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FUZZY PARAMETRIC GEOMETRIC PROGRAMMING WITH APPLICATION IN FUZZY EPQ MODEL UNDER FLEXIBILITY AND RELIABILITY CONSIDERATION

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Abstract. An economic production quantity model with demand dependent unit production cost in fuzzy environment has been developed. Flexibility and reliability consideration are introduced in the production process. The models are developed under fuzzy goal and fuzzy restrictions on budgetary cost. The inventory related costs and other parameters are taken as fuzzy in nature. The problem is solved by parametric geometric programming technique. The model is illustrated through numerical example. The sensitivity analyses of the cost function due to different measures are performed and presented graphically.

Keywords: parametric geometric programming technique; production process

1. Introduction

Since late 1960's Geometric Programming (GP) has been very popular in various fields of science and engineering. Duffin, Peterson and Zener [1] discussed the basic theories on GP with engineering application in their books. Peterson [2], Rijckaert [3], Jefferson and Scott [4] have presented informative surveys on GP. The parameter used in the GP problem may not be fixed. It is more fruitful to use fuzzy parameter instead of crisp parameter. In that case we introduced the concept of fuzzy parametric GP technique, where the parameters are fuzzy.

Application of GP can be observed in many aspects of inventory/production, there appears only few papers concerned with the solution of inventory problems using GP (Cheng [5, 6, 7]; Jung and Klein [8]; Kochenberger [9]; Lee [10]; Worrall and Hall [11]).

The determination of the most cost-effective production quantity under rather stable conditions is commonly known as classical economic production quantity (EPQ) inventory problem. Fabulous amount of research effort has been expended on topic leading to the publication of many interesting results in the literature ([Clark [12], Urgelleti [13], Velnott [14]).

A basic assumption in the classical EPQ model is that the production set-up cost is fixed. In addition the model also implicitly assumes that items produced are of perfect quality (Hax and Canadea [15]). However, in reality product quality is not always perfect but directly affected by the reliability of the production process employed to manufacturer the product. Thus a high-level of product quality can only be consistently achieved with substantial investment in improving the reliability of production process. Furthermore, while the set-up time, hence set-up cost, will be fixed in short term, it will tend to decrease in the long term because of the possibility of investment in new machineries that are highly flexible, e.g. flexible manufacturing system. Van and Putten [16] have addressed extensively the issue of flexibility improvement production and inventory management under various scenarios, while the issues of process reliability, quality improvement and set-up time reduction have been discussed by Porteus [17, 18], Rosenblatt and Lee [19]

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and Zangwill [20]. Cheng [5] proposed a general equation to model the relationship between production set-up cost and process reliability and flexibility. Cheng [6] also introduced demand dependent unit production cost in an EOQ model. Tripathy et al. [21] developed an EOQ model with imperfect production process and the unit production cost is directly related to process reliability and inversely related to the demand rate. Islam and Roy [22] developed an EPQ model with flexibility and reliability consideration in fuzzy environment and the model is solved by fuzzy geometric programming technique. Leung [23] considered an EPQ model with flexibility and reliability considerations using GP based on the arithmetic-geometric mean inequality.

In this paper we introduced the concept of fuzzy parametric GP technique. Here we have considered the coefficients of the problem are fuzzy and taken these in parametric form and solve it by GP technique which is formed as a fuzzy parametric GP. An economic production quantity model with demand dependent unit production cost in fuzzy environment has been developed. Flexibility and reliability consideration are introduced in the production process. The models are developed under fuzzy goal and fuzzy restrictions on budgetary cost. The inventory related costs and other parameters are taken as fuzzy in nature. The problem is solved by parametric geometric programming technique.

2. Mathematical model

Let a company produces a single product using a conventional production process with a certain level of reliability. The process reliability depends on a number of factors such as machine capability, use of on-line monitoring devices, skill level of the operating personal and maintenance and replacement policies. The process, thereby reducing the costs of scrap and rework of substandard products, wasted material and labor hours, more consistently produces higher reliability means products with acceptable quality. However, high reliability can only be achieved with substantial capital investment that will increase the cost of interest and depreciation of the production process.

A modern flexible production process that substantially reduces the production set-up time can produce the product more efficiently. It is thus economical to produce in smaller batch sizes with flexible process, thereby reducing the inventory holding cost. Also, substantial capital expenditure due to illustration of the new production process will give rise to might interest changes and great depreciation cost.

2.1. Notations

To construct a model for this problem, we define the following variables and parameters:

S	set-up cost per batch (a decision variable),
h	inventory carrying cost per item per unit time,
D	demand rate (a decision variable),
q	production quantity per batch (a decision variable),
r	production process reliability (a decision variable),
f(S, r)	total cost of interest and depreciation for a production process per production cycle,
TC(D, S, q, r)	total average cost,
P	unit production cost,
B	total budgetary cost.

2.2. Assumptions

The following assumptions are made for developing the mathematical production quantity model:

1) The rate of demand D is uniform over time
2) Shortages are not allowed
3) The time horizon is infinite
4) Total cost of interest and depreciation per production cycle is inversely related to a set-up cost and directly related to process reliability according to the following equation

$$f(S, r) = aS^{-b} r^{c} \tag{1.1}$$

where a, b, c > 0 are constant real numbers chosen to provide the best fit of the estimated cost function.

5) The unit production cost (p) is a continuous function of demand (D) and takes the following form

$$p = \gamma D^{-\beta} \tag{1.2}$$

where β(>1) is called price elasticity and γ (>0) is a scaling constant.

The first three assumptions are the basic assumptions used in the classical EPQ model. The fourth assumption is based on the fact that to reduce the costs of production set-up and scrap and rework on shoddy protects substantial investment in improving the flexibility and reliability of the production process is necessary. The fifth assumption is mainly based on the unit variable production is demand dependent. When the demand of an item increases then the production/purchasing cost spread all over the items and hence the unit purchasing cost reduces and varies inversely with demand.

The process reliability level r means of all the items produced in a production run only r % are acceptable quality that can be used to meet demand. The situation of the inventory model is illustrated in figure below:

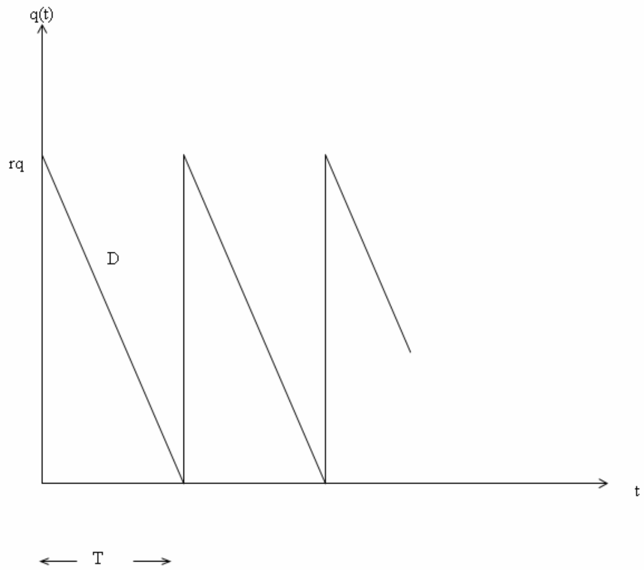

Figure.1 Schematic of the situation of the economic production quantity model

2.3. Crisp Model

If q(t) is the inventory level at time t over the time period (0, T), then

$$\frac{dq(t)}{dt} = -D \quad for\ 0 \le t \le T \tag{1.3}$$

with initial and boundary conditions q(0) = rq, q(T) = 0.

The solution of this differential equation is obtained as

$$q(t) = rq - Dt \tag{1.4}$$

$$\text{and } T = (rq) / D \tag{1.5}$$

Now, the inventory-carrying cost is given by

$$= h\int_0^T q(t)dt \;= h\int_0^T (rq - Dt)dt \;= \frac{hr^2q^2}{2D} \tag{1.6}$$

It is evident that the length of the production cycle is the sum of set-up, production, inventory carrying and interest and depreciation costs, that is total cost per cycle is

$$S + pq + hq^2 r^2 / 2D + f(S, r) \tag{1.7}$$

Our objective is to minimize the total cost per unit time under limited budgetary cost.

So TC (D, S, q, r) = (total cost per cycle) / (qr / D) (1.8)

After substituting (1.1), (1.2) and (1.7) in (1.8) which becomes

$$TC(D,S,q,r) = DSq^{-1}r^{-1} + \gamma D^{1-\beta}r^{-1} + \frac{Hqr}{2} + aDS^{-b}q^{-1}r^{(c-1)} \tag{1.9}$$

It is natural to expect the cost a product to be more if it is more sophisticated and reliable (except possible in the event of some major technological breakthrough). So, one can consider the budgetary function as an increasing function of reliability. Let the production cost per unit is Pr^x and the total budgetary cost of the process is less or equal to B. Here we have considered budgetary cost as a constraint function as follows:

$$Pr^x q \le B \qquad x \in (0,1) \tag{1.10}$$

Hence the inventory model can be written as follows

$$\begin{aligned} &Minimize\;\; TC(D,S,q,r) = DSq^{-1}r^{-1} + \gamma D^{1-\beta}r^{-1} + \frac{Hqr}{2} + aDS^{-b}q^{-1}r^{(c-1)} \\ &\text{subject to} \quad Pr^x q \le B \qquad x \in (0,1) \\ &\qquad\qquad D, S, q, r > 0 \end{aligned} \tag{1.11}$$

The above problem (1.11) can be treated as a Posynomial Geometric Programming problem with zero Degree of Difficulty.

2.4. Fuzzy Model

If the coefficients of objective function and constraint goal of (1.11) are fuzzy [24] in nature then crisp model (1.11) transformed into a fuzzy model as follows

$$\begin{aligned} &Minimize\;\; TC(D,S,q,r) = DSq^{-1}r^{-1} + \tilde{\gamma} D^{1-\beta}r^{-1} + \frac{\tilde{H}qr}{2} + \tilde{a}DS^{-b}q^{-1}r^{(c-1)} \\ &subject\ to \qquad Pr^x q \lessapprox B \qquad x \in [0,1] \\ &\qquad\qquad D, S, q, r>0. \end{aligned} \tag{1.12}$$

where $\tilde{\gamma}, \tilde{h}, \tilde{a}, \tilde{P}$ and $\tilde{B}$ are fuzzy in nature.

3. Prerequisite Mathematics

Definition 3.1 Fuzzy Set: A fuzzy set $\tilde{A}$ in a universe of discourse X is defined as the following set of pairs $\tilde{A} = \{(\mathrm{x}, \mu_{\tilde{A}}(\mathrm{x})) : \mathrm{x} \in \mathrm{X}\}$. Here $\mu_{\tilde{A}} : X \to [0,1]$ is a mapping called the membership function of the fuzzy set $\tilde{A}$ and $\mu_{\tilde{A}}(\mathrm{x})$ is called the membership value or degree of membership of x∈ X in the fuzzy set $\tilde{A}$. The larger $\mu_{\tilde{A}}(\mathrm{x})$ is the stronger the grade of membership in $\tilde{A}$.

Definition 3.2 Normal Fuzzy Set: A fuzzy set $\tilde{A}$ of the universe of discourse X is called a normal fuzzy set implying that there exists at least one $x \in X$ such that $\mu_{\tilde{A}}(\mathrm{x}) = 1$. Otherwise the fuzzy set is subnormal.

Definition 3.3 α-Level Set or α-cut of a Fuzzy Set: The α-level set (or interval of confidence at level α or α-cut) of the fuzzy set $\tilde{A}$ of X is a crisp set A_α that contains all the elements of X that have membership values in $\tilde{A}$ greater than or equal to α i.e. $A_\alpha = \{x : \mu_{\tilde{A}}(x) \ge \alpha, x \in X, \alpha \in [0,1]\}$

Definition 3.4 Fuzzy Number: A fuzzy number $\tilde{A}$ is a fuzzy set of the real line $\Re$ whose membership function $\mu_{\tilde{A}}(x)$ has the following characteristics with $-\infty < a_1 \le a_2 \le a_3 \le a_4 < \infty$

$$\mu_{\tilde{A}}(x) = \begin{cases} \mu_L(x) & for\, a_1 \le x \le a_2 \\ 1 & for\ a_2 \le x \le a_3 \\ \mu_R(x) & for\ a_3 \le x \le a_4 \\ 0 & for\, otherwise \end{cases}$$

where $\mu_L(x)$:$[a_1,a_2] \to [0,1]$ is continuous and strictly increasing; $\mu_R(x)$:$[a_3, a_4] \to [0, 1]$ is continuous and strictly decreasing.

4. Mathematical analysis

Consider a particular non-linear programming problem

$$\begin{aligned} &Min\ g_0(x) \\ &s.t.\ g_i(x) \le 1 \qquad (1 \le i \le n) \\ &\qquad \mathrm{x} > 0. \end{aligned} \tag{4.1}$$

Its objective and constraint functions are of the form

$$g_i(x) = \sum_{k=1}^{T_i} c_{ik} \prod_{j=1}^{m} x_j^{\rho_{ikj}} \quad (0 \le i \le n)$$

where $x_j > 0$; and c_{ik}, ρ_{ikj} are real numbers.

The constraint in (4.1) needs softening and considering the problem of fuzzy objective and constraint with fuzzy coefficients, we transform (4.1) into a fuzzy geometric programming [25] as follows:

$$\begin{aligned} &\tilde{M}in\ g_0(x) \\ &subject\ to\quad g_i(x) \lesssim 1 \qquad (1 \le i \le n), \\ &\qquad \mathrm{x} > 0, \end{aligned} \tag{4.2}$$

where $x=\left(x_1,x_2,...,x_m\right)^T$ is a variable vector,

$g_i(x)=\sum_{k=1}^{T_i} c_{ik}\prod_{j=1}^{m} x_j^{\rho_{ikj}}$ $(0\le i\le n)$ are all posynomial of x in which coefficients c_{ik} are fuzzy numbers.

Here for fuzzy numbers $\tilde{c}_{ik}=\left(c_{1ik},c_{2ik},c_{3ik}\right)$ containing the coefficients $\tilde{c}_{ik}\left(0\le i\le n;1\le k\le T_i\right)$, with the membership function as follow

$$\mu_{\tilde{c}_{ik}}\left(t\right)=\begin{cases}\dfrac{c_{2ik}-t}{c_{2ik}-c_{1ik}} & for\ c_{1ik}\le t\le c_{2ik}\\ \dfrac{t-c_{2ik}}{c_{3ik}-c_{2ik}} & for\ c_{2ik}\le t\le c_{3ik}\\ 0 & otherwise\end{cases} \tag{4.3}$$

Here α-cut of $\tilde{c}_{ik}\left(0\le i\le n;1\le k\le T_i\right)$ is given by

$$c_{ik}\left(\alpha\right)=\left[c_{ikL}\left(\alpha\right),c_{ikR}\left(\alpha\right)\right]=\left[c_{1ik}+\alpha\left(c_{2ik}-c_{1ik}\right),c_{3ik}-\alpha\left(c_{3ik}-c_{2ik}\right)\right] \tag{4.4}$$

Proposition 4.1 If the coefficients of the fuzzy geometric programming problem are taken as fuzzy numbers then the problem (4.2) reduces to

$$Min\sum_{k=1}^{T_0} c_{0kL}\left(\alpha\right)\prod_{j=1}^{m} x_j^{\rho_{0kj}} \tag{4.5}$$

$$\text{Subject to}\quad \sum_{k=1}^{T_i} c_{ikL}\left(\alpha\right)\prod_{j=1}^{m} x_j^{\rho_{ikj}}\le 1,\qquad \left(1\le i\le n\right)$$

$$x_j\ge 0.$$

If the coefficients are taken as fuzzy numbers then the fuzzy geometric programming problem (4.2) will take the form:

$$\tilde{M}in\sum_{k=1}^{T_{0i}} \tilde{c}_{0k}\prod_{j=1}^{m} x_j^{\rho_{0kj}}$$

$$\text{Subject to}\quad \sum_{k=1}^{T_i} \tilde{c}_{ik}\prod_{j=1}^{m} x_j^{\rho_{ikj}}\ \tilde{\le}\ 1,\ \left(1\le i\le n\right)$$

$$x_j>0.$$

Using α-cut of the fuzzy numbers coefficients, the above problem is reduces to

$$Min\sum_{k=1}^{T_0}\left[c_{0kL}\left(\alpha\right),c_{0kR}\left(\alpha\right)\right]\prod_{j=1}^{m} x_j^{\rho_{0kj}} \tag{4.6}$$

$$\text{Subject to}\quad \sum_{k=1}^{T_i}\left[c_{ikL}\left(\alpha\right),c_{ikR}\left(\alpha\right)\right]\prod_{j=1}^{m} x_j^{\rho_{ikj}}\ \tilde{\le}\ 1,\ \left(1\le i\le n\right)$$

$$x_j>0.$$

Which is equivalent to

$$Min\sum_{k=1}^{T_0} c_{0kL}\left(\alpha\right)\prod_{j=1}^{m} x_j^{\rho_{0kj}} \tag{4.7}$$

$$\text{Subject to} \quad \sum_{k=1}^{T_i} c_{ikL}(\alpha)\prod_{j=1}^{m} x_j^{\rho_{ikj}} \le 1, \qquad (1 \le i \le n)$$

$$x_j \ge 0.$$

4.1. Solution procedure of fuzzy parametric geometric programming

Solution of parametric problem (4.7) using fuzzy parametric geometric programming problem is discussed here. Problem (4.7) is a constrained posynomial GP problem. The number of terms in each posynomial constraint function varies and it is denoted by T_r for each r=0,1,2,…,l. Let $T=T_0+T_1+T_2+\ldots+T_l$ be the total number of terms in the primal program. The Degree of Difficulty =T-(m+1).

The dual problem of the primal problem (4.7) is as follows

$$\text{Maximize } d(\delta) = \prod_{r=0}^{l}\prod_{k=1}^{T_r}\left(\frac{c_{rkL}(\alpha)}{\delta_{rk}}\right)^{\delta_{rk}}\left(\sum_{s=1+T_{r-1}}^{T_r}\delta_{rs}\right)^{\delta_{rk}} \tag{4.8}$$

$$\text{subject to } \sum_{k=1}^{T_0}\delta_{0k} = 1, \qquad \text{(Normality condition)}$$

$$\sum_{r=0}^{l}\sum_{k=1}^{T_r}\rho_{rkj}\delta_{rk} = 0, \text{ (j=1,2,...,m)} \qquad \text{(Orthogonality conditions)}$$

$$\delta_{rk} > 0, \text{ (r=0,1,2,…,l; k=1,2,..,}T_r\text{).} \qquad \text{(Positivity conditions)}$$

Case I. For $T \ge m+1$, the dual program presents a system of linear equations for the dual variables. A solution vector exists for the dual variables.

Case II. For T<m+1, in this case generally no solution vector exists for the dual variables.

The solution of the GP problem is obtained by solving the system of linear equations of dual problem (4.8). Ones optimal dual variable vector δ^* are known, the corresponding values of the primal variable vector t is found from the following relations:

$$c_{ikL}(\alpha)\prod_{j=1}^{m} x_j^{\rho_{ikj}} = \delta_i^* d^*(\delta^*) \qquad \text{(i=0,1,2,…,}T_0\text{)} \tag{4.9}$$

Taking logarithms in (4.9), T_0 log-linear simultaneous equations are transformed as

$$\sum_{j=1}^{m}\rho_{ikj}(\log x_j) = \log\left(\frac{\delta_i^* d^*(\delta^*)}{c_{ikL}(\alpha)}\right) \qquad \text{(i=0,1,2,….,}T_0\text{)} \tag{4.10}$$

It is a system of linear equations in t_j (=log x_j) for j=1,2,...,n. Since there are more primal variables x_j than the number of terms T_0, many solutions x_j (j=1,2,...,n) may exist. So, to find the optimal primal variables x_j (j=1,2,...,n), it remains to minimize the primal objective function with respect to reduced m-T_0 ($\neq 0$) variables. When m-T_0=0 i.e. number of primal variables = number of log-linear equations, primal variables can be determined uniquely from log-linear equations.

For different value of $\alpha \in [0,1]$, equ.(4.10) will return different solution set of δ_i^*. And hence different solution set for dual as well as primal problem will be obtained. These solutions sets will help decision-maker to take apt decision.

5. Parametric Geometric Programming Technique on EPQ Model

According to section 4, the fuzzy EPQ model (1.11) reduces to a fuzzy parametric geometric programming by replacing $\tilde{\gamma} = \gamma_0 + (1-\alpha)\gamma_1$, $\tilde{H} = H_0 + (1-\alpha)H_1$, $\tilde{a} = a_0 + (1-\alpha)a_1$, $\tilde{P} = P_0 + (1-\alpha)P_1$ and $\tilde{B} = B_0 + (1-\alpha)B_1$ where $\alpha \in [0,1]$ in (1.11). The model takes the reduces form as follows

$$Minimize\ \ TC(D,S,q,r) = DSq^{-1}r^{-1} + \left(\gamma_0 + (1-\alpha)\gamma_1\right)D^{1-\beta}r^{-1} + \frac{\left(H_0 + (1-\alpha)H_1\right)qr}{2} + \left(a_0 + (1-\alpha)a_1\right)DS^{-b}q^{-1}r^{(c-1)} \tag{5.1}$$

$$subject\ to \qquad r^x q \le \left(W_0 + (1-\alpha)W_1\right) \qquad x \in [0,1]$$

D, S, q, r>0.

Applying GP technique the dual programming of the problem (5.1) is

$$Max\ d(\delta) = \left(\frac{1}{\delta_1}\right)^{\delta_1}\left(\frac{\left(\gamma_0+(1-\alpha)\gamma_1\right)}{\delta_2}\right)^{\delta_2}\left(\frac{\left(H_0+(1-\alpha)H_1\right)}{\delta_3}\right)^{\delta_3}\left(\frac{\left(a_0+(1-\alpha)a_1\right)}{\delta_4}\right)^{\delta_4}\left(\frac{1}{\left(W_0+(1-\alpha)W_1\right)}\right)^{\delta_5} \tag{5.2}$$

subject to

$$\begin{aligned} &\delta_1 + \delta_2 + \delta_3 + \delta_4 = 1 \\ &\delta_1 + (1-\beta)\delta_2 + \delta_4 = 0 \\ &\delta_1 - b\delta_4 = 0 \\ &-\delta_1 + \delta_3 - \delta_4 + \delta_5 = 0 \\ &-\delta_1 - \delta_2 + \delta_3 + (c-1)\delta_4 + x\delta_5 = 0 \end{aligned} \tag{5.3}$$

This is a system of five linear equations in five 5 unknowns. Solving we get the optimal values as follows

$$\delta_1^* = \frac{b(x+1)(\beta-1)}{(1+b)(2x\beta - x + 2) + c(\beta-1)} \tag{5.4}$$

$$\delta_2^* = \frac{(x+1)(1+b)}{(1+b)(2x\beta - x + 2) + c(\beta-1)} \tag{5.5}$$

$$\delta_3^* = \frac{(1+b)\left\{(\beta-1)(x+c) + 2 - \beta\right\}}{(1+b)(2x\beta - x + 2) + c(\beta-1)} \tag{5.6}$$

$$\delta_4^* = \frac{(x+1)(\beta-1)}{(1+b)(2x\beta - x + 2) + c(\beta-1)} \tag{5.7}$$

$$\delta_5^* = \frac{\left\{c(1-\beta) - (1+b)\right\}(x+1)}{(x-1)\left\{(1+b)(2x\beta - x + 2) + c(\beta-1)\right\}} \tag{5.8}$$

Putting these values in (5.1) we get the optimal solution of dual problem. The values of D, S, q, r is obtained by using the primal dual relation as follows

From primal dual relation we get

$$DSq^{-1}r^{-1} = \delta_1^* \times d^*(\delta)$$

$$\left(\gamma_0 + (1-\alpha)\gamma_1\right)D^{1-\beta}r^{-1} = \delta_2^* \times d^*(\delta)$$

$$\left(H_0 + (1-\alpha)H_1\right)qr = \delta_3^* \times d^*(\delta)$$

$$\left(a_0 + (1-\alpha)a_1\right)DS^{-b}q^{-1}r^{(c-1)} = \delta_4^* \times d^*(\delta)$$

$$\frac{1}{\left(W_0 + (1-\alpha)W_1\right)}r^x q = \delta_5^*$$

The optimum solution of the model (1.12) through parametric approach is given by

$$d^*(\delta)=\left(\frac{1}{\delta_1}\right)^{\delta_1}\left(\frac{\gamma_0+(1-\alpha)\gamma_1}{\delta_2}\right)^{\delta_2}\left(\frac{H_0+(1-\alpha)H_1}{\delta_3}\right)^{\delta_3}\left(\frac{a_0+(1-\alpha)a_1}{\delta_4}\right)^{\delta_4}\left(\frac{1}{W_0+(1-\alpha)W_1}\right)^{\delta_5}$$

$$q^*=\delta_5^*\left(W_0+(1-\alpha)W_1\right)\left(\frac{c(H_0+(1-\alpha)H_1)\delta_5^*}{2d^*(\delta)\times\delta_3^*}\right)^x$$

$$r^*=\frac{2\times d^*(\delta)\times\delta_3^*}{c(H_0+(1-\alpha)H_1)\delta_5^*}$$

$$D^*=\left[\frac{2\times(d^*(\delta))^2\times\delta_3^*\times\delta_2^*}{(\gamma_0+(1-\alpha)\gamma_1)(H_0+(1-\alpha)H_1)c\delta_5^*}\right]^{\frac{1}{1-\beta}}$$

$$S^*=\frac{2\times d^*(\delta)\times\delta_3^*}{(H_0+(1-\alpha)H_1)}\left[\frac{2\times(d^*(\delta))^2\times\delta_3^*\times\delta_2^*}{(\gamma_0+(1-\alpha)\gamma_1)(H_0+(1-\alpha)H_1)c\delta_1^*}\right]^{-\frac{1}{1-\beta}}$$

Note that optimal solution of GP technique in parametric approach is depends on α.

6. An illustrative example of the EPQ model

A manufacturing company produces a machine. It is given that the inventory carrying cost of the machine is \$10.5 per unit per year. The production cost of the machine varies inversely with the demand. From the past experience, the production cost of the machine is 15000D-3.6, where D is the demand rate. The total cost of interest and depreciation per production cycle is 1500S-1.6r, where S and r are set-up cost per batch and production process reliability respectively. Let production cost per unit is $8r^{0.6}$ and total budgetary cost (C) is \$ 58. Determine the demand rate (D), set-up cost (S), production quantity (q), production process reliability (r), and optimum total average cost (TC) of the production system.

Formulation of the said model is presented as follows:

$$\begin{aligned}&Min\ TC(D,S,q,r)=DSq^{-1}r^{-1}+15000D^{1-3.6}r^{-1}+\frac{10.5qr}{2}+1500DS^{-1.6}q^{-1}r^{(1.5-1)}\\&subject\ to\qquad 8r^{0.6}q\le 58\qquad\qquad x\in[0,1]\end{aligned}\tag{6.1}$$

The optimum solution of the problem (6.1) by Non-Linear programming (NLP) and Geometric Programming (GP) using LINGO [26] are presented in Table 1.

Table 1 Optimal solution of EPQ model (6.1)

Method	TC^*(\$)	D^*	S^*(\$)	q^*	r^*
GP	114.1258	14.56821	14.91823	9.679971	0.6053934
NLP	114.1618	14.65152	14.93629	9.799638	0.6051736

When the coefficient are taken as fuzzy number i.e. $1\tilde{5}000=14900+(1-\alpha)250$, $\tilde{5}.25=5+(1-\alpha)0.5$, $1\tilde{5}00=1475+(1-\alpha)50$, $\tilde{8}=8+(1-\alpha)0.275$ and $5\tilde{8}=58+(1-\alpha)2,\ \alpha\in[0,1]$ the optimal solutions of the fuzzy model by fuzzy parametric geometric programming is presented in table 2.

Table 2 Optimal solution of fuzzy EPQ model of (6.1)

α	TC^*($)	D^*	S^*($)	q^*	r^*
0.1	114.9443	15.54715	13.74584	11.07385	0.5194229
0.2	114.6381	15.40587	13.92585	10.87414	0.5324381
0.3	114.3301	15.26491	14.10965	10.67673	0.5458799
0.4	114.0201	15.12427	14.29737	10.48161	0.5597660
0.5	113.7083	14.98395	14.48912	10.28876	0.5741147
0.6	113.3945	14.84396	14.68502	10.09816	0.5889454
0.7	113.0788	14.70428	14.88520	9.909811	0.6042784
0.8	112.7610	14.56492	15.08978	9.723689	0.6201351
0.9	112.4412	14.42587	15.29892	9.539781	0.6365380
1.0	112.1193	14.28714	15.51275	9.358073	0.6535106

The solution of objective function and decision variables for different value of α is shown by graphical presentation in figure 2.

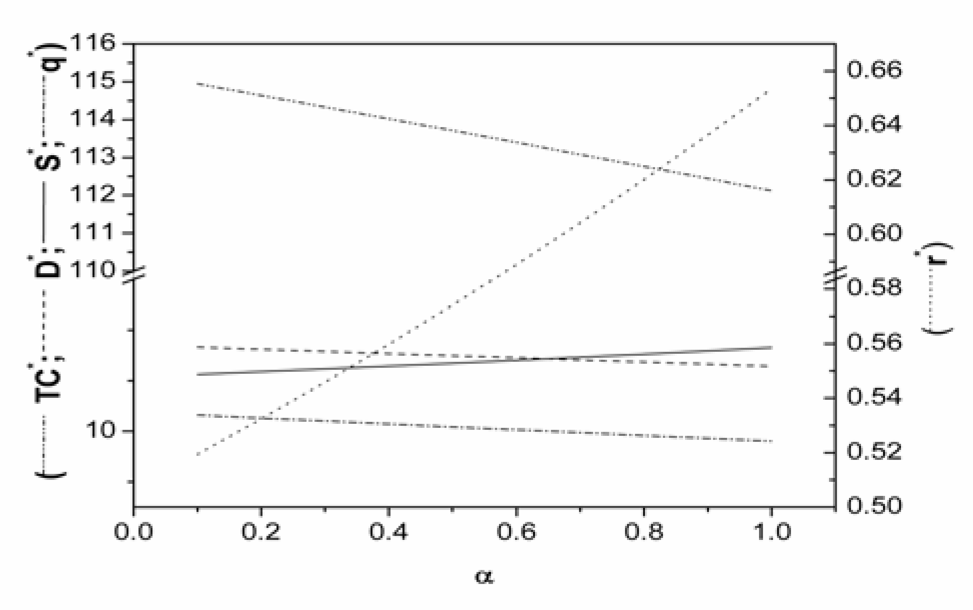

Figure 2 Optimal objective value, decision variables D, S, q and r vs. α

7. Sensitivity analysis

The change of optimal solutions of the problem for fuzzy model with small change of tolerance of constraint goal when α change is, given in Table 3.

Table 3 shows that as B changes increasingly the total average cost of the given problem slightly decreases, which is expected. So it is clear from the sensitivity analysis that if the management traced on proceed reliability, the D and q will be less, so they should not the over 1000 demand. Another fact is that if production management decided on the fact they should try to fulfill the demand then the fact is on to be relation on the production process reliability and setup cost with changing of tolerance of constraint goal, decision variables are also changed. It is noted that the demand and order quantity are increasing with increasing tolerance of constraint goal but setup cost and process reliability are decreasing when tolerance increasing.

Table 3 Change of value of objective function and decision variables for change of α

α	Tolerance of B	TC^*($)	D^*	S^*($)	Q^*	r^*
0.1	0.25	114.9443	15.54715	13.74584	11.07385	0.5194229
	0.5	114.0867	16.15895	13.02879	12.06080	0.4733597
	0.75	113.2600	16.77616	12.36821	13.10344	0.4325368
0.3	0.25	114.3301	15.26491	14.10965	10.67673	0.5458799
	0.5	113.6595	15.73480	13.52818	11.41730	0.5074780
	0.75	113.0079	16.20798	12.98311	12.19061	0.4725617
0.5	0.25	113.7083	14.98395	14.48912	10.28876	0.5741147
	0.5	113.2267	15.31532	14.05581	10.79881	0.5446813
	0.75	112.7551	15.64837	13.64229	11.32508	0.5172066
0.7	0.25	113.0788	14.70428	14.88520	9.909811	0.6042784
	0.5	112.7883	14.90053	14.61377	10.20473	0.5853072
	0.75	112.5014	15.09737	14.34996	10.50531	0.5671140
0.9	0.25	112.4412	14.42587	15.29892	9.539781	0.6365380
	0.5	112.3438	14.49043	15.20440	9.634461	0.6297368
	0.75	112.2469	14.55505	15.11079	9.729751	0.6230312

8. Conclusion

In this paper, an economic production quantity model with investment costs for set-up reduction and quality improvement is formulated. The model has involved one budgetary constraint. The problem is solved by Fuzzy parametric GP method. The Fuzzy parametric GP method provides an alternative approach to this problem. The method, as illustrated, is efficient and reliable. Here decision maker may obtain the optimal results according to his expectation. The method presented is quite general and can be applied to the model in other areas of operation research and other field of optimization involvement.

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