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Dual lag quasi-synchronization of a class of chaotic systems with parameter mismatch

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Abstract. This paper studies the effect of parameter mismatch on the dual-lag synchronization of a class of coupled chaotic systems. Based on the Lyapunov stability theory, a new definition for global dual lag quasi-synchronization is introduced and used to analyze the synchronous behavior of coupled chaotic systems in the presence of parameter mismatch. Numerical simulations on the Ikeda oscillator are presented to verify the theoretical results

Keywords: Dual lag quasi-synchronization; Time delay system; Parameter mismatch.

1. Introduction

Chaos synchronization, which was firstly introduced by Pecora and Carrols [1], has attracted increased interest for the applications of secure communications and spread spectrum communications. For chaotic communication systems, it would also be of great interest to exploit the property of multiplexing chaotic signals in one communication channel. In 1996, Tsimring and Sushchik [2] investigated multiplexing chaos synchronization in a simple map and an electronic circuit model for the first time. Then in 2000 Liu and Davids raised the concept of “dual synchronization", which refers to using a scale signal to simultaneously synchronize two different pairs of chaotic oscillator (two masters and two slaves) [3].

Many studies on dual synchronization of chaotic systems have been reported. For example, Ref. [4] considered the dual synchronization in Colpitts electronic oscillators. Ref.[5] studied the dual synchronization of the Lorenz and the Rössler systems. Dual and cross dual synchronization of chaotic external cavity laser diodes were investigated in [6]. In[7] experimental and numerical dual synchronization of chaos in two pairs of one-way coupled microchip lasers using only one transmission channel were studied. Dual synchronization in modulated time delay system using delay feedback controller was proposed in [8]. Based on Lyapunov stability theory, a general method to achieve the dual-anticipating, dual, dual-lag synchronization of time-delayed chaotic systems was suggested.

It is well known that parameter mismatch is inevitable in practical implementations of chaos synchronization because of noise or other artificial factors. In certain cases parameter mismatches are detrimental to the synchronization quality: in the case of small parameter mismatches the synchronization error does not decay to zero with time, but can show small fluctuations about zero or even a non-zero mean value; larger values of parameter mismatches can result in the loss of synchronization [9].

Recently, there are some reports on chaos synchronization in the presence of parameter mismatch. In Ref. [10] the authors investigated the robustness of the synchronization with respect to parameter mismatches or noise. In Ref. [11], the authors studied the synchronization between two nonidentical unidirectionally linearly coupled chaotic systems with time delay and showed that parameter mismatch is of crucial importance in achieving synchronization. The effect of parameter mismatch on lag synchronization of chaotic systems was studied in Ref. [12]. Ref. [9] considered the effect of parameter mismatch on anticipating synchronization of chaotic systems with time delay in the framework of the master-slave configuration. However, to the best of our knowledge, only a few studies have addressed the effects of parameter mismatches on dual lag synchronization theoretically.

In this paper, we present theoretical analysis and numerical simulations of the parameter-mismatch effect on dual lag quasi-synchronization for a class of coupled chaotic systems. A new definition for global dual lag quasi-synchronization is introduced and a global dual lag synchronization error bound together with a sufficient condition is derived. Numerical simulations on the Ikeda oscillator are presented to verify the theoretical results

The rest part of the paper is organized as follows: In the next section, the problem to be studied is formulated and some preliminaries are presented. In Sec. 3, a sufficient condition for dual lag quasi-synchronization in the presence of parameter mismatch is derived. An illustrating example is then given in Sec. 4, and some conclusions are finally drawn in Sec. 5.

2. Problem formulation and preliminaries

Consider a class of delay chaotic system as

$$\dot{x}_1(t) = A_1 x_1 + B_1 f(x_1(t-\tau_1(t))), \tag{1}$$

where $x_1(t) \in R^n$ is the state vector, A_1 is an $n\times n$ symmetric matrix, B_1 is an $n\times n$ matrix, $f: R^n \in R^n$ is a nonlinear function with $f(0)=0$. $\tau_1(t)$ is the delay time of the feedback loop, where $0 \le \tau_1(t) \le \tau_1$.

Many chaotic systems with delays are of the form of (1), for example the Ikeda oscillator [13], the Mackey-Glass oscillator [14], the Vallee system [15], etc.

We take another system with parameter mismatch from (1) as

$$\dot{y}_1(t) = A_2 y_1 + B_2 f(y_1(t-\tau_2(t))), \tag{2}$$

where $y_1(t) \in R^n$ is the state vector, A_2 is an $n\times n$ symmetric matrix and B_2 is an $n\times n$ matrix. $\tau_2(t)$ is the delay time of the feedback loop, where $0 \le \tau_2(t) \le \tau_2$.

By using a combination of systems (1) and (2), we have the following drive system:

$$\begin{cases} \dot{x}_1(t) = A_1 x_1 + B_1 f(x_1(t-\tau_1(t))) + C_1 g(y_1(t-\tau_2(t))), \\ \dot{y}_1(t) = A_2 y_1 + B_2 f(y_1(t-\tau_2(t))) + C_2 g(x_1(t-\tau_1(t))), \end{cases} \tag{3}$$

where C_1, C_2 are $n\times n$ matrices, $g: R^n \to R^n$ is a nonlinear function with $g(0)=0$.

To synchronize system (3) using feedback control in the framework of the drive-response configuration, we design the response system as:

$$\begin{cases} \dot{x}_2(t) = \bar{A}_1 x_2 + \bar{B}_1 f(x_2(t-\tau_1(t))) + \bar{C}_1 g(y_2(t-\tau_2(t))) + K(x_1(t-\tau(t)) - y_1(t)), \\ \dot{y}_2(t) = \bar{A}_2 y_2 + \bar{B}_2 f(y_2(t-\tau_2(t))) + \bar{C}_2 g(x_2(t-\tau_1(t))) + K(x_2(t-\tau(t)) - y_2(t)), \end{cases} \tag{4}$$

where $x_2 \in R^n, y_2 \in R^n$ are the response states, $\tau(t)$ is coupling delay which is bounded and K is the coupling strength.

Ref.[8] investigated the dual lag synchronization between systems (3) and (4) with $A_1 = \bar{A}_1 = -a_1, B_1 = \bar{B}_1 = -b_1, A_2 = \bar{A}_2 = -a_2, B_2 = \bar{B}_2 = b_2, C_1 = \bar{C}_1 = b_2, C_2 = \bar{C}_2 = b_1, g(x) = f(x),$ and $\tau(t) = \tau_p$, where τ_p is a constant. In this paper, we focus on the case of $A_i \ne \bar{A}_i, B_i \ne \bar{B}_i, C_i \ne \bar{C}_i, i=1,2.$ We use $\Delta A_i = \bar{A}_i - A_i, \Delta B_i = \bar{B}_i - B_i, \Delta C_i = \bar{C}_i - C_i, i=1,2,$ to denote the parameter mismatch errors, and let $e_1(t) = x_2(t) - x_1(t-\tau(t)), e_2(t) = y_2(t) - y_1(t-\tau(t))$ be the synchronization errors between the states of drive system (3) and response system (4). By subtracting Eq.(3) from Eq. (4), we obtain the following error system:

$$\begin{cases} \dot{e}_1(t) = \bar{A}_1 e_1(t) + \bar{B}_1[f(x_2(t-\tau_1(t))) - f(x_1(t-\tau_1(t)-\tau(t)))] \\ \qquad + \bar{C}_1[g(y_2(t-\tau_2(t))) - g(y_1(t-\tau_2(t)-\tau(t)))] + \Delta A_1 x_1(t-\tau(t)) \\ \qquad + \Delta B_1 f(x_1(t-\tau_1(t)-\tau(t))) + \Delta C_1 g(y_1(t-\tau_2(t)-\tau(t))) \\ \qquad + [A_1 x_1(t-\tau(t)) + B_1 f(x_1(t-\tau_1(t)-\tau(t))) + C_1 g(y_1(t-\tau_2(t)-\tau(t)))]\dot{\tau}(t) - K e_1(t), \\ \dot{e}_2(t) = \bar{A}_2 e_2(t) + \bar{B}_2[f(y_2(t-\tau_2(t))) - f(y_1(t-\tau_2(t)-\tau(t)))] \\ \qquad + \bar{C}_2[g(x_2(t-\tau_1(t))) - g(x_1(t-\tau_1(t)-\tau(t)))] + \Delta A_2 y_1(t-\tau(t)) \\ \qquad + \Delta B_2 f(y_1(t-\tau_2(t)-\tau(t))) + \Delta C_2 g(x_1(t-\tau_1(t)-\tau(t))) \\ \qquad + [A_2 y_1(t-\tau(t)) + B_2 f(y_1(t-\tau_2(t)-\tau(t))) + C_2 g(x_1(t-\tau_1(t)-\tau(t)))]\dot{\tau}(t) - k e_2(t). \end{cases} \tag{5}$$

It is easy to observe that $e(t)=(e_1^T(t),e_2^T(t))^T=0$ is not an equilibrium point of system (5), which means that dual-lag synchronization is impossible. Therefore, we study the dual-lag quasi-synchronization between systems (3) and (4). First of all, we present the definition of dual-lag quasi-synchronization.

Definition. For the drive system (3) and the response system (4), it is said that system (3) and (4) are dual lag quasi-synchronized with error bound ε if there exists a $T\geq t_0$ such that $\|x_1(t-\tau(t))-y_1(t)\|\leq\varepsilon$, $\|x_2(t-\tau(t))-y_2(t)\|\leq\varepsilon$ for all $t\geq T$.

Before give our main results, we introduce two Lemmas which are needed in the proof of the main theorem.

Lemma 1. Let $\tau_1=max\{\tau_1(t)\},\tau_2=max\{\tau_2(t)\},\tau=max\{\tau_1,\tau_2\},t\in[0,\infty)$. Suppose that function $x(t)$ is non-negative when $t\in[-\tau,\infty)$ and satisfies the following inequality:

$$\dot{x}(t)\leq -k_0x(t)+k_1x(t-\tau_1(t))+k_2x(t-\tau_2(t))+\alpha,$$

where α,k_0,k_1, and k_2 are nonnegative constants, and $k_0>k_1+k_2$. Then

$$x(t)<\|x(0)\|e^{-rt}+\frac{\alpha}{r},\quad t\geq 0, \tag{6}$$

where $\|x(0)\|=\max\limits_{-\tau\leq s\leq 0}|x(s)|$ and r is the unique positive solution of

$$-r=-k_0+k_1e^{r\tau_1}+k_2e^{r\tau_2}.$$

Proof: Let $y(t)=\|x(0)\|e^{-rt}+\frac{\alpha}{r},t\geq 0$. From (6), we need only to prove that $x(t)<y(t),t\geq 0$. Assume it is not true, then there must be a positive number t_0 such that

$$\begin{cases} x(t)<y(t), & t<t_0, \\ x(t_0)=y(t_0), & t=t_0, \\ \dot{x}(t_0)-\dot{y}(t_0)\geq 0. \end{cases}$$

Note that

$$\begin{aligned}
\dot{x}(t_0)&\leq -k_0x(t_0)+k_1x(t_0-\tau_1(t_0))+k_2x(t_0-\tau_2(t_0))+\alpha\\
&=-k_0y(t_0)+k_1x(t_0-\tau_1(t_0))+k_2x(t_0-\tau_2(t_0))+\alpha\\
&<-k_0y(t_0)+k_1y(t_0-\tau_1(t_0))+k_2y(t_0-\tau_2(t_0))+\alpha\\
&=-k_0(\|x(0)\|e^{-rt_0}+\frac{\alpha}{r})+k_1(\|x(0)\|e^{-r(t_0-\tau_1(t_0))}+\frac{\alpha}{r})+k_2(\|x(0)\|e^{-r(t_0-\tau_2(t_0))}+\frac{\alpha}{r})+\alpha\\
&=-k_0\|x(0)\|e^{-rt_0}+k_1\|x(0)\|e^{-r(t_0-\tau_1(t_0))}+k_2\|x(0)\|e^{-r(t_0-\tau_2(t_0))}+\frac{-k_0+k_1+k_2}{r}\alpha+\alpha\\
&\leq(-k_0+k_1e^{r\tau_1(t_0)}+k_2e^{r\tau_2(t_0)})\|x(0)\|e^{-rt_0}\\
&\leq(-k_0+k_1e^{r\tau_1}+k_2e^{r\tau_2})\|x(0)\|e^{-rt_0}\\
&=-r\|x(0)\|e^{-rt_0}=\dot{y}(t_0),
\end{aligned}$$

we have $\dot{x}(t_0)<\dot{y}(t_0)$, which is in contrast with $\dot{x}(t_0)-\dot{y}(t_0)\geq 0$. Therefore, one gets $x(t)<y(t),t\geq 0$ and the proof of lemma 1 is completed .

Remark 1. Lemma 1 is the extension of that presented in Ref.[12].

Lemma 2 (Ref. 13). For any vectors $x,y\in R^n$ and positive-definite matrix $Q\in R^{n\times n}$, the following matrix inequality holds:

$$2x^Ty\leq x^TQx+y^TQ^{-1}y.$$

Throughout this paper, we assume that f and $g: R^n \to R^n$ are Lipschitz continuous, i.e. there exist positive constants L_f and L_g such that for all $x, y \in R^n$, $\| f(x) - f(y) \| \le L_f \| x - y \|, \| g(x) - g(y) \| \le L_g \| x - y \|$.

3. Criterion for dual lag quasi-synchronization in the presence of parameter mismatch

Now, we are in a position to give our main theorem which provides a criterion for dual lag quasi-synchronization of the chaotic systems (3) and (4) with parameter mismatch.

Theorem 1. Suppose that $\Omega_1 = \{x_1 \in R^n \| \| x_1 \| \le \omega_1\}$, $\Omega_2 = \{y_1 \in R^n \| \| y_1 \| \le \omega_2\}$ and $| \Delta A_1 \| \omega_1 + \| \Delta B_1 \| L_f \omega_1 + \| \Delta C_1 \| L_g \omega_2 \le \mu_1, (\| A_1 \| \omega_1 + \| B_1 \| L_f \omega_1 + \| C_1 \| L_g \omega_2)\tau \le \nu_1$, $\| \Delta A_2 \| \omega_2 + \| \Delta B_2 \| L_f \omega_2 + \| \Delta C_2 \| L_g \omega_1 \le \mu_2, (\| A_2 \| \omega_2 + \| B_2 \| L_f \omega_2 + \| C_2 \| L_g \omega_1)\tau \le \nu_2$. Also, suppose there exists a symmetric and positive-definite matrix $P > 0$, and positive scalars $\alpha_i, \beta_i, \gamma_i, \delta_i, K_i$ $(i = 1, 2)$, and K_0, such that the following conditions hold:

(1) $P\overline{A}_1 + \overline{A}_1 P + \alpha_1 P \overline{B}_1 \overline{B}_1^T P + \beta_1 P \overline{C}_1 \overline{C}_1^T P + \gamma_1 P^2 + \delta_1 P^2 - K_0 P \le 0,$

(2) $P\overline{A}_2 + \overline{A}_2 P + \alpha_2 P \overline{B}_2 \overline{B}_2^T P + \beta_2 P \overline{C}_2 \overline{C}_2^T P + \gamma_2 P^2 + \delta_2 P^2 - K_0 P \le 0,$

(3) $\alpha_1^{-1} L_f^2 I + \beta_2^{-1} L_g^2 I - K_1 P \le 0,$

(4) $\beta_1^{-1} L_g^2 I + \alpha_2^{-1} L_f^2 I - K_2 P \le 0,$

(5) $2K - K_0 - K_1 - K_2 > 0.$

Then the dual lag quasi-synchronization with error bound $\varepsilon + \sqrt{\frac{\gamma_1^{-1}\mu_1^2 + \delta_1^{-1}\nu_1^2 + \gamma_2^{-1}\mu_2^2 + \delta_2^{-1}\nu_2^2}{\lambda_m(P) r}}$ between the systems (3) and (4) is achieved, where ε is any arbitrary small positive number and r is the unique positive solution of equation $-r = K_0 - 2K + K_1 e^{r\tau_1} + K_2 e^{r\tau_2}$.

Proof. Construct the following Lyapunov function:

$$V(e(t)) = e_1^T(t) P e_1(t) + e_2^T(t) P e_2(t),$$

where $e(t) = (e_1^T(t), e_2^T(t))^T$. Differentiating $V(e(t))$ with respect to t along the trajectory of error system (5) yields

$$\begin{aligned}
\dot{V}(t) = {} & 2e_1(t)^T P\{\overline{A}_1 e_1(t) + \overline{B}_1[f(x_2(t-\tau_1(t))) - f(x_1(t-\tau_1(t)-\tau(t)))] \\
& + \overline{C}_1[g(y_2(t-\tau_2(t))) - g(y_1(t-\tau_2(t)-\tau(t)))] + \Delta A_1 x_1(t-\tau(t)) \\
& + \Delta B_1 f(x_1(t-\tau_1(t)-\tau(t))) + \Delta C_1 g(y_1(t-\tau_2(t)-\tau(t))) \\
& + [A_1 x_1(t-\tau(t)) + B_1 f(x_1(t-\tau_1(t)-\tau(t))) + C_1 g(y_1(t-\tau_2(t)-\tau(t)))]\dot{\tau}(t) - K e_1(t)\} \qquad (7) \\
& + 2e_2(t)^T P\{\overline{A}_2 e_2(t) + \overline{B}_2[f(y_2(t-\tau_2(t))) - f(y_1(t-\tau_2(t)-\tau(t)))] \\
& + \overline{C}_2[g(x_2(t-\tau_1(t))) - g(x_1(t-\tau_1(t)-\tau(t)))] + \Delta A_2 y_1(t-\tau(t)) \\
& + \Delta B_2 f(y_1(t-\tau_2(t)-\tau(t))) + \Delta C_2 g(x_1(t-\tau_1(t)-\tau(t))) \\
& + [A_2 y_1(t-\tau(t)) + B_2 f(y_1(t-\tau_2(t)-\tau(t))) + C_2 g(x_1(t-\tau_1(t)-\tau(t)))]\dot{\tau}(t) - K e_2(t)\}.
\end{aligned}$$

In view of Lemma 2, we have

$$\begin{aligned}
& 2e_1(t) P \overline{B}_1[f(x_2(t-\tau_1(t))) - f(x_1(t-\tau_1(t)-\tau(t)))] \le \alpha_1 e_1^T(t) P \overline{B}_1 \overline{B}_1^T P e_1(t) \\
& + \alpha_1^{-1}[f(x_2(t-\tau_1(t))) - f(x_1(t-\tau_1(t)-\tau(t)))]^T [f(x_2(t-\tau_1(t))) - f(x_1(t-\tau_1(t)-\tau(t)))] \\
& = \alpha_1 e_1^T(t) P \overline{B}_1 \overline{B}_1^T P e_1(t) + \alpha_1^{-1} \| f(x_2(t-\tau_1(t))) - f(x_1(t-\tau_1(t)-\tau(t))) \|^2 \\
& \le \alpha_1 e_1^T(t) P \overline{B}_1 \overline{B}_1^T P e_1(t) + \alpha_1^{-1} L_f^2 e_1^T(t-\tau_1(t)) e_1(t-\tau_1(t)),
\end{aligned}$$

$$2e_1(t)P\overline{C}_1[g(y_2(t-\tau_2(t)))-g(y_1(t-\tau_2(t)-\tau(t)))]\le \beta_1 e_1^T(t)P\overline{C}_1\overline{C}_1^T Pe_1(t)$$
$$+\beta_1^{-1}[g(y_2(t-\tau_2(t)))-g(y_1(t-\tau_2(t)-\tau(t)))]^T[g(y_2(t-\tau_2(t)))-g(y_1(t-\tau_2(t)-\tau(t)))]$$
$$=\beta_1 e_1^T(t)P\overline{C}_1\overline{C}_1^T Pe_1(t)+\beta_1^{-1}\|[g(y_2(t-\tau_2(t)))-g(y_1(t-\tau_2(t)-\tau(t)))]\|^2$$
$$\le\beta_1 e_1^T(t)P\overline{C}_1\overline{C}_1^T Pe_1(t)+\beta_1^{-1}L_g^2 e_2^T(t-\tau_2(t))e_2(t-\tau_2(t)),$$
$$2e_1(t)P[\Delta A_1 x_1(t-\tau(t))+\Delta B_1 f(x_1(t-\tau_1(t)-\tau(t)))+\Delta C_1 g(y_1(t-\tau_2(t)-\tau(t)))]$$
$$\le\gamma_1 e_1^T(t)P^2 e_1(t)+\gamma_1^{-1}\|\Delta A_1 x_1(t-\tau(t))+\Delta B_1 f(x_1(t-\tau_1(t)-\tau(t)))+\Delta C_1 g(y_1(t-\tau_2(t)-\tau(t)))\|^2$$
$$\le\gamma_1 e_1^T(t)P^2 e_1(t)+\gamma_1^{-1}(\|\Delta A_1\|\omega_1+\|\Delta B_1\|L_f\omega_1+\|\Delta C_1\|L_g\omega_2)^2$$
$$\le\gamma_1 e_1^T(t)P^2 e_1(t)+\gamma_1^{-1}\mu_1^2,$$
$$2e_1(t)P[A_1 x_1(t-\tau(t))+B_1 f(x_1(t-\tau_1(t)-\tau(t)))+C_1 g(y_1(t-\tau_2(t)-\tau(t)))]\dot{\tau}(t)]$$
$$\le\delta_1 e_1^T(t)P^2 e_1(t)+\delta_1^{-1}(\|A_1\|\omega_1+\|B_1\|L_f\omega_1+\|C_1\|L_g\omega_2)^2\tau^2$$
$$\le\delta_1 e_1^T(t)P^2 e_1(t)+\delta_1^{-1}\nu_1^2.$$

Similary, we can derive the following inequalities:

$$2e_2(t)P\overline{B}_2[f(y_2(t-\tau_2(t)))-f(y_1(t-\tau_2(t)-\tau(t)))]\le\alpha_2 e_2^T(t)P\overline{B}_2\overline{B}_2^T Pe_2(t)+\alpha_2^{-1}L_f^2 e_2^T(t-\tau_2(t))e_2(t-\tau_2(t)),$$
$$2e_2(t)P\overline{C}_2[g(x_2(t-\tau_1(t)))-g(x_1(t-\tau_1(t)-\tau(t)))]\le\beta_2 e_2^T(t)P\overline{C}_2\overline{C}_2^T Pe_2(t)+\beta_2^{-1}L_g^2 e_1^T(t-\tau_1(t))e_1(t-\tau_1(t)),$$
$$2e_2(t)P[\Delta A_2 y_1(t-\tau(t))+\Delta B_2 f(y_1(t-\tau_2(t)-\tau(t)))+\Delta C_2 g(x_1(t-\tau_1(t)-\tau(t)))]\le\gamma_1 e_2^T(t)P^2 e_2(t)+\gamma_2^{-1}\mu_2^2,$$
$$2e_2(t)P[A_2 y_1(t-\tau(t))+B_2 f(y_1(t-\tau_2(t)-\tau(t)))+C_2 g(x_1(t-\tau_1(t)-\tau(t)))]\dot{\tau}(t)]\le\delta_2 e_2^T(t)P^2 e_2(t)+\delta_2^{-1}\nu_2^2.$$

Substituting these into Eq.(7) yields

$$\begin{aligned}\dot{V}(t)&\le e_1(t)^T[P\overline{A}_1+\overline{A}_1P+\alpha_1P\overline{B}_1\overline{B}_1^TP+\beta_1P\overline{C}_1\overline{C}_1^TP+\gamma_1P^2+\delta_1P^2-K_0P]e_1(t)\\&\quad+e_2(t)^T[P\overline{A}_2+\overline{A}_2P+\alpha_2P\overline{B}_2\overline{B}_2^TP+\beta_2P\overline{C}_2\overline{C}_2^TP+\gamma_2P^2+\delta_2P^2-K_0P]e_2(t)\\&\quad+e_1^T(t-\tau_1(t))(\alpha_1^{-1}L_f^2I+\beta_2^{-1}L_g^2I-K_1P)e_1(t-\tau_1(t))\\&\quad+e_2^T(t-\tau_2(t))(\beta_1^{-1}L_g^2I+\alpha_2^{-1}L_f^2I-K_2P)e_2(t-\tau_2(t))\\&\quad+(K_0-2K)e_1(t)^TPe_1(t)+(K_0-2K)e_2(t)^TPe_2(t)+K_1e_1^T(t-\tau_1(t))Pe_1(t-\tau_1(t))\\&\quad+K_2e_2^T(t-\tau_2(t))Pe_1(t-\tau_2(t))+(\gamma_1^{-1}\mu_1^2+\delta_1^{-1}\nu_1^2+\gamma_2^{-1}\mu_2^2+\delta_2^{-1}\nu_2^2)\\&\le(K_0-2K)V(e(t))+K_1V(e(t-\tau_1(t)))+K_2V(e(t-\tau_2(t)))\\&\quad+(\gamma_1^{-1}\mu_1^2+\delta_1^{-1}\nu_1^2+\gamma_2^{-1}\mu_2^2+\delta_2^{-1}\nu_2^2)\\&=(K_0-2K)V(e(t))+K_1V(e(t-\tau_1(t)))+K_2V(e(t-\tau_2(t)))+\alpha,\end{aligned}$$

where $\alpha=\gamma_1^{-1}\mu_1^2+\delta_1^{-1}\nu_1^2+\gamma_2^{-1}\mu_2^2+\delta_2^{-1}\nu_2^2$.

By Lemma 1, we have

$$V(e(t))<\|V(e(0))\|e^{-rt}+\frac{\alpha}{r},\quad t\ge 0,$$

where r is the unique positive solution of $-r=K_0-2K+K_1e^{r\tau_1}+K_2e^{r\tau_2}$. Note that

$$\lambda_m(P)\|e(t)\|^2\le V(e(t)),$$

where $\lambda_m(P)$ is the minimal eigenvalue of matrix P, one gets $\lambda_m(P)\|e(t)\|^2\le\|V(e(0))\|e^{-rt}+\frac{\gamma_1^{-1}\mu_1^2+\delta_1^{-1}\nu_1^2+\gamma_2^{-1}\mu_2^2+\delta_2^{-1}\nu_2^2}{r}$. Accordingly, we obtain

$$\begin{aligned}\|e(t)\|&\le\sqrt{\tfrac{1}{\lambda_m(P)}(\|V(e(0))\|e^{-rt}+\tfrac{\gamma_1^{-1}\mu_1^2+\delta_1^{-1}\nu_1^2+\gamma_2^{-1}\mu_2^2+\delta_2^{-1}\nu_2^2}{r})}\\&\le\sqrt{\tfrac{1}{\lambda_m(P)}\|V(e(0))\|}e^{-\frac{r}{2}t}+\sqrt{\tfrac{\gamma_1^{-1}\mu_1^2+\delta_1^{-1}\nu_1^2+\gamma_2^{-1}\mu_2^2+\delta_2^{-1}\nu_2^2}{\lambda_m(P)r}}.\end{aligned}$$

Obviously, $\sqrt{\frac{1}{\lambda_m(P)}\|V(e(0))\|e^{-\frac{r}{2}t}}\to 0$ as $t\to+\infty$. Thus for any arbitrary small positive number ε, there is a positive T such that for any $t\ge T$, $\|e(t)\|\le\varepsilon+\sqrt{\frac{\gamma_1^{-1}\mu_1^2+\delta_1^{-1}\nu_1^2+\gamma_2^{-1}\mu_2^2+\delta_2^{-1}\nu_2^2}{\lambda_m(P)r}}$. Therefore, the dual lag quasi-synchronization occurs between system (3) and system (4) with error bound $\varepsilon+\sqrt{\frac{\gamma_1^{-1}\mu_1^2+\delta_1^{-1}\nu_1^2+\gamma_2^{-1}\mu_2^2+\delta_2^{-1}\nu_2^2}{\lambda_m(P)r}}$ for any arbitrary small positive number ε. This completes the proof.

Remark 2. It is clear that if the parameter mismatch vanishes and $\tau(t)$ is constant, then the dual-lag synchronization will occur.

Remark 3. The first two conditions in Theorem 1 are a set of LMIs since we are able to transform them to the following equivalent LMIs, respectively:

$$\begin{pmatrix} P\bar{A}_1+\bar{A}_1P+\gamma_1P^2+\delta_1P^2-K_0P & -\alpha_1P\bar{B}_1 & -\beta_1P\bar{C}_1 \\ -\alpha_1\bar{B}_1^TP & -\alpha_1I & 0 \\ -\beta_1\bar{C}_1^TP & 0 & -\beta_1I \end{pmatrix}\le 0,$$

and

$$\begin{pmatrix} P\bar{A}_2+\bar{A}_2P+\gamma_2P^2+\delta_2P^2-K_0P & -\alpha_2P\bar{B}_2 & -\beta_2P\bar{C}_2 \\ -\alpha_2\bar{B}_2^TP & -\alpha_2I & 0 \\ -\beta_2\bar{C}_2^TP & 0 & -\beta_2I \end{pmatrix}\le 0,$$

where I is an identity matrix.

Let $P=I,\alpha_1=\alpha_2=\beta_1=\beta_2=\gamma_1=\gamma_2=\delta_1=\delta_2=1,$ one has the following Corollary.

Corollary 1. Suppose that $\Omega_1=\{x_1\in R^n\,|\,\|x_1\|\le\omega_1\}$, $\Omega_2=\{y_1\in R^n\,|\,\|y_1\|\le\omega_2\}$ and $\|\Delta A_1\|\omega_1+\|\Delta B_1\|L_f\omega_1+\|\Delta C_1\|L_g\omega_2\le\mu_1,(\|A_1\|\omega_1+\|B_1\|L_f\omega_1+\|C_1\|L_g\omega_2)\tau\le\nu_1,$ $\|\Delta A_2\|\omega_2+\|\Delta B_2\|L_f\omega_2+\|\Delta C_2\|L_g\omega_1\le\mu_2,(\|A_2\|\omega_2+\|B_2\|L_f\omega_2+\|C_2\|L_g\omega_1)\tau\le\nu_2.$ If the following conditions hold:

(1) $\bar{A}_1+\bar{A}_1+\bar{B}_1\bar{B}_1^T+\bar{C}_1\bar{C}_1^T+2I-K_0I\le 0,$

(2) $\bar{A}_2+\bar{A}_2+\bar{B}_2\bar{B}_2^T+\bar{C}_2\bar{C}_2^T+2I-K_0I\le 0,$

(3) $L_f^2+L_g^2-K_1\le 0,$ (4) $L_g^2+L_f^2-K_2\le 0,$

(5) $2K-K_0-K_1-K_2>0.$

Then the dual lag quasi-synchronization with error bound $\varepsilon+\sqrt{\frac{\gamma_1^{-1}\mu_1^2+\delta_1^{-1}\nu_1^2+\gamma_2^{-1}\mu_2^2+\delta_2^{-1}\nu_2^2}{r}}$ between the systems (3) and (4) is achieved, where ε is any arbitrary small positive number and r is the unique positive solution of equation $-r=k_0-2k+k_1e^{r\tau_1}+k_2e^{r\tau_2}$.

Remark 4. From Corollary 1 it is easy to see that if K,K_0,K_1,K_2 are sufficient large, then conditions (1)-(5) are satisfied, which means as long as K is large enough the dual lag quasi-synchronization with error bound $\varepsilon+\sqrt{\frac{\gamma_1^{-1}\mu_1^2+\delta_1^{-1}\nu_1^2+\gamma_2^{-1}\mu_2^2+\delta_2^{-1}\nu_2^2}{r}}$ between the systems (3) and (4) will occur.

4. Simulation and results

In the following, we will confirm that the numerical simulations fully support the analytical results presented above. For simplicity, we take the Ikeda time-delay system[13] to show the effectiveness of the proposed results. The Ikeda oscillator is of the form:

$$\dot{x}_1(t) = -a_1 x_1 + b_1 \sin(x_1(t - \tau_1(t))), \tag{8}$$

where x_1 is the phase lag of the electric field across the resonator, a_1 is the relaxation coefficient for the dynamical variable, and b_1 is the laser intensity injected into the system. $\tau_1(t)$ is the round-trip time of the light in the resonator or feedback delay time in the coupled systems. The Ikeda model was introduced to describe the dynamics of an optical bistable resonator and is well known for delay-induced chaotic behavior. When $a_1 = 1, b_1 = 4,$ and $\tau_1(t) = 1.5$, the Ikeda model is chaotic, as shown in Fig. 1.

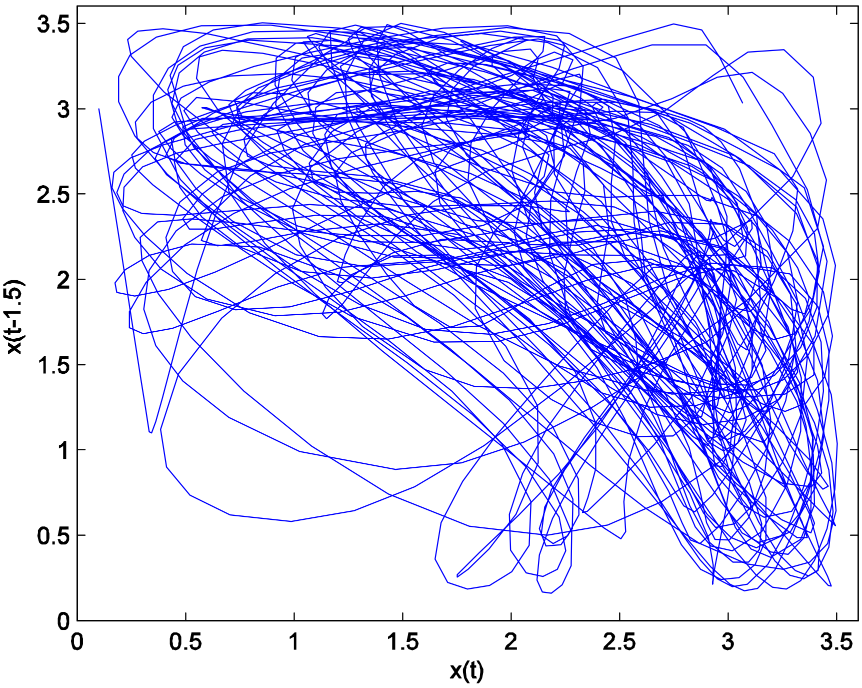

Figure 1. The chaotic attractor of Ikeda system (8).

For numerical simulations, we assume that the drive system associated with Eq. (8) is of the form

$$\begin{cases} \dot{x}_1(t) = -a_1 x_1 + b_1 \sin(x_1(t - \tau_1(t))) + c_1 \sin(\dfrac{y_1(t - \tau_2(t))}{2}), \\ \dot{y}_1(t) = -a_2 y_1 + b_2 \sin(y_1(t - \tau_2(t))) + c_2 \sin(\dfrac{x_1(t - \tau_1(t))}{2}). \end{cases} \tag{9}$$

System (9) exhibits chaotic behavior for the set of parameter values $a_1 = 1, b_1 = 4, c_1 = 6, a_2 = 1.2, b_2 = 6, c_2 = 4, \tau_1(t) = 1.5$, and $\tau_2(t) = 1$. The chaotic attractors of the drive system (9) is shown in Fig. 2.

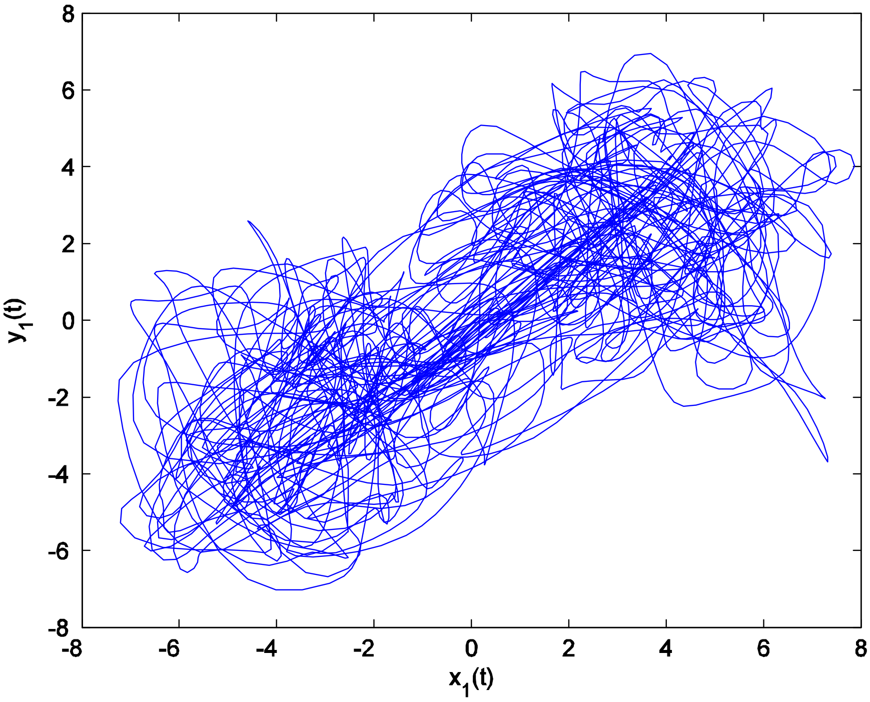

Figure 2. The chaotic attractor of system (9).

Using the proposed delay feedback controller the response system corresponding with system (9) is designed as

$$\begin{cases} \dot{x}_2(t) = -\bar{a}_1 x_2 + \bar{b}_1 \sin(x_2(t-\tau_1(t))) + \bar{c}_1 \sin(\frac{y_2(t-\tau_2(t))}{2}) + K(x_1(t-\tau(t)) - y_1(t)), \\ \dot{y}_2(t) = -\bar{a}_2 y_2 + \bar{b}_2 \sin(y_2(t-\tau_2(t))) + \bar{c}_2 \sin(\frac{x_2(t-\tau_1(t))}{2}) + K(x_2(t-\tau(t)) - y_2(t)). \end{cases} \quad (10)$$

In the process of simulation, we set

$a_1 = 1, b_1 = 4, c_1 = 6, a_2 = 1.2, b_2 = 6, c_2 = 4, \bar{a}_1 = 0.999, \bar{b}_1 = 4.001, \bar{c}_1 = 5.999, \bar{a}_2 = 1.201,$
$\bar{b}_2 = 5.999, \bar{c}_2 = 4.001, \tau_1(t) = 1.5$, $\tau_2(t) = 1,$ and $\tau(t) = 2$. Thus $\|\Delta A_i\| = \|\Delta B_i\| = \|\Delta C_i\| = 0.001,\ i = 1, 2$
According to Fig. 2, we have $\omega_1 = \omega_2 = 8.$ Notice that $L_f = 1, L_g = 1, \dot{\tau}(t) = 0,$ one gets $\mu_1 = \mu_2 = 0.021, \nu_1 = \nu_2 = 0.$ Suppose $K_0 = 51.9980, K_1 = K_2 = 2,$ then conditions (1)-(4) are satisfied. If we take $r = 1,$ then we obtain $K = 33.6990,$ and condition (5) is also satisfied. In virtue of Corollary 1 we can state that the dual lag quasi-synchronization between the systems (9) and (10) is achieved and the estimated error bound is $D = \{d \in R \mid d \le \sqrt{\frac{\gamma_1^{-1}\mu_1^2 + \delta_1^{-1}\nu_1^2 + \gamma_2^{-1}\mu_2^2 + \delta_2^{-1}\nu_2^2}{r}}\} = \{d \in R \mid d \le 0.021 * \sqrt{2} \approx 0.0297\}.$ The synchronization error curve with the control strength $K = 33.6990$ is shown in Fig.3. From this figure, it is easy to see that the numerical simulation is in good agreement with the theoretical analysis.

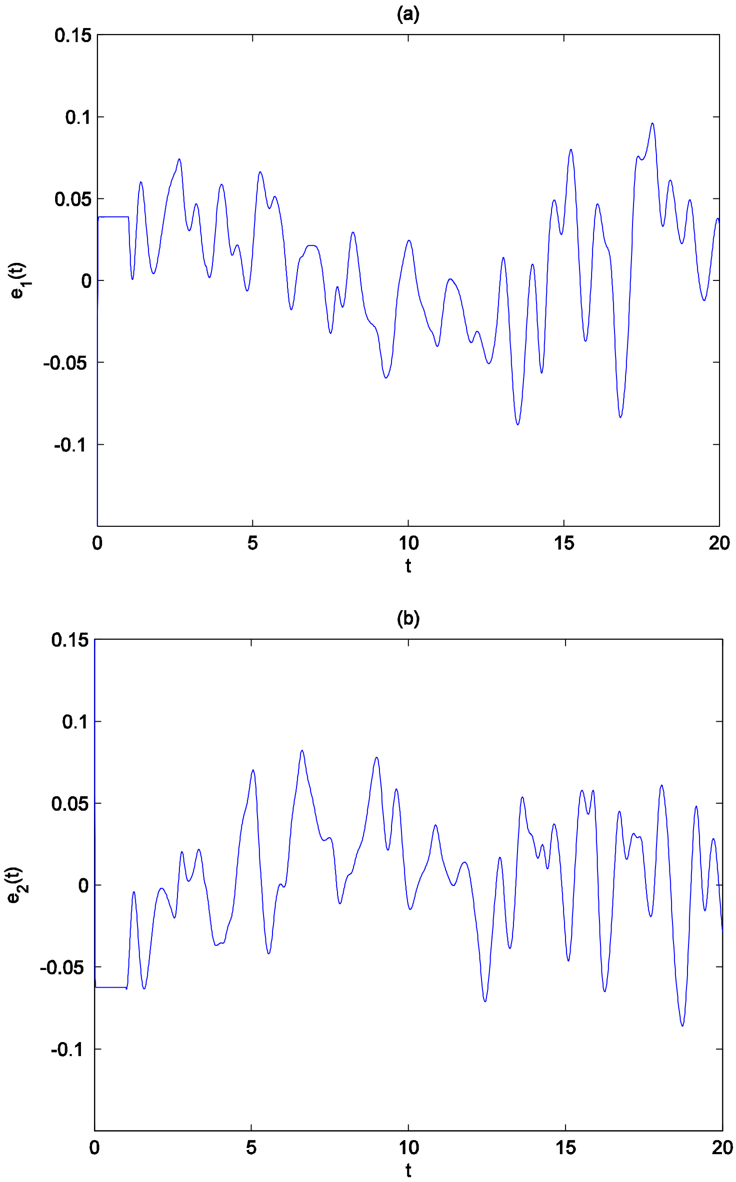

Figure 3. Synchronization error curve with the control strength K=33.6990.

5. Conclusion

This paper discusses the effect of parameter mismatch on the dual-lag synchronization of a class of coupled chaotic systems. Based on the Lyapunov stability theory, we suggest a general method to achieve dual-lag synchronization. As an example, numerical simulations for the Ikeda systems are conducted, which is in good agreement with the theoretical analysis.

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