

# Automatic Segmentation Approach Based Data Aggregation for the Classification of Brain Tissues

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**Abstract.** The paper presents a study and an evaluation of a novel unsupervised segmentation technique based aggregation approach and some of possibility theory concepts. Firstly, the MPFCM (Modified Possibilistic Fuzzy C-Means) algorithm is used to extract information from each of MR images modalities. In second step, an obtained data are combined with an operator in order to exploiting the uncertainty and ambiguity in the images. Finally, the segmented image is constructed using a decision rule. The efficiency of the proposed method is demonstrated by segmentation experiments using simulated MR images with different noise levels.

**Keywords:** aggregation, possibility theory, segmentation, MPFCM, MR images.

## 1. Introduction

Magnetic resonance (MR) imaging has been widely applied in biological research and diagnostics, primarily because of its excellent soft tissues contrast resolution, non-invasive character, high spatial resolution and easy slice selection at any orientation. In many applications, its segmentation serves a significant role on the following areas : (a) identifying anatomical areas of interest for diagnosis, treatment, or surgery planning paradigms; (b) preprocessing for multimodality image registration ; and (c) improved correlation of anatomical areas of interest with localized functional metrics [1]. Fully automatic brain tissue classification from magnetic resonance images (MRI) is of great importance for research and several clinical study of much neurological pathology. The accurate segmentation of MR images into different components, such as gray matter (GM), white matter (WM) and cerebrospinal fluid (CSF), is a vital role both in image analysis and computer vision.

In medical imaging domain, segmenting MR images has been found a difficult task due to the limited spatial resolution, noise and intensity in homogeneities variation, partial volume effects and a remarkable amount and largest set of data to be processed. To handle these difficulties, an enormous number of approaches have been reported in the literature, including fuzzy logic methods [3], neural networks [4], markov random field methods with the maximum expectation [5], statistical methods [5], and data fusion methods [6]. Here, the evaluation of a full automatic and robust approach for the segmentation of the human brain tissues using a multispectral aggregation technique is presented. This approach consists of the computation of fuzzy tissue maps generated by each of the three modalities of MR images T1, T2 and PD as an information source, the creation of fuzzy maps by a combination operator and a segmented image is computed in decision step.

The rest of this paper is organized as follows : In section 2, review of related research is briefly cited. Section 3 summarize the fuzzy clustering algorithm employed in the proposed method. In section 4, we describe the principals of possibility theory reasoning. The proposed process is detailed in section 5. Simulation results and discussions are introduced in Section 6. Finally, the conclusion is summed up in Section 7.

## 2. Review of Related Research

Copious number of works of fuzzy information fusion field is found in the literature. Let us review some

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of them. Waltz [11] presented three basic levels of image data fusion named as : pixel level, feature level and decision level, espacially to three processing architectures. Some concepts of Dempster-Shafer evidence theory have been outlined by I. Bloch [2], which can very useful for medical image fusion for classification, segmentation or recognition goals. Examples were given to indicate its ability to take into account a various of situations. Registration-based methods are considered as pixel-level fusion, such as MRI-PET (Positron Emission Tomography) data fusion[12]. Some techniques of knowledge-based segmentation can be stated as the feature-level fusion such as the methods proposed in [16].

One of belief functions, uncertainty theory and Dempster-Shafer theory are often used for decision-level fusion such as in [14]. I. Bloch [17] proposed an unified framework of information fusion in the medical field based on the fuzzy sets, allow to represent and process the numerical data as well as symbolic systems. V. Barra and J. Y. Boire [9] have discussed a general framework of the fusion of anatomical and functional medical images. The purpose of their research is to fuse functional and anatomical information obtained from medical imaging, the fusion process is realized in possibilistic logic frame, which allows for the management of uncertainty and imprecision inherent to the images. A new class of operators based on information theory and the whole process is finally illustrated in two clinical cases : the study of Alzheimer's disease by MR/SPECT fusion and the study of epilepsy with MR/PET/SPECT. The obtained results was very encouraging.

V. Barra and J. Y. Boire [15] proposed a new effective scheme of information fusion to segment intern cerebral structures. The information is provided by both expert knowledge and MR images, and consists of constitution, morphological and topological characteristics of tissues. The fusion of multimodality images is described in [13]. In [8], the authors have presented a framework of fuzzy information fusion to segment automatically tumor areas of human brain from multispectral magnetic resonance imaging (MRI); in this method three fuzzy models are used to represent tumor features for different MR image sequences and the fuzzy region growing is exploited to improve the fused result.

Maria del C. and al [10] proposed a new multispectral MRI data fusion technique for white matter lesion segmentation, in that an approach is detailed and compared with thresholding method in FLAIR images. In [19], Hongwei Zhu et al. have proposed an adaptive fuzzy evidential reasoning scheme for segmenting multi-modality MR brain images. Recently, The authors in [20] have presented a new framework of fuzzy information fusion using T2-weighted and proton density (PD) images in order to improve the quality of brain tissue segmentation.

### 3. The MPFCM Algorithm Clustering

Typically, clustering is a process of partitioning an unlabeled data set  $X=\{x_1, x_2, x_3, \dots, x_n\} \in \mathcal{F}^p$  into  $1 < c < n$  non-overlapped, consistent regions called classes with respect to some characteristics, by assigning labels to the vectors in  $X$ . A cluster contains similar patterns placed together. One of the most widely used clustering methods is the MPFCM (*Modified Possibilistic Fuzzy C-Means*) algorithm [21]. The MPFCM algorithm uses both the information of pixels and their neighborhoods, membership and typicality for classification. The MPFCM clustering algorithm minimizes the objective function :

$$J(U, T, V, X) = \sum_{i=1}^C \sum_{k=1}^N (au_{ik}^m + bt_{ik}^\eta) D_{ik} + \sum_{i=1}^C \gamma_i \sum_{k=1}^N (1 - t_{ik})^\eta + \beta \sum_{i=1}^C \sum_{k=1}^N (au_{ik}^m + bt_{ik}^\eta) S_{ik} \quad (1)$$

where  $m > 1$  is the weighting exponent,  $\lambda \in [3, 5]$  is the typicality exponent  $D_{jk}$  is the Euclidean distance between data  $x_j$  and cluster center  $v_i$ ,  $S_{ik} = \sum_{w=1}^{n_w} \|x_w - v_i\|$  where  $x_w$  is a neighbor pixel of  $x_k$  in a window around  $x_k$  and  $n_w$  is the number of neighbours in this window.,  $[U]_{C \times N}$  is the fuzzy matrix where  $\forall k, \sum_{i=1}^C u_{ik} \leq 1$ .  $[T]_{C \times N}$  is the typicality matrix where  $\forall k, t_{ik} \leq 1$ ,  $a > 0$ ,  $b > 0$  are user defined constants and the parameter  $\gamma_i$  is given by :

$$\gamma_i = \frac{\sum_{k=1}^N D_{ik}}{\sum_{k=1}^N u_{ik}^m}, K > 1$$

The minimization of objective function  $J(U, T, V, X)$  can be brought by an iterative process in which updating of membership degrees  $u_{ij}$ , typicality degrees  $t_{ij}$  and the cluster centers are done for each iteration by :

$$u_{ik} = \sum_{j=1}^c \left( \frac{D_{ik} + \beta S_{ik}}{D_{jk} + \beta S_{jk}} \right)^{1/(1-m)} \tag{2}$$

$$t_{ik} = \frac{1}{1 + \left( \frac{b}{\gamma_i} D_{ik} + \beta S_{ik} \right)^{1/(\eta-1)}} \tag{3}$$

$$v_i = \frac{\sum_{k=1}^N (au_{ik}^m + bt_{ik}^\eta)(x_k + \beta R_k)}{(1 + \alpha) \sum_{k=1}^N (au_{ik}^m + bt_{ik}^\eta)} \tag{4}$$

where :  $\alpha$  and  $\beta$  are a given values and :

$$R_k = \sum_{w=1}^{n_w} x_w \tag{5}$$

The algorithm of the MPFCM consists then of the reiterated application of (2), (3) and (4) until stability of the solutions.

### 4. The Possibility Theory

Possibilistic logic was introduced by Zadeh (1978) following its former works in fuzzy logic (Zadeh, 1965) so as to simultaneously represent imprecise and uncertain knowledge. In fuzzy set theory, a fuzzy measure is a representation of the uncertainty, giving for each subset  $Y$  of the universe of discourse  $X$  a coefficient in  $[0,1]$  assessing the degree of certitude for the realization of the event  $Y$ . In possibilistic logic, this fuzzy measure is modeled as a measure of possibility  $\Pi$  satisfying:

$$\begin{aligned} \Pi(X) &= 1 \quad \text{et} \quad \Pi(\phi) = 0 \\ (\forall Y_i) \Pi(\cup_i Y_i) &= \text{Sup}_i \Pi(Y_i) \end{aligned}$$

An event  $Y$  is completely possible if  $\Pi(Y) = 1$  and is impossible if  $\Pi(Y) = 0$ . Zadeh showed that  $\Pi$  could completely be defined from the assessment of the certitude on each singleton of  $X$ . Such a definition relies on the definition of a distribution of possibility  $\pi$  satisfying :

$$\begin{aligned} \pi : X &\rightarrow [0,1] \\ x &\rightarrow \pi(x) / \text{Sup}\{\pi(x) = 1\} \end{aligned}$$

Fuzzy sets  $F$  can then be represented by distributions of possibility, from the definition of their characteristic function  $\mu_F : (\forall x \in X) \mu_F(x) = \pi(x)$

Distributions of possibility can mathematically be related to probabilities, and they moreover offer the capability to declare the ignorance about an event. Considering such an event  $A$  (e.g., voxel  $v$  belongs to tissue  $T$ , (where  $v$  is at the interface between two tissues), the probabilities would assign  $P(A) = P(\bar{A}) = 0.5$ , whereas the possibility theory allows fully possible  $\Pi(A) = \Pi(\bar{A}) = 1$ . We chose to model all the information using distributions of possibility, and equivalently we represented this information using fuzzy sets [21]. The three-steps fusion can be therefore described as :

- Modeling of information in the same theoretical frame ;
- The extracted information is then aggregated with a fusion operator  $F$ . This operator must affirm redundancy and manage the complementarities and conflicts.

In the decision step, we pass from information provided by the sources to the choice of a decision.

## 5. Proposed Method

### 5.1. Modeling Step

Particularly, in our study this step consists in the creation of WM, GM, CSF and background (BG) fuzzy maps for both T1, T2 and PD images using the MPFCM algorithm.

### 5.2. Fusion Step

In this step, If  $\pi_T^{T1}(v), \pi_T^{T2}(v), \pi_T^{PD}(v)$  are the memberships of a voxel  $v$  to tissue  $T$  resulting from step 1 then a fusion operator  $F$  combine these values to generate a new membership value and can managing the existing ambiguity and redundancy. The possibility theory propose a wide range of operators for the combination of memberships [7]. I. Bloch [18] classified these operators in three classes defined as: Context independent and constant behavior operators (CICB), Context independent and variable behavior operators (CIVB) and Context dependent operators (CD). For our MR images fusion, we chose a context-based conjunctive operator because in the medical context, both images were supposed to be almost everywhere concordant, except near boundaries between tissues and in pathologic areas. In addition, the context-based behavior allowed to take into account these ambiguous but diagnosis-relevant areas. Then we retained an operator of this class, this one is introduced in [18]:

If  $\pi_T^{T1}(v), \pi_T^{T2}(v)$  and  $\pi_T^{PD}(v)$  are the gray-levels possibility distributions of tissue  $T$  extracted from  $T_{T1}, T_{T2}$  and  $T_{PD}$  fuzzy maps respectively and  $F$  design the fusion operator, then the fused possibility distribution is defined for any gray level  $v$  as :

$$\pi_T(v) = \max\left(\frac{\min(\pi_T^{I_i}(v), \pi_T^{I_j}(v))}{h}, 1 - h\right)$$

where  $I_i, I_j \in \{T1, T2, PD\}$ , and  $h$  is a measure of agreement between  $\pi_T^{I_i}$  and  $\pi_T^{I_j}$  :

$$h = 1 - \sum_{v \in \text{Image}} \left| \pi_T^{I_i}(v) - \pi_T^{I_j}(v) \right| / |\text{Image}|$$

### 5.3. Decision Step

A segmented image was finally obtained using the four maps computed in step 2 by assigning to the tissue  $T$  any voxel for which it had the greatest degree of membership.

The general algorithm using for fusion process is :

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General algorithm

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Modeling of the image

    For a in  $\{I_i, I_j\}$  do

        MPFCM (a)

    End For

Fusion

    Possibilistic fusion

Decision

    Segmented image

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Three models of fusion are generated by this algorithm : T1/T2 fusion, T1/PD fusion and T2/PD fusion.

## 6. Validations

Brainweb provides simulated brain datasets which contains a set of realistic MRIs created using an MRI simulator. In this section, T1-weighted, T2-weighted and PD-weighted brain MR images with a slice thickness of 1 mm, and a volume size of 217x181x181 with three noise levels (0%, 3%, 5%) are employed to investigate the proposed method. These images are obtained from the Brainweb Simulated Brain Database at the McConnell Brain Imaging Centre of the Montreal Neurological Institute(MNI), McGill University.

To compare the performance of these three models of fusion produced by  $F$  operator, we compute different coefficients reflecting how well two segmented volumes match. We use a different performance

measures :

$$Overlap (Ovrl) = \frac{TP}{TP + FN + FP}$$

$$Similarity (SI) = \frac{2 \times TP}{2 \times TP + FN + FP}$$

where TP and FP for true positive and false positive (voxels correctly and incorrectly classified as brain tissue). TN and FN for true negative and false negative, which were defined as the number of voxels correctly and incorrectly classified as non-brain tissue by the automated algorithm. The comparative results are depicted in table 1 below :

TABLE 1. Comparative Results.

	T1/T2 Fusion			T1/PD Fusion			T2/PD Fusion		
	CSF	WM	GM	CSF	WM	GM	CSF	WM	GM
Overl.	<b>0.88</b>	<b>0.92</b>	<b>0.87</b>	0.74	0.85	0.82	0.70	0.88	0.72
SI	<b>0.96</b>	<b>0.95</b>	<b>0.90</b>	0.88	0.90	0.86	0.81	0.90	0.83

The results in Table 1 show considerable improvement for all tissues using T1/T2 fusion model than T1/PD and T2/PD models. In addition, the results obtained from T1/T2 fusion are compared to the results obtained with a fuzzy segmentation computed by the algorithm of classification MPFCM on the T1 image alone, T2 image alone and the PD image alone. An example of segmentation result for the slice number 95 of Brainweb is portrayed in figure 1 below:

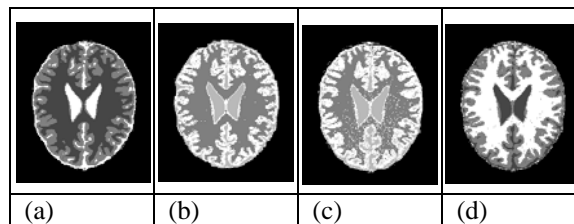


Fig. 1. (a) T1 segmented with MPFCM algorithm. (b) T2 segmented with MPFCM algorithm. (c) PD segmented with MPFCM algorithm. (d) Image of T1/T2 fusion with F operator.

The results for each one of the segmentation for all tissues CSF, WM and GM are reported in figures 2 and 3 below :

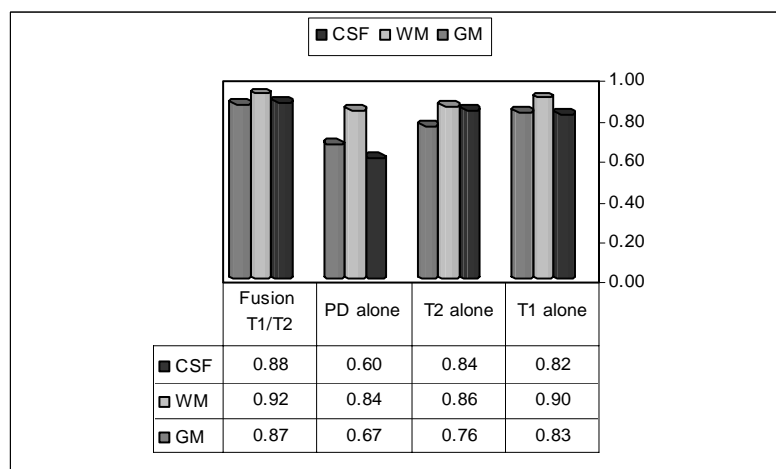


Fig. 2. Overlap measurement for different segmentations with 3% noise.

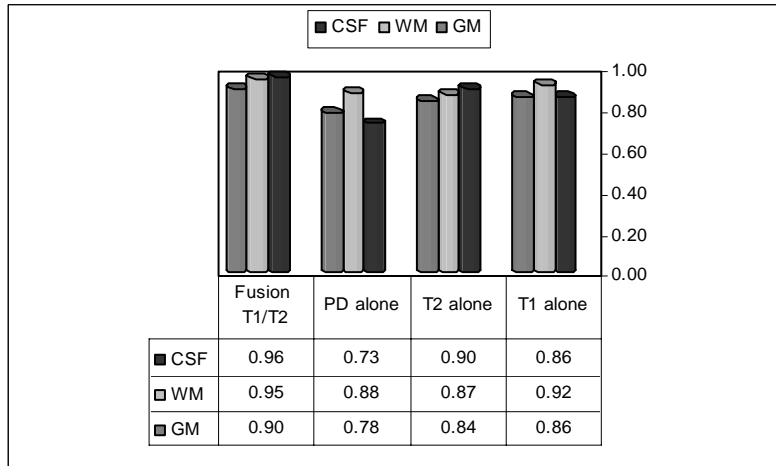


Fig. 3. Similarity measurement for different segmentations with 3% noise.

Table 2. Results on Brainweb phantom images for five methods and the approach we propose.

Measurement	Approach	Inu. Noise Tissue	20%		
			0%	3%	5%
Jaccard coefficient	Published work in [24]	CSF	-	0.58	0.59
		WM	-	0.88	0.87
		GM	-	0.83	0.84
Dice coefficient	Published work in [23]	CSF	-	-	-
		WM	-	0.77	0.76
		GM	-	0.82	0.83
Accuracy	Published work in [19](FDS1)	Min Acc.	-	95.95	-
		Acc.	-	96.95	-
		Max Acc.	-	97.51	-
	Published work in [19](FDS2)	Min Acc.	-	96.11	-
		Acc.	-	97.04	-
		Max Acc.	-	97.58	-
Jaccard coefficient	Our proposed approach	CSF	0.88	0.88	0.86
		WM	0.96	0.92	0.89
		GM	0.87	0.87	0.84
Dice coefficient	Our proposed approach	CSF	0.93	0.96	0.92
		WM	0.97	0.95	0.94
		GM	0.93	0.90	0.91
Accuracy	Our proposed approach	Min Acc.	-	96.89	-
		Acc.	-	97.06	-
		Max Acc.	-	97.99	-

The graphics of figures 2 and 3 underline the advantages of the multispectral fusion images within the fuzzy possibilistic framework to improve the segmentation results clearly. Indeed all measurement values obtained with fusion of T1 and T2 images for CSF, WM and GM tissues are greater than ones obtained when taking into account of only one weighting in MR image segmentation. Finally, we have also compared the performance of our proposed algorithm to that of well-known methods and other published reports that have recently been applied on brain tissue segmentation on Brainweb datasets for the segmentation of MR images

in CSF, WM and GM tissues. They are summarized in table 2, where the four methods can be categorized into two groups: non-fusion methods, these include the published works in [23][24] respectively, and fusion based methods, these include the proposed work in [19]. The results are reported in table 2 below using Accuracy coefficient [19], Jaccard coefficient [24] and Dice coefficient [25].

The methods compared in table 2 have been run on images which have 0%, 3% and 5% of noise, 20% of intensity inhomogeneity (Inu.) and voxel size of  $1\text{mm}^3$ .

As can be seen from this table, the proposed system of fusion which does not use any training data outperforms tested methods the multi-agent based approach in [24] and the published work in [23] for all tissues CSF, GM and WM. Regarding the performance of the fusion based methods, the proposed evidential fusion approach described in [19] is the worst (in terms of average accuracy  $\overline{Acc}$ , minimum accuracy Min acc. and maximum accuracy Max acc.), because it is based on focal elements and masses to represent data and the Dempster-Shafer rule to combine evidence. However, our approach is close to those proposed in [19] FDS1 and FDS2. Results of comparison show clearly the potential interest of our approach for magnetic resonance imaging (MRI) brain segmentation.

## 7. Conclusion

In this paper, a study and an evaluation of a novel technique for a brain MRI segmentation based on a fusion approach and possibility theory concepts are discussed. In the proposed method the pixel intensity, its neighbourhood, memberships and typicality are used in the modelling step to generate data to fusion step. This method offers a considerable improvement in brain MRI segmentation and demonstrates the superior capabilities of fusion approach compared to the taking into account of only one weighting in MR image segmentation. The presented approach has been found robust against noise levels.

As a future perspective of this work other more robust algorithms against noise or hybrid algorithms to modelling a data are desired. In addition, we can integrate other numerical, symbolic information, experts' knowledge or images coming from other imaging devices in order to improve the segmentation of the MR images or to detect anomalies in the pathological images.

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