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Adaptive control and synchronization of the Newton-Leipnik systems

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Abstract. This paper firstly introduces the chaotic system Newton---Leipnik system which possesses two strange attractors.Effective adaptive controllers are proposed for stabilizing chaos to unstable equilibria. In addition, Chaos synchronizations achieved by employing active control scheme. Numerical simulations are provided to verify the feasibility and effectiveness, so the result of the control is mutually verified with the theoretical analyses and numerical simulations.

Keywords: adaptive control, synchronization, Newton-Leipnik systems, numerical simulations

1. Introduction

Since the seminal work of Ott, Grebogi and Yorke (OGY) [1], there has been an increasing interest in recent years in the study of controlling chaotic systems in physics, mathematics and engineering community, etc. Different techniques and methods [2-7] have been proposed over the last decade. Moreover, some of these methods have been successfully applied to experimental systems. Therefore, controlling and synchronization of the chaos have been very important goals and subjects of much current research.

In 1981, Newton and Leipnik constructed a set of differential equations from Euler's rigid body equations which were modified with a linear feedback [8]. Then in 2002, B. Marlin established the existence of closed orbits which were not asymptotically stable for this system [9].In 2002, Chen et al.[10]studied chaos control and synchronization of the Newton–Leipnik system for the first time by using a stable-manifold-based method. Afterwards, Richter further studied the stabilization of a desired motion within one attractor as well as taking the system dynamics from one attractor to another applying the Lyapunov function method [11]. More recent studies by Wang and Tian[12] showed that this chaotic system can be controlled to unstable period orbits and torus with a suited linear controller.

In this paper, Newton–Leipnik system is controlled with adaptive chaos control method. At the same time we use the same method to enable stabilization of chaotic motion to a steady state as well as synchronization between two identical systems. Computer simulation is also given for the purpose of illustration and verification.

2. Adaptive control of Newton–Leipnik system

The Newton–Leipnik system is described by

$$\begin{cases} \dot{x} = -ax + y + 10yz \\ \dot{y} = -x - 0.4y + 5xz \\ \dot{z} = bz - 5xy \end{cases} \tag{1}$$

Where a, b are positive parameters. The Newton–Leipnik system is a chaotic system with two strange attractors. For the system parameter $a = 0.4, b = 0.175$ and initial states $(0.349, 0, -0.160)$ and $(0.349, 0, -0.180)$, we can obtain the two strange attractors which are demonstrated in Fig.1and 2.

This chaotic system has five equilibria:

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$O = (0,0,0)$

$E_1 = (0.06325/(a\sqrt{ab(4a+15+\sqrt{225+80a})}), -0.1581/(a\sqrt{ab(4a+15+\sqrt{225+80a})})(1-(0.25+0.05\sqrt{225+80a}), 0.05+0.01\sqrt{225+80a})$

$E_2 = (-0.06325/(a\sqrt{ab(4a+15+\sqrt{225+80a})}), 0.1581/(a\sqrt{ab(4a+15+\sqrt{225+80a})})(1-(0.25+0.05\sqrt{225+80a}), 0.05+0.01\sqrt{225+80a})$

$E_3 = (0.06325/(a\sqrt{ab(4a+15-\sqrt{225+80a})}), -0.1581/(a\sqrt{ab(4a+15-\sqrt{225+80a})})(1-(0.25-0.05\sqrt{225+80a}), 0.05-0.01\sqrt{225+80a})$

$E_4 = (-0.06325/(a\sqrt{ab(4a+15-\sqrt{225+80a})}), 0.1581/(a\sqrt{ab(4a+15-\sqrt{225+80a})})(1-(0.25-0.05\sqrt{225+80a}), 0.05-0.01\sqrt{225+80a})$

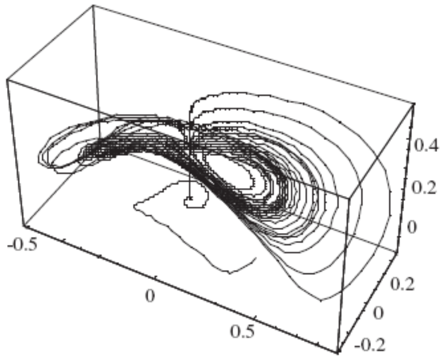


Fig. 1. Chaotic attractor of Newton–Leipnik system, initial point in $(0.349, 0, -0.160)$.

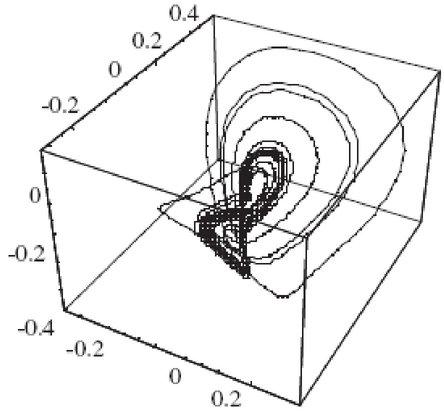


Fig. 2. Chaotic attractor of Newton–Leipnik system, initial point in $(0.349, 0, -0.180)$.

Let us assume that the equations of the controlled Newton-leipnik system are given by

$$\begin{cases} \dot{x} = -ax + y + 10yz + u_1 \\ \dot{y} = -x - 0.4y + 5xz + u_2 \\ \dot{z} = bz - 5xy + u_3 \end{cases} \tag{2}$$

Where u_1, u_2 and u_3 are external control inputs. It will be suitably designed to drive the trajectory of the system, specified by (x, y, z) to each of the five unstable equilibrium points of the uncontrolled (i.e., $u_1 = u_2 = u_3 = 0$) system (1) O, E_1, E_2, E_3 and E_5. For this purpose the goal of control is to find a controller $u = [u_1, u_2, u_3]^T$ and the parameters estimation update law for Eq. (2) such that each of the five equilibria is asymptotically stable.

2.1. Stabilizing the equilibrium $O = (0,0,0)$

Assume the parameters a and b are unknown constant parameters, we choose Lyapunov function for (2) as follows:

$$V(X, \tilde{a}, \tilde{b}) = \frac{1}{2}(X^T X + \tilde{a}^2 + \tilde{b}^2)$$

Where $X = (x, y, z)^T$, $\tilde{a} = a - a_1$ and $\tilde{b} = b - b_1$, a_1, b_1 are estimate values of the unknown parameters a, b, respectively. The time derivative of V along trajectories (2) is

$$\dot{V}(X, \tilde{a}, \tilde{b}) = x(-ax + y + 10yz + u_1) + y(-x - 0.4y + 5xz + u_2) + z(bz - 5xy + u_3) + \tilde{a}(-\dot{a}_1) + \tilde{b}(-\dot{b}_1)$$

We choose the controller u as follows:

$$\begin{cases} u_1 = (a_1 - 1)x - 10yz \\ u_2 = 0 \\ u_3 = -(b_1 + 1)z \end{cases} \tag{3}$$

and the parameters estimation update law $\dot{a}_1, \dot{b}_1$ as follows

$$\begin{cases} \dot{a}_1 = -x \\ \dot{b}_1 = z \end{cases} \tag{4}$$

With this choice, the time derivative of $V(X, \tilde{a}, \tilde{b})$ is given by

$$\dot{V}(X, \tilde{a}, \tilde{b}) = -x^2 - 0.4y^2 - z^2$$

It is clear that $V(X, \tilde{a}, \tilde{b})$ is positive definite and $\dot{V}(X, \tilde{a}, \tilde{b})$ is negative definite in the neighborhood of the zero solution for the system (2). Therefore, the equilibrium solution $O = (0,0,0)$ of the controlled system is asymptotically stable.

2.2. Stabilizing non-zero equilibria E_1, E_2, E_3 and E_4

The equilibrium points of the controlled Newton-Leipnik system (2) are determined from the solution of the following system:

$$\begin{cases} -a\bar{x} + \bar{y} + 10\bar{y}\bar{z} + \bar{u}_1 = 0 \\ -\bar{x} - 0.4\bar{y} + 5\bar{x}\bar{z} + \bar{u}_2 = 0 \\ b\bar{z} - 5\bar{x}\bar{y} + \bar{u}_2 = 0 \end{cases}$$

Where $\bar{x}, \bar{y}, \bar{z}$ and $\bar{u}_i, i = 1,2,3$, are the state variables and control functions at the equilibrium points of the controlled system (2). Now, we proceed to obtain the perturbed equations of the controlled system (2) about its equilibrium points $\bar{x}, \bar{y}, \bar{z}$ and $\bar{u}_i, i = 1,2,3$. For this purpose, we introduce the following variables:

$$e_1 = x - \bar{x}, e_2 = y - \bar{y}, e_3 = z - \bar{z} \text{ And } v_i = u_i - \bar{u}_i, i = 1,2,3 \tag{5}$$

Substituting Eq.(5) into (2). We obtain the perturbed equations about the equilibrium points of the system (2)in the following form:

$$\begin{cases} \dot{e}_1 = -ae_1 + e_2 + 10(e_2e_3 + e_2\bar{z} + e_3\bar{y}) + v_1 \\ \dot{e}_2 = -e_1 - 0.4e_2 + 5(e_1e_3 + e_3\bar{x} + e_1\bar{z}) + v_2 \\ \dot{e}_3 = be_3 - 5(e_1e_2 + \bar{x}e_2 + \bar{y}e_1) + v_3 \end{cases} \tag{6}$$

This system admits the special solution $e_i = 0, v_i = 0, i = 1,2,3$. This solution describes the equilibrium points of the system (2). Assume that some of equilibrium points are unstable, then the control can be made such that these states become asymptotically stable.

Let the parameters a and b are unknown constant parameters, we choose Lyapunov function for (6) as follows:

$$V(e, \tilde{a}, \tilde{b}) = \frac{1}{2}(e^T e + \tilde{a}^2 + \tilde{b}^2)$$

Where $e = (e_1, e_2, e_3)^T$, $\tilde{a} = a - a_1$ and $\tilde{b} = b - b_1$, a_1, b_1 are estimate values of the unknown parameters a, b, respectively. The time derivative of V along trajectories (6) is

$$\begin{aligned} \dot{V}(e, \tilde{a}, \tilde{b}) = {} & e_1(-ae_1 + e_2 + 10(e_2e_3 + e_2\bar{z} + e_3\bar{y}) + v_1) \\ & + e_2(-e_1 - 0.4e_2 + 5(e_1e_3 + e_3\bar{x} + e_1\bar{z}) + v_2) \\ & + e_3(be_3 - 5(e_1e_2 + \bar{x}e_2 + \bar{y}e_1) + v_3) + \tilde{a}(-\dot{a}_1) + \tilde{b}(-\dot{b}_1) \end{aligned}$$

We choose the controller $v = [v_1, v_2, v_3]^T$ as follows:

$$\begin{cases} v_1 = a_1 e_1 - e_1 - 10 e_2 e_3 - 15 e_2 \bar{z} \\ v_2 = 0 \\ v_3 = -b_1 e_3 - e_3 - 5 e_1 \bar{y} \end{cases} \tag{7}$$

and the parameters estimation update law $\dot{a}_1, \dot{b}_1$ as follows

$$\begin{cases} \dot{a}_1 = -e_1^2 \\ \dot{b}_1 = e_3^2 \end{cases} \tag{8}$$

With this choice, the time derivative of $V(e, \tilde{a}, \tilde{b})$ is given by

$$\dot{V}(e, \tilde{a}, \tilde{b}) = -e_1^2 - 0.4 e_2^2 - e_3^2$$

This translates to

$$\lim_{t \to \infty} \| e(t) \| = 0, \forall a, b \in R$$

Therefore, the equilibrium solution $O = (0,0,0)$ of the syste(6) is asymptotically stable. According to (5) and choosing $\bar{u}_i = 0, i = 1,2,3$, the equilibrium solutions $\bar{x}, \bar{y}, \bar{z}$ of system (2) are asymptotically stable if the controller u is

$$\begin{cases} u_1 = a_1 e_1 - e_1 - 10 e_2 e_3 - 15 e_2 \bar{z} \\ u_2 = 0 \\ u_3 = -b_1 e_3 - e_3 - 5 e_1 \bar{y} \end{cases} \tag{9}$$

and the parameters estimation update law is (8). Therefore, non-zero equilibria E_1, E_2, E_3 and E_4 of (2) are asymptotically stable if the controller u is (9) and the parameters estimation update law is (8).

3. Numerical simulations

In this section, numerical simulations are carried out . In addition, a time step size 0.01 is employed. We select the parameters of Newton-leipnik system as $a = 0.4, b = 0.175$ so that Newton-leipnik system exhibits a chaotic behavior. The initial states of the controlled Newton-leipnik systems (2) are $x_0(0) = 10, y_0(0) = -10, z_0(0) = 10$ and the initial values of the parameters estimation update laws are $a_1(0) = 5, b_1(0) = 5$. Fig.3 shows that the chaos in Newton-leipnik system is controlled to equilibrium point $O = (0,0,0)$ in presence of system's uncertain parameters a, b, with the control law (3) and the parameters estimation update law (4). Fig. 4 (a)~(d) shows that the chaos in Newton-leipnik system is controlled to equilibrium point E_1, E_2, E_3 and E_4 in presence of system's unknown uncertain parameters a, b, with the control law (9) and the parameters estimation update law (8).

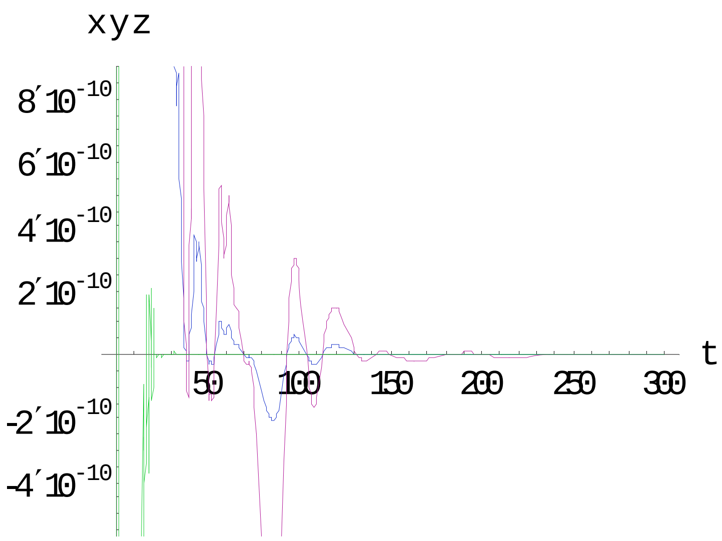


Fig. 3. The time response of states (x, y, z) for the controlled system (2) with control law (3) and the parameters estimation update law (4): stabilizing the equilibrium *O*.

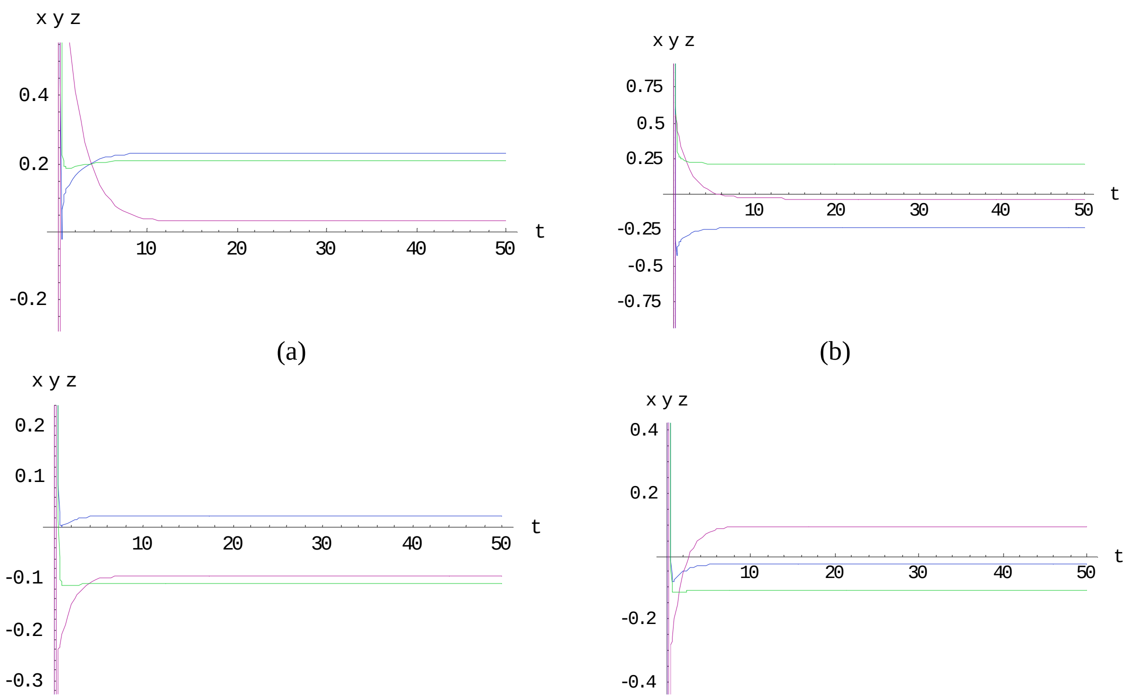


Fig. 4. The time response of states (x, y, z)) for the controlled system (2) with control law (9) and the parameters estimation update law (8): stabilizing the equilibriums E_1, E_2, E_3 and E_4, respectively.

4. Adaptive synchronization of Newton-leipnik system

In order to observe the synchronization behavior in Newton-leipnik system, we assume that we have two identical Newton-leipnik systems and that the drive system with the subscript 1 is to control the response system with subscript 2. The drive and response systems are defined as follows:

$$\begin{cases} \dot{x}_1 = -ax_1 + y_1 + 10y_1z_1 \\ \dot{y}_1 = -x_1 - 0.4y_1 + 5x_1z_1 \\ \dot{z}_1 = bz_1 - 5x_1y_1 \end{cases} \tag{9}$$

where the parameters a and b are unknown or uncertain, and the response system is

$$\begin{cases} \dot{x}_2 = -a_1 x_2 + y_2 + 10 y_2 z_2 - u_1 \\ \dot{y}_2 = -x_2 - 0.4 y_2 + 5 x_2 z_2 - u_2 \\ \dot{z}_2 = b_1 z_2 - 5 x_2 y_2 - u_3 \end{cases} \tag{10}$$

where a_1 and b_1 are parameters of the response system which need to be estimated, and $u = [u_1, u_2, u_3]^T$ is the controller we introduced in Eq. (10). Suppose that

$$u_1 = k_1 e_1, u_2 = k_2 e_2, u_3 = k_3 e_3 \tag{11}$$

where e_1, e_2 and e_3 are the error states which are defined as follows:

$$e_1 = x_2 - x_1, e_2 = y_2 - y_1, e_3 = z_2 - z_1 \tag{12}$$

And

$$\begin{cases} \dot{a}_1 = f_a = \alpha e_1 x_2 \\ \dot{b}_1 = f_b = -\beta e_3 z_2 \end{cases} \tag{13}$$

where $k_1, k_2, k_3 > 0$ and α, β are positive real constants.

Theorem The two systems (9) and (10) can be synchronized under the controller (11) and a parameter estimation update law (13), if the following conditions are satisfied:

$$\begin{cases} (1)\mathrm{k}_1 + a > 0 \\ (2)(\mathrm{k}_1 + a)(k_2 + 0.4) - \dfrac{225 M_1^2}{4} > 0 \\ (3)(\mathrm{k}_1 + a)(k_2 + 0.4)(k_3 - b) - \dfrac{225 M_1^2}{4} - (k_2 + 0.4)\dfrac{M_2^2}{4} > 0 \end{cases} \tag{14}$$

where $M_1 > |z_1|$ and $M_2 > |10 y_2 - 5 y_1|$ are positive real constants.

Proof. It is easy to see from (9)-(11) that the error dynamical system can be obtained as follows:

$$\begin{cases} \dot{e}_1 = -a_1 x_2 + a x_1 + e_2 + 10(e_2 z_1 + e_3 y_2) - k_1 e_1 \\ \dot{e}_2 = -e_1 - 0.4 e_2 + 5(e_3 x_2 + e_1 z_1) - k_2 e_2 \\ \dot{e}_3 = b_1 z_2 - b z_1 - 5(x_2 e_2 + y_1 e_1) - k_3 e_3 \end{cases} \tag{15}$$

Let $e_a = a_1 - a, e_b = b_1 - b$. Choose the following Lyapunov function as follows:

$$V(e_1, e_2, e_3) = \frac{1}{2}(e_1^2 + e_2^2 + e_3^2 + \frac{1}{\alpha} e_a^2 + \frac{1}{\beta} e_b^2)$$

then differentiation of V along trajectories (15) is

$$\begin{aligned} \dot{V} &= e_1 \dot{e}_1 + e_2 \dot{e}_2 + e_3 \dot{e}_3 + \frac{1}{\alpha} e_a f_a + \frac{1}{\beta} e_b f_b \\ &= e_1(-a_1 x_2 + a x_1 + e_2 + 10(e_2 z_1 + e_3 y_2) - k_1 e_1) + e_2(-e_1 - 0.4 e_2 + 5(e_3 x_2 + e_1 z_1) - k_2 e_2) \\ &\quad + e_3(b_1 z_2 - b z_1 - 5(x_2 e_2 + y_1 e_1) - k_3 e_3) + \frac{1}{\alpha} e_a f_a + \frac{1}{\beta} e_b f_b \\ &= -(k_1 + a) e_1^2 - (k_2 + 0.4) e_2^2 - (k_3 - b) e_3^2 + 15 e_1 e_2 z_1 + e_1 e_3 (10 y_2 - 5 y_1) \\ &\quad + e_a(\frac{1}{\alpha} f_a - e_1 x_2) + e_b(\frac{1}{\beta} f_b + e_3 z_2) \end{aligned}$$

choose, $f_a = \alpha e_1 x_2$, $f_b = -\beta e_3 z_2$, $M_1 > |z_1|$ and $M_2 > |10 y_2 - 5 y_1|$, then

$$\dot{V} < -(k_1 + a)e_1^2 - (k_2 + 0.4)e_2^2 - (k_3 - b)e_3^2 + 15|e_1 e_2| M_1 + |e_1 e_3| M_2$$

Where $e = [|e_1|, |e_2|, |e_3|]^T$, and

$$P = \begin{bmatrix} k_1 + a & -\frac{15M_1}{2} & -\frac{M_2}{2} \\ -\frac{15M_1}{2} & k_2 + 0.4 & 0 \\ -\frac{M_2}{2} & 0 & k_3 - b \end{bmatrix}$$

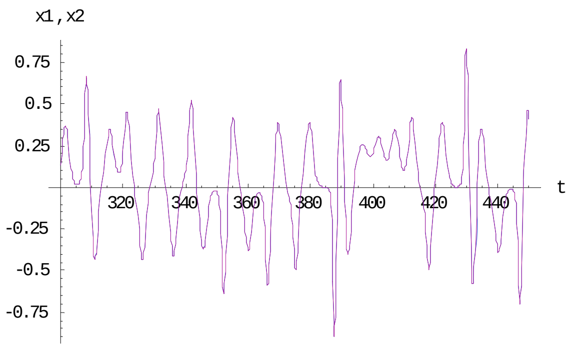
Obviously, to ensure that the origin of error system (15) is asymptotically stable, the matrix P should be positive definite, which implies that $\dot{V}$ is negative definite. This case is satisfied if and only if the following inequalities hold:

$$\begin{cases} (1)\mathrm{k}_1 + a > 0 \\ (2)(\mathrm{k}_1 + a)(k_2 + 0.4) - \frac{225M_1^2}{4} > 0 \\ (3)(\mathrm{k}_1 + a)(k_2 + 0.4)(k_3 - b) - \frac{225M_1^2}{4} - (k_2 + 0.4)\frac{M_2^2}{4} > 0 \end{cases}$$

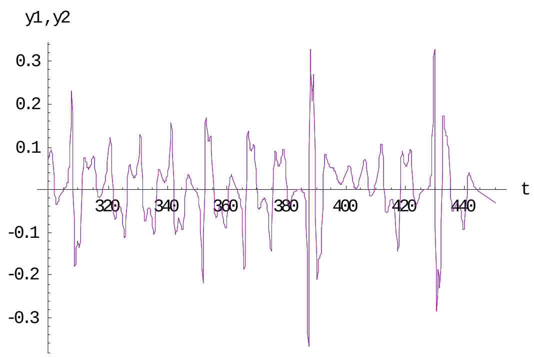
Accordingly, the response system (10) is synchronizing with the drive system (9) under the controller (11) and a parameter estimation update law (13), if the conditions (14) are satisfied. The proof is completed.

5. Numerical simulations

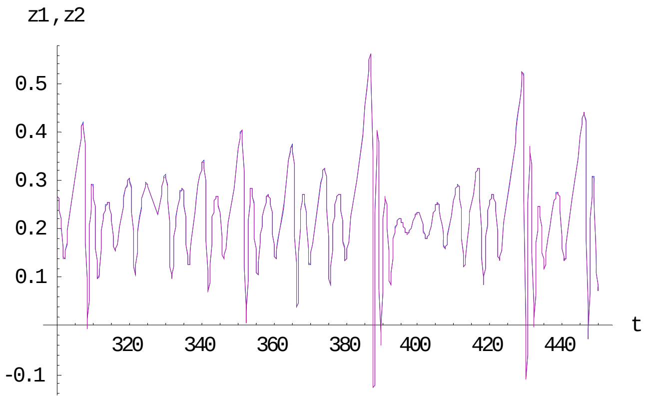
In this section, numerical simulations are carried out. In addition, a time step size 0.01 is employed. We select the parameters of Newton-leipnik system as $a = 0.4, b = 0.175$ so that Newton-leipnik system exhibits a chaotic behavior. The initial values of the parameters a_1, b_1, c_1 are zero The initial states of the drive and response systems (9)and(10) are $x_1(0) = 0.349, y_1(0) = 0, z_1(0) = -0.160$ and $x_2(0) = 1, y_2(0) = -1, z_2(0) = 1$. In order to choose the control parameters $M_1 > |z_1|$, and $M_2 > |10y_2 - 5y_1|$ need to be estimated. Through simulations, we obtain $M_1 = 1, M_2 = 10$. The control parameters are chosen as follows $k_1 = 6, k_2 = 10, k_3 = 10$. Choose $\alpha = \beta = 1$. The state trajectories for the drive system (9) and the response system (10) with the control laws (11) and parameters estimation update law (13) are shown in Fig.5(a)–(c). The dynamics of changing parameters a_1, b_1 is shown in Fig.6.



(a)



(b)



(c)

Fig. 5. The time response of states for drive system (9) and response system (10) with the control law (11) and parameters estimation update law (13); (a) signals x_1 and x_2, (b) signals y_1 and y_2, (c) signals z_1 and z_2.

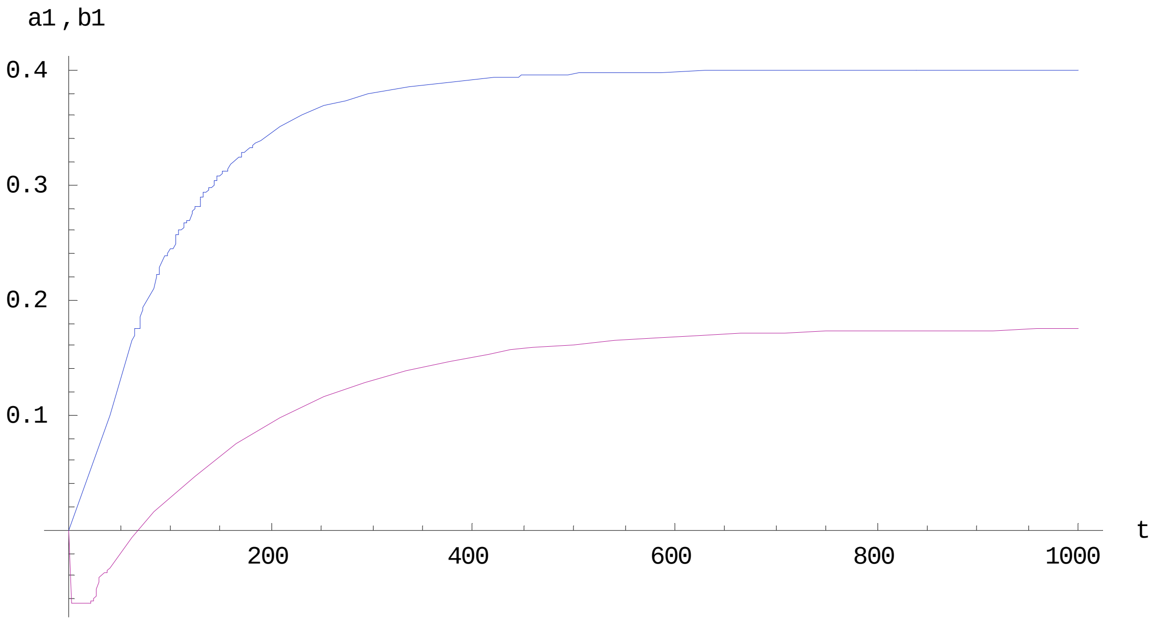


Fig. 6. Changing parameters a_1, b_1 of system (10) with time t.

6. Conclusion

This work addresses adaptive chaos control and synchronization of Newton-leipnik system when the parameters of the drive system are fully uncertain and different with those of the response system. Based on Lyapunov stability theory, the sufficient conditions for the synchronization have been obtained analytically. Numerical simulations are shown to verify the proposed method.

7. Acknowledgements

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