, A non-interior-point comtinuation method for linear complementarity problems, *SIAM J. Matrix Anal. Appl.* 14 (1993), pp, 1168-1190.

[12] Chen, B, Xiu, N, A superlinear noninterior one-step continuation method for monotone LCP in the absence of strict complementarity. Journal of Optim,ization Theory and Application, 108(2001), pp, 317-332.

[13] Qi, L., Sun, D., Zhou, G., A new look at smoothing Newton methods for nonlinear complementarity problems and box constrained axiational inequalities. *Math. Prog.* 87 (2000), pp, 1-35.

[14] Chen, X, Smoothing methods for complementarity problems and their applications: A survey. Journal of the Operation Reseach Society of Japan, 43 (2000), pp, 32-47.

[15] R. Fletcher, S. Leyffer, Nonlinear programming without a penalty function. Mathematical Programming. 91(2002), pp, 239-269.

[16] C. Audet, J. E. Dennis, A pattern search filter method for nonlinear programming without derivatives. SIAM Journal on Optimization. 14(2004), pp, 980-1010.

[17] C. M. Chin, R. Fletcher, On the global convergence of an SLP-filter algorithm that takes EQP steps, Mathematical Programming. 23(2003), pp, 161-177.

[18] M. Ulbrich, S. Ulbrich, L. N. Vicente, A globally convergent primal-dual interior-point filter method for nonlinear programming, Mathematical Programming. 100(2004), pp, 379-410.

[19] E.Karas, A. Ribeiro, C. Sagastizabal, A bundle filter method for nonsmooth convex constrained optimization, Mathematical Programming. 116(2009), pp, 297-320.

[20] N. I. M. Gould, S. Leyffer,P. L. Toint, A multidimensional filter algorithm for nonlinear equations and nonlinear leastsquares, SIAM Journal on Optimization. 15(2004), pp, 17-38.

[21] N.I.M. Gould, C. Sainvitu, P. L. Toint, A filter-trust region method for unconstrained optimization, SIAM Journal on Optimization. 16(2006), pp, 341-357.

[22] L. Grippo, F. Lampariello, S. Lucidi, A nonmonotone line search technique for Newton's method, SIAM Journal on Numerical Analysis.23(1986), pp, 707–716.

[23] J.L. Zhang, J. Chen, A constrained optimization approach for LCP, Journal of Computational Mathematics. 22(2004), pp, 509-522.

[24] Fischer, A., Solution of monotone complementarity problems with locally lipschitzian functions. *Math. Prog.* 76 (1997), pp, 513-532.

[25] K.Su,Z.S.Yu, A modified SQP method with nonmonotone technique and its global convergence, Computers and Mathematics with Applications. 57(2009), pp, 240-247.

[26] Z.S. Yu, D.G. Pu, A new nonmonotone line search technique for unconstrained optimization, Journal of Computational and Applied Mathematics. 219(2008), pp, 134-144.