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Chebyshev Semi-iterative Method to Solve Fully Fuzzy linear Systems

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Abstract. In this paper, semi-iterative method is applied to find solution of the fully fuzzy linear systems. The convergence of this method is discussed in details. Furthermore, we show that in some situations that the existing methods such as Jacobi, Gauss-Seidel, JOR, SOR and are divergent, our proposed method is applicable. Finally, numerical computations are presented based on a particular linear system, which clearly show the reliability and efficiency of our algorithms

Keywords: iterative methods, semi-iterative methods, Chebyshev, fuzzy numbers, fuzzy arithmetic, fuzzy linear equations.

1. Introduction

Let us consider the following linear systems

$$Ax=b, \tag{1}$$

where $A\in R^{n\times n}, b, x\in R^n$. These method often occur in a wide variety of area including numerical differential equation, eigenvalue problems, economics models, design and computer analysis of circuits, power system networks, chemical engineering processes, physical and biological sciences ; see [1-12] and the references therein.

However, when the estimation of the system coefficients is imprecise and only some vague knowledge about the actual values of the parameters is available, it may be convenient to represent some or all of them with fuzzy numbers [13]. Fuzzy data is being used as a natural way to describe uncertain data. Fuzzy concept was introduced by Zadeh [13- 14]. We refer the reader to [15] for more information on fuzzy numbers and fuzzy arithmetic. Fuzzy systems are used to study a variety of problems including fuzzy metric spaces [16], fuzzy differential equations [17], particle physics [18- 19] , Game theory [20], optimization [21] and fuzzy linear systems[22-25].

Fuzzy number arithmetic is widely applied and useful in computation of linear system whose parameters are all or partially represented by fuzzy numbers. Dubois and Prade [26-27] investigated two definitions of a system of fuzzy linear equations, consisting of system of tolerance constraints and system of approximate equalities. The simplest method for finding a solution for this system is creating scenarios for the fuzzy system, which is a realization of fuzzy systems. Based on these actual scenarios, Buckley and Qu [28] extended several methods for this category and proved their approaches are not practicable, because infinite number of scenarios can be driven for a fully fuzzy linear system (FFLS). Friedman *et al*. [22] introduced a general model for solving a fuzzy n × n linear system whose coefficient matrix is crisp and the right-hand side column is an arbitrary fuzzy number vector. They used the parametric form of fuzzy numbers and replaced the original fuzzy n × n linear system by a crisp 2n × 2n linear system and studied duality in fuzzy linear systems $AX = BX+Y$ where A , B are real $n\times n$ matrix, the unknown vector X is vector consisting of n fuzzy numbers and the constant Y is vector consisting of n fuzzy numbers, in [29]. There are many other numerical methods for solving fuzzy linear systems such as Jacobi, Gauss-Seidel, Adomiam decomposition method and SOR iterative method [30-35]. In addition, another important kind of fuzzy linear systems are the fully fuzzy linear systems (FFLS) in which all the parameters are fuzzy numbers. Dehghan and Hashemi [36-37] proposed the Adomian decomposition method, and other iterative methods to find the positive fuzzy vector solution of $n \times n$ fully fuzzy linear system. Dehghan *et al.* [38] proposed some computational methods such as Cramer's rule, Gauss elimination method, *LU* decomposition method and linear programming approach for finding the approximated solution of FFLS. Nasseri *et al.* [39] used a certain decomposition methods of the coefficient matrix for solving fully fuzzy linear system of equations. Kumar *et al.* in [40] obtained exact solution of fully fuzzy linear system by solving a linear programming. In this paper, we propose Semi-iterative method for solving fully fuzzy linear systems.

This paper is organized as follows:

In Section 2 some basic definitions and arithmetic are reviewed. In Section 3 a new method is proposed for solving FFLS and we respectively give the semi-iterative method and some convenient iterative methods. In section 4 numerical results are considered to show the efficiency of the proposed method. Section 5 ends this paper with a conclusion.

2. Some Basic Definition and Arithmetic Operations

In this section, an appropriate brief introduction to preliminary topics such as fuzzy numbers and fuzzy calculus will be introduced and the definition for FFLS will be provided. For details, we refer to [26, 37].

Definition 2.1 Let X denote a universal set. Then a fuzzy subset $\widetilde{A}$ of X is defined by its membership function $\mu_{\widetilde{A}} : X \to [0,1]$; which assigns a real number $\mu_{\widetilde{A}}(x)$ in the interval [0,1], to each element $x \in X$, where the value of $\mu_{\widetilde{A}}(x)$ at x shows the grade of membership of x in $\widetilde{A}$.

A fuzzy subset $\widetilde{A}$ can be characterized as a set of ordered pairs of element x and grade $\mu_{\widetilde{A}}(x)$ and is often written $\widetilde{A} = \{(x, \mu_{\widetilde{A}}(x)); x \in X\}$. The class of fuzzy sets on X is denoted with $\Gamma(X)$.

Definition 2.2 A fuzzy set with the following membership function is named a triangular fuzzy number and in this paper we will use these fuzzy numbers.

$$\mu_{\widetilde{A}}(x) = \begin{cases} 1 - \dfrac{m-x}{\alpha}, & m-\alpha \le x \le m, \alpha > 0, \\ 1 - \dfrac{x-m}{\beta}, & m \le x \le m+\beta, \beta > 0, \\ 0, & \textit{else.} \end{cases}$$

Definition 2.3 A fuzzy number $\tilde{A}$ is said to be positive (negative) by $\tilde{A} > 0 (\tilde{A} < 0)$ if its membership function $\mu_{\tilde{A}}(x)$ satisfies $\mu_{\tilde{A}}(x) = 0, \forall x \le 0 (\forall x \ge 0)$.

Using its mean value and left and right spreads, and shape functions, such a fuzzy number $\tilde{A}$ is symbolically written $\tilde{A} = (m, \alpha, \beta)$. Obviously, $\tilde{A}$ is positive, if and only if $m - \alpha \ge 0$.

Definition 2.4 Two fuzzy numbers $\tilde{A} = (m, \alpha, \beta)$ and $\tilde{B} = (n, \gamma, \delta)$ are said to be equal, if and only if $m = n$, $\alpha = \gamma$ and $\beta = \delta$.

Definition 2.5 Let $\tilde{A} = (m, \alpha, \beta)$, $\tilde{B} = (n, \gamma, \delta)$ be two triangular fuzzy numbers then;

(i) $\tilde{A} \oplus \tilde{B} = (m, \alpha, \beta) \oplus (n, \gamma, \delta) = (m+n, \alpha+\gamma, \beta+\delta)$,

(ii) **)** $-\tilde{A} = -(m, \alpha, \beta) = (-m, \beta, \alpha)$,

(iii)if $\tilde{A}, \tilde{B}$ be a positive fuzzy number then: $(m, \alpha, \beta) \otimes (n, \gamma, \delta) \cong (mn, n\alpha + m\gamma, n\beta + m\delta)$,

(iv) For scalar multiplication we have;

$$\lambda \otimes (m,\alpha,\beta) = \begin{cases} (\lambda m, \lambda\alpha, \lambda\beta), & \lambda \geq 0, \\ (\lambda m, -\lambda\beta, -\lambda\alpha), & \lambda < 0. \end{cases}$$

Definition 2.6 A matrix $\tilde{A} = (\tilde{a}_{ij})$ is called a fuzzy matrix, if each element of $\tilde{A}$ is a fuzzy number. A fuzzy matrix $\tilde{A}$ will be positive and denoted by $\tilde{A} > 0$, if each element of $\tilde{A}$ be positive. We may represent n × n fuzzy matrix $\tilde{A} = (\tilde{a}_{ij})_{n\times n}$, such that $\tilde{a}_{ij} = (a_{ij}, \alpha_{ij}, \beta_{ij})$, with the new notation $\tilde{A} = (A, M, N)$, where $A = (a_{ij})$, $M = (\alpha_{ij})$ and $N = (\beta_{ij})$ are three n × n crisp matrices.

3. Chebyshev Semi-iterative Method for FFLS

Consider Fully fuzzy linear system (FFLS) $\tilde{A} \otimes \tilde{x} = \tilde{b}$. In this paper we are going to obtain a positive solution of FFLS, where, $\tilde{A} = (A, M, N) > \tilde{0}, \tilde{b} = (b, g, h) > \tilde{0}$ and $\tilde{x} = (x, y, z) > \tilde{0}$.

So we have;

$$(A, M, N) \otimes (x, y, z) = (b, g, h). \tag{2}$$

Then by Definition 2.5 we have;

$$(Ax, Ay + Mx, Az + Nx) = (b, g, h). \tag{3}$$

And by Definition 2.4, concludes that;

$$\begin{cases} Ax = b, \\ Ay + Mx = g, \\ Az + Nx = h. \end{cases} \tag{4}$$

Then,

$$\underbrace{\begin{pmatrix} A & 0 & 0 \\ M & A & 0 \\ N & 0 & A \end{pmatrix}}_{\Lambda} \underbrace{\begin{pmatrix} x \\ y \\ z \end{pmatrix}}_{X} = \underbrace{\begin{pmatrix} b \\ g \\ h \end{pmatrix}}_{\Xi}.$$

So, by assuming that A be a nonsingular matrix we have;

$$\begin{cases} x = A^{-1}b, \\ y = A^{-1}(g - Mx), \\ z = A^{-1}(h - Nx). \end{cases} \tag{5}$$

Dehghan *et al.* [36] applied some iterative techniques such as Richardson, Jacobi, Jacobi overrelaxation (JOR), Gauss–Seidel, successive overrelaxation (SOR), accelerated overrelaxation (AOR), symmetric and unsymmetric SOR (SSOR and USSOR) and extrapolated modify ed Aitken (EMA) for solving FFLS. First, we review their work.

Consider Eq. (4) and let $A=Q- P$ be a proper splitting of crisp matrix A and Q, called the splitting matrix, be a nonsingular crisp matrix. Thus, the iterative method for FFLS is as follows;

$$\begin{pmatrix} x^{(k+1)} \\ y^{(k+1)} \\ z^{(k+1)} \end{pmatrix} = T \begin{pmatrix} x^{(k)} \\ y^{(k)} \\ z^{(k)} \end{pmatrix} + \xi, (k \geq 0). \tag{6}$$

T is called the iteration matrix and ξ is a vector and;

$$T = \begin{pmatrix} Q^{-1}P & 0 & 0 \\ -Q^{-1}M & Q^{-1}P & 0 \\ -Q^{-1}N & 0 & Q^{-1}P \end{pmatrix}, \xi = \begin{pmatrix} Q^{-1}b \\ Q^{-1}g \\ Q^{-1}h \end{pmatrix}. \tag{7}$$

Therefore by choose special parameters in Q we can obtain the popular iterative method. For example, if $A=D-L-U$, where D is diagonal , L is lower triangular and U is upper triangular part of A, then we have;

1) Jacobi method for $Q=D$.
2) JOR(Jacobi Overrelaxation) method for $Q = \frac{1}{w} D, (w \in R)$.
3) Gauss-Seidel method for $Q=D-L$.
4) SOR method for $Q = (\frac{1}{w} D - L), (w \in R)$.

For details, we refer to [36].

Next, we apply another acceleration method called Chebyshev semi-iterative method for FFLS. Based on above demonstration, semi-iterative method is as follows;

$$\begin{pmatrix} x^{(k+1)} \\ y^{(k+1)} \\ z^{(k+1)} \end{pmatrix} = \omega_{k+1} (\begin{pmatrix} Q^{-1}P & 0 & 0 \\ -Q^{-1}M & Q^{-1}P & 0 \\ -Q^{-1}N & 0 & Q^{-1}P \end{pmatrix} \begin{pmatrix} x^{(k)} \\ y^{(k)} \\ z^{(k)} \end{pmatrix} + \xi - \begin{pmatrix} x^{(k-1)} \\ y^{(k-1)} \\ z^{(k-1)} \end{pmatrix}) + \begin{pmatrix} x^{(k-1)} \\ y^{(k-1)} \\ z^{(k-1)} \end{pmatrix}, \tag{8}$$

where,

$$\omega_{k+1} = \frac{2C_k(\frac{1}{\rho(T)})}{\rho(T) C_{K+1}(\frac{1}{\rho(T)})},$$

$$\begin{pmatrix} x^{(k)} \\ y^{(k)} \\ z^{(k)} \end{pmatrix} \in R^{3n}, \begin{pmatrix} x^{(1)} \\ y^{(1)} \\ z^{(1)} \end{pmatrix} = T \begin{pmatrix} x^{(0)} \\ y^{(0)} \\ z^{(0)} \end{pmatrix} + \xi,$$

and,

$$C_k(x) = 2xC_{k-1}(x) - C_{k-2}(x), C_0(x) = 1, C_1(x) = x,$$

are the Chebyshev polynomials of the first kind and also $\rho(T)$ is called spectral radius of T; see [1- 2, 41] . For example, by Eq. (8), Chebyshev-SOR semi-iterative method is as follows;

$$\begin{cases} x^{(k+1)} = \omega_{k+1}(\kappa x^{(k)} + w(D - wL)^{-1} b) + (1 - \omega_{k+1}) x^{(k-1)}, \\ y^{(k+1)} = \omega_{k+1}(\kappa y^{(k)} - w(D - wL)^{-1} M x^{(k)} + w(D - wL)^{-1} g) + (1 - \omega_{k+1}) y^{(k-1)}, \\ z^{(k+1)} = \omega_{k+1}(\kappa y^{(k)} - w(D - wL)^{-1} N x^{(k)} + w(D - wL)^{-1} h) + (1 - \omega_{k+1}) y^{(k-1)}. \end{cases} \tag{9}$$

Where,

$$\kappa = (D - wL)^{-1}[(1-w)D + wU].$$

However, since the spectral radius of T is not known in advance, $\rho(T)$ is usually replaced by the lower and upper bounds (see [41]), that is;

$$\begin{pmatrix} x^{(k+1)} \\ y^{(k+1)} \\ z^{(k+1)} \end{pmatrix} = \omega_{k+1}(\gamma \begin{pmatrix} m^{(k)} \\ n^{(k)} \\ o^{(k)} \end{pmatrix} + \begin{pmatrix} x^{(k)} \\ y^{(k)} \\ z^{(k)} \end{pmatrix} - \begin{pmatrix} x^{(k-1)} \\ y^{(k-1)} \\ z^{(k-1)} \end{pmatrix}) + \begin{pmatrix} x^{(k-1)} \\ y^{(k-1)} \\ z^{(k-1)} \end{pmatrix}, \tag{10}$$

where,

$$\omega_{i+1} = 2.\upsilon.\frac{C_i(\upsilon)}{C_{i+1}(\upsilon)},$$

$$\begin{pmatrix} m^{(k)} \\ n^{(k)} \\ o^{(k)} \end{pmatrix} = \begin{pmatrix} Q^{-1}b \\ Q^{-1}g \\ Q^{-1}h \end{pmatrix} - \Lambda \begin{pmatrix} x^{(k)} \\ y^{(k)} \\ z^{(k)} \end{pmatrix}, \gamma = \frac{2}{2-(\beta+\alpha)}, \upsilon = \frac{2-(\beta+\alpha)}{(\beta-\alpha)},$$

$$-1 \le \alpha \le \lambda \le \beta \le 1, \quad \beta > \alpha, \quad \lambda \in eigenvalues(T).$$

Furthermore, after some calculation, from Eq. (10), we have; see [2, 42]:

$$\begin{pmatrix} x^{(k+1)} \\ y^{(k+1)} \\ z^{(k+1)} \end{pmatrix} = \frac{\rho_{k+1}}{2-(\alpha+\beta)}\{[2T-(\alpha+\beta)I]\begin{pmatrix} x^{(k)} \\ y^{(k)} \\ z^{(k)} \end{pmatrix} + 2\begin{pmatrix} Q^{-1}b \\ Q^{-1}g \\ Q^{-1}h \end{pmatrix}\} + (1-\rho k_{i+1})\begin{pmatrix} x^{(k-1)} \\ y^{(k-1)} \\ z^{(k-1)} \end{pmatrix}, \tag{11}$$

where,

$$\rho_1 = 1, \quad \rho_2 = 2\upsilon^2/(2\upsilon^2-1), \quad for \quad n \ge 2; \rho_{k+1} = (1-\rho_k/4\upsilon^2)^{-1}.$$

Theorem 3.1. Chebyshev semi-iterative method (8) for solving fully fuzzy linear system $\tilde{A} \otimes \tilde{x} = \tilde{b}$, converges if and only if its classical version converges for solving the crisp linear system $Ax = b$ derived from the corresponding FFLS.

Proof. By above demonstrations and based on Eq. (7) it is easy to see that spectrum of T is equal to spectrum of $Q^{-1}P$. Therefore, the proof is complete. □

Theorem 3.2. Let $P_1(T) = [2T - (\alpha+\beta)I]/[2-(\alpha+\beta)]$. Then Chebyshev semi-iterative method converges, if $\rho(P_1(T)) < 1$.

Proof. Using Theorem 3.1 of this paper and Theorem 4.11[42], the result is trivial. □

4. Numerical Experiments

In this section, we give some numerical experiments to illustrate the results obtained in previous sections. All the numerical experiments presented in this section were computed in double precision using a MATLAB 7 on a PC with a 1.86GHz 32-bit processor and 1GB memory.

Example 4.1. Consider the Consider the following FFLS:

$$\tilde{A}=(A,M,N);\begin{cases} A=tridiag(1,4,1), \\ M=tridiag(0.1,1,0.1), \\ N=tridiag(0.2,1,0.1). \end{cases}$$

And ,

$$\tilde{b}=(b,g,h);b_i=i,g_i=\frac{i}{n},h_i=\frac{i}{n+1}.$$

The following table shows the numerical results of above example with the tolerance $\varepsilon=10^{-6}$ and the initial approximation zero vector. In the Table 1, we reported the number of iterations (**Iter**) and Elapsed time (**ELP**) for the SOR iterative method and Chebyshev-SOR semi-iterative method with different n and *w=1.1*.

Table1 Shows the results of example 4.1 for SOR and Chebyshev-SOR methods.

Method	**SOR method**		**Chebyshev-SOR method**	
n	**Iter**	**ELP**	**Iter**	**ELP**
50	18	0.049826	16	0.008087
100	19	0.251584	17	0.145590
200	20	0.251584	18	0.298130
300	20	0.292463	19	0.374779

Example 4.2. Consider the following FFLS:

$$\begin{bmatrix} (1,0.2,0.2) & (1,0.4,0.3) & (2,0.3,0.4) & (4,0.2,0.1) \\ (4,0.3,0.1) & (3,0.4,0.2) & (2,0.2,0.3) & (1,0.1,0.3) \\ (1,0.3,0.2) & (1,0.5,0.2) & (3,0.3,0.1) & (1,0.2,0.3) \\ (2,0.4,0.5) & (4,0.5,0.2) & (2,0.6,1.2) & (3,0.3,0.3) \end{bmatrix}\begin{pmatrix} (x_1,y_1,z_1) \\ (x_2,y_2,z_2) \\ (x_3,y_3,z_3) \\ (x_4,y_4,z_4) \end{pmatrix}=\begin{pmatrix} (6.9,5,4.1) \\ (5,3,4) \\ (4.9,4,3.2) \\ (7,5,6) \end{pmatrix}.$$

If we use iterative methods [36] for this problem we can see that all of the Jacobi, Gauss-Sidel and JOR, SOR methods are divergent (since $\rho(Q^{-1}P)>1$).

However, by using the initial values $x=y=z=s=(0,0,0,0)^t$ and stopping criterion $tol\le 10^{-6}$, the Chebyshev-Gauss-Seidel semi-iterative method($Q=D-L$) converges in **29** iterations to the following solution;

$$\begin{pmatrix} (x_1,y_1,z_1) \\ (x_2,y_2,z_2) \\ (x_3,y_3,z_3) \\ (x_4,y_4,z_4) \end{pmatrix}=\begin{pmatrix} (0.1961,0.0072,0.1448) \\ (0.3143,0.0393,0.3681) \\ (1.1169,0.8533,0.5744) \\ (1.0390,0.6348,0.4386) \end{pmatrix}.$$

5. Conclusion

In this paper, the fully fuzzy linear systems, *i.e*., fuzzy linear systems with fuzzy coefficients involving fuzzy variables are investigated and semi-iterative method is applied for solving these systems. The proposed method is easy to understand and apply in real life situations. Furthermore, we show that our algorithm compare with some other algorithms works better . Finally, from theoretical speaking and numerical examples, it may be concluded that this method is efficient and convenient.

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