

Hyperchaos Synchronization Between two Different Hyperchaotic Systems

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Abstract. This paper is involved with a new hyperchaotic Lorenz system and the hyperchaotic Rössler system. Synchronization of the two hyperchaotic systems is achieved via active control technique. Based on the Lyapunov stability theory, adaptive control theory is applied to realize synchronization of the two different hyperchaotic systems with unknown parameters. Numerical simulations are given for the purpose of illustration and verification.

Keywords: hyperchaos; synchronization; hyperchaotic Lorenz system; hyperchaotic Rössler system

1. Introduction

Synchronization of chaotic system has attracted lots of attentions from various fields since the revolutionary work of Pecora and Carroll [1]. Applications of chaos synchronization theory can be found in secure communication, modeling brain activity and neural network et al. Great efforts have been devoted to develop the theories and techniques of synchronization between two chaotic systems owning only one positive Lyapunov exponent, and the matured methods and techniques for chaos control and synchronization has been applied to problems of hyperchaos [2].

The hyperchaotic system, possessing at least two positive Lyapunov exponents, has more complex behavior than common chaotic system. Studies of hyperchaos control and hyperchaos synchronization are distinctively interesting and challenging works. Problems of suppressing hyperchaos and realizing hyperchaos synchronization have attracted many mathematicians, physicists and engineers [3-11, 12-18]. Many hyperchaotic systems, including the hyperchaotic Rössler system [4], the hyperchaotic Chen system [5], the hyperchaotic Lü system [6], and the hyperchaotic unified system [7], have been constructed and investigated. Lots of literatures have studied the synchronization between two identical hyperchaotic systems via classical control techniques, including feedback control [8], active control [13], backstepping design [16] and adaptive control [18] et al.

In this paper, we will realize synchronization between two different hyperchaotic systems, even between two systems with unknown parameters.

2. System descriptions

In [9], we constructed a new hyperchaotic system via introducing an additional state to the celebrated Lorenz chaotic system. For simplicity, we call it hyperchaotic Lorenz system in this context. The hyperchaotic Lorenz system is described by

$$\begin{cases} \dot{x} = -a(x - y) + u \\ \dot{y} = -xz + cx - y \\ \dot{z} = xy - bz \\ \dot{u} = -xz + du \end{cases} \quad (1)$$

in which $(a, b, c, d) \in R^4$. When $a = 10, b = 8/3, c = 28$ and $d = 1.3$, this system (1) has two positive Lyapunov exponents as follows $\lambda_1 = 0.39854$ and $\lambda_2 = 0.24805$. Thus, the system (1) is hyperchaotic under

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these parameters. The hyperchaotic attractor is given in fig.1.

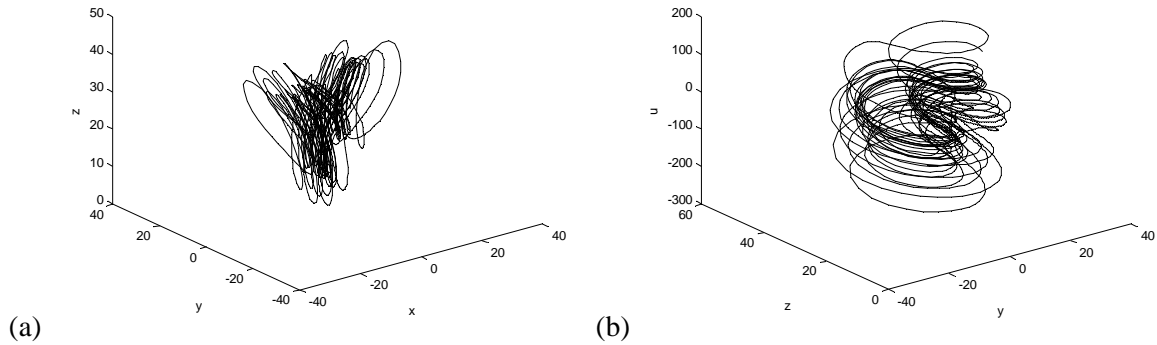


Figure 1: projection of the attractor of hyperchaotic Lorenz system. (a) xyz and (b) yzu .

The hyperchaotic Rössler system [4] is given by

$$\begin{cases} \dot{x} = fy + gx \\ \dot{y} = 3 + yz \\ \dot{z} = -y - u \\ \dot{u} = x + z + eu \end{cases} \quad (2)$$

in which $(e, f, g) \in R^3$. When parameters $e = 0.25$, $f = -0.5$ and $g = 0.05$, the two positive Lyapunov exponents of the system (2) are $\lambda_1 = 0.109$ and $\lambda_2 = 0.024$. The system (2) shows hyperchaotic behavior with suitable initial conditions. Starting from initial condition $(14, 2, -17, 7)$, the hyperchaotic system (2) converges to the attractor given in fig.2.

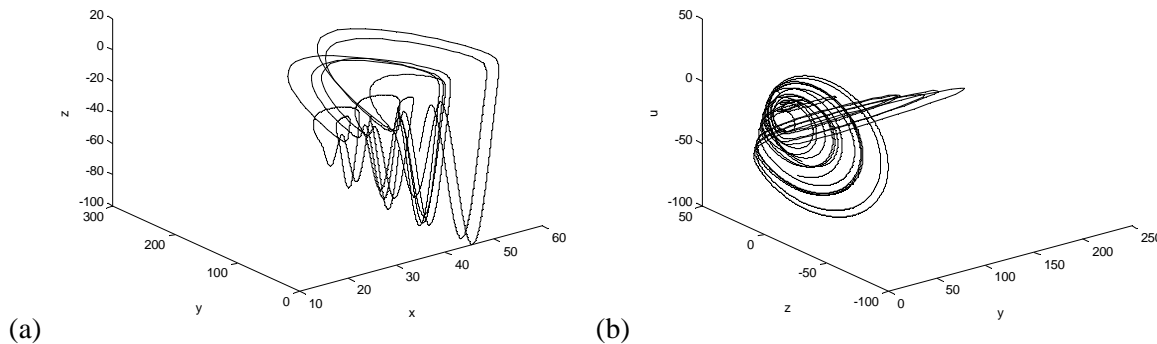


Figure 2: projection of the attractor of hyperchaotic Rössler system with initial condition $(14, 2, -17, 7)$.

(a) xyz and (b) yzu .

3. Synchronization between two different hyperchaotic systems

In this section, synchronization between the hyperchaotic Rössler system and the hyperchaotic Lorenz system is achieved. To realize synchronization of the two hyperchaotic systems, an effective controller will be designed via active control technique.

Choose the hyperchaotic Lorenz system as the drive system

$$\begin{cases} \dot{x}_1 = -a(x_1 - y_1) + u_1 \\ \dot{y}_1 = -x_1 z_1 + cx_1 - y_1 \\ \dot{z}_1 = x_1 y_1 - bz_1 \\ \dot{u}_1 = -x_1 z_1 + du_1 \end{cases} \quad (3)$$

And let the hyperchaotic Rössler system be the response one

$$\begin{cases} \dot{x}_2 = fy_2 + gx_2 + v_1 \\ \dot{y}_2 = 3 + y_2 z_2 + v_2 \\ \dot{z}_2 = -y_2 - u_2 + v_3 \\ \dot{u}_2 = x_2 + z_2 + eu_2 + v_4 \end{cases} \quad (4)$$

in which v_i ($i = 1, 2, 3, 4$) are active control functions to be designed.

Subtracting system (3) from the system (4), we obtain the error dynamical system between the drive system and the response one given by

$$\begin{cases} \dot{e}_1 = (f - a)y_2 + (g + a)x_2 + ae_2 - ae_1 - u_1 + v_1 \\ \dot{e}_2 = y_2 z_2 + x_1 z_1 - cx_1 + y_2 - e_2 + v_2 + 3 \\ \dot{e}_3 = -y_2 - u_2 + v_3 - x_1 y_1 + bz_2 - be_3 \\ \dot{e}_4 = x_2 + z_2 + (e - d)u_2 + v_4 + x_1 z_1 + de_4 \end{cases} \quad (5)$$

in which $e_1 = x_2 - x_1, e_2 = y_2 - y_1, e_3 = z_2 - z_1$ and $e_4 = u_2 - u_1$.

To ensure that the error dynamical system (5) asymptotically stable at the origin, we choose the active control function as follows

$$\begin{cases} v_1 = (a - f)y_2 - (g + a)x_2 - ae_2 + u_1 \\ v_2 = -y_2 z_2 - x_1 z_1 + cx_1 - y_2 - 3 \\ v_3 = y_2 + u_2 + x_1 y_1 - bz_2 \\ v_4 = -x_2 - z_2 + (d - e)u_2 - x_1 z_1 - (d + 1)e_4 \end{cases} \quad (6)$$

Thus, the error dynamical system becomes

$$\begin{cases} \dot{e}_1 = -ae_1 \\ \dot{e}_2 = -e_2 \\ \dot{e}_3 = -be_3 \\ \dot{e}_4 = -e_4 \end{cases} \quad (7)$$

The characteristic values of the Jacobin matrix of the error dynamical system (7) are $-a, -1, -b$ and -1 . Hence, in light of the linear system theory, the error dynamical system (5) is asymptotically stable at the origin with the controller (6). Consequently, the synchronization of the drive system (3) and the response system (4) is achieved.

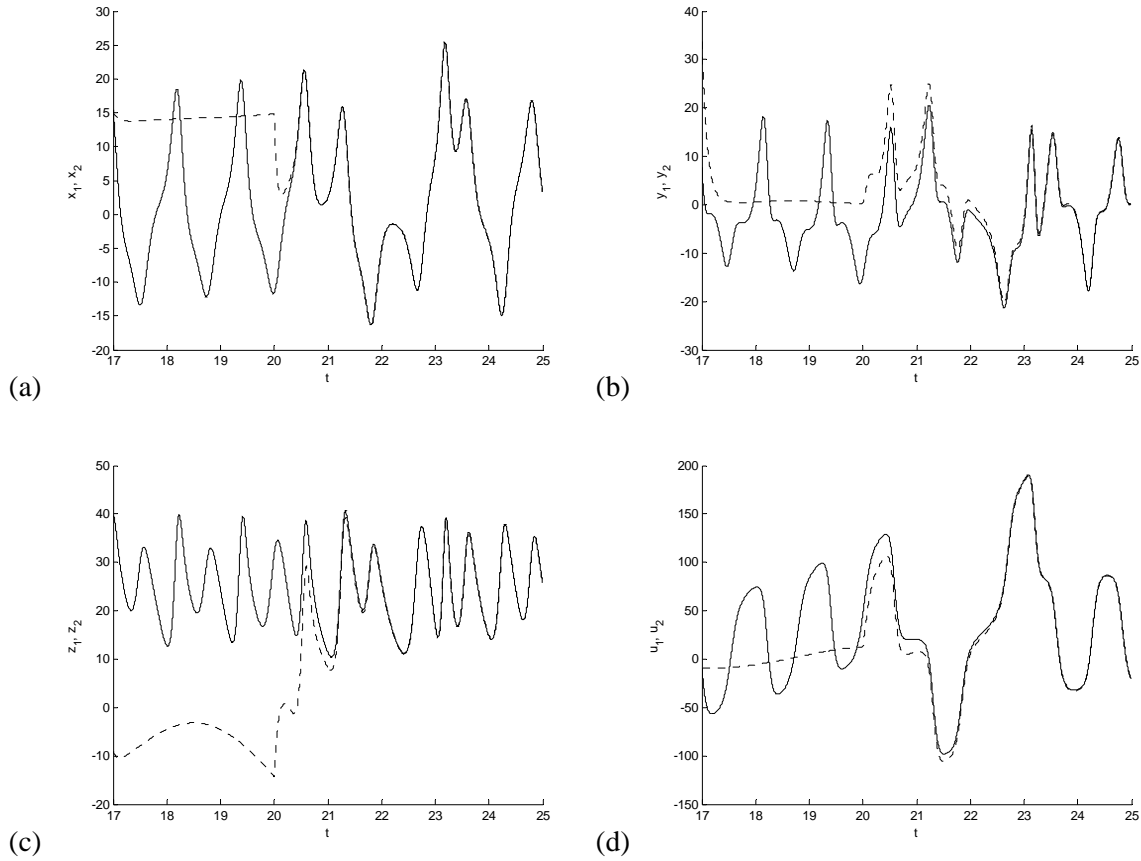


Figure 3: time responses of system (3) (the solid line) and controlled system (4) (the dotted line).

(a) $x_1 - x_2$, (b) $y_1 - y_2$, (c) $z_1 - z_2$ and (d) $u_1 - u_2$. The controller is activated at $t = 20$.

Numerical simulations are performed to verify the efficiency and feasibility of the controller (6). The Runge-Kutta method is employed with the time step $\Delta t = 0.001$. Let the initial conditions of the hyperchaotic Lorenz system and the hyperchaotic Rössler system be $(-2, 8, 2, -8)$ and $(14, 2, -17, 7)$, respectively. The controller is activated at $t = 20$. From fig.3, we can see that the synchronization between the two hyperchaotic systems is achieved immediately after the controller is activated.

4. Synchronization between two hyperchaotic systems with unknown parameters

In general, it is different even impossible to obtain the system parameters. An important and necessary work is how to realize the synchronization of the hyperchaotic system with unknown parameters.

In this section, adaptive control theory is applied to achieve the synchronization of two hyperchaotic systems with seven unknown parameters.

Similarly, choose the hyperchaotic Lorenz system as the drive system

$$\begin{cases} \dot{x}_1 = -a(x_1 - y_1) + u_1 \\ \dot{y}_1 = -x_1 z_1 + c x_1 - y_1 \\ \dot{z}_1 = x_1 y_1 - b z_1 \\ \dot{u}_1 = -x_1 z_1 + d u_1 \end{cases} \quad (8)$$

in which a, b, c and d are uncertain parameters.

And let the hyperchaotic Rössler system be the response one

$$\begin{cases} \dot{x}_2 = fy_2 + gx_2 + v_1 \\ \dot{y}_2 = 3 + y_2z_2 + v_2 \\ \dot{z}_2 = -y_2 - u_2 + v_3 \\ \dot{u}_2 = x_2 + z_2 + eu_2 + v_4 \end{cases} \quad (9)$$

in which e, f and g are uncertain parameters, v_i ($i = 1, 2, 3, 4$) are active control functions to be designed.

Subtracting the system (8) from the system (9), we obtain the error dynamical system between the drive system (8) and the response system (9) which is described by

$$\begin{cases} \dot{e}_1 = (f - a)y_2 + (g + a)x_2 + ae_2 - ae_1 - u_1 + v_1 \\ \dot{e}_2 = y_2z_2 + x_1z_1 - cx_1 + y_2 - e_2 + v_2 + 3 \\ \dot{e}_3 = -y_2 - u_2 + v_3 - x_1y_1 + bz_2 - be_3 \\ \dot{e}_4 = x_2 + z_2 + (e - d)u_2 + v_4 + x_1z_1 + de_4 \end{cases} \quad (10)$$

where parameters a, b, c, d, e, f and g are unknown constants. Active control functions and parameters estimation update rules will be designed to guarantee that the error dynamical system (10) is asymptotically stable at the origin.

Construct the following Lyapunov candidate function

$$V = 1/2(e_1^2 + e_2^2 + e_3^2 + e_4^2 + \tilde{a}^2 + \tilde{b}^2 + \tilde{c}^2 + \tilde{d}^2 + \tilde{e}^2 + \tilde{f}^2 + \tilde{g}^2)$$

where $\tilde{a} = \hat{a} - a, \tilde{b} = \hat{b} - b, \tilde{c} = \hat{c} - c, \tilde{d} = \hat{d} - d, \tilde{e} = \hat{e} - e, \tilde{f} = \hat{f} - f$ and $\tilde{g} = \hat{g} - g$, and $\hat{a}, \hat{b}, \hat{c}, \hat{d}, \hat{e}, \hat{f}, \hat{g}$ are the estimate values of the parameters a, b, c, d, e, f and g , respectively.

The time derivation of the Lyapunov candidate function is

$$\begin{aligned} \dot{V} &= e_1\dot{e}_1 + e_2\dot{e}_2 + e_3\dot{e}_3 + e_4\dot{e}_4 + \tilde{a}\dot{\tilde{a}} + \tilde{b}\dot{\tilde{b}} + \tilde{c}\dot{\tilde{c}} + \tilde{d}\dot{\tilde{d}} + \tilde{e}\dot{\tilde{e}} + \tilde{f}\dot{\tilde{f}} + \tilde{g}\dot{\tilde{g}} \\ &= [(f - a)y_2 + (g + a)x_2 + ae_2 - ae_1 - u_1 + v_1]e_1 + (y_2z_2 + x_1z_1 - cx_1 + y_2 - e_2 + v_2 + 3)e_2 \\ &= (-y_2 - u_2 + v_3 - x_1y_1 + bz_2 - be_3)e_3 + [x_2 + z_2 + (e - d)u_2 + v_4 + x_1z_1 - de_4]e_4 + \tilde{a}\dot{\tilde{a}} + \tilde{b}\dot{\tilde{b}} \\ &\quad + \tilde{c}\dot{\tilde{c}} + \tilde{d}\dot{\tilde{d}} + \tilde{e}\dot{\tilde{e}} + \tilde{f}\dot{\tilde{f}} + \tilde{g}\dot{\tilde{g}}. \end{aligned}$$

To guarantee the time derivation of the Lyapunov function be negative, we define the following controller v_i ($i = 1, 2, 3, 4$)

$$\begin{cases} v_1 = -(\hat{f} - \hat{a})y_2 - (\hat{g} + \hat{a})x_2 - \hat{a}e_2 + u_1 \\ v_2 = \hat{c}x_1 - y_2z_2 - x_1z_1 - y_2 - 3 - 4e_2 \\ v_3 = y_2 + u_2 + x_1y_1 - \hat{b}z_2 \\ v_4 = (\hat{d} - \hat{e})u_2 - (\hat{d} + 1)e_4 - x_2 - z_2 - x_1z_1 \end{cases} \quad (11)$$

and the parameter estimate update law as follows

$$\begin{cases} \dot{\hat{a}} = x_2e_1 - y_2e_1 + e_1e_2 - \tilde{a} \\ \dot{\hat{b}} = z_2e_3 - \tilde{b} \\ \dot{\hat{c}} = -x_1e_2 - \tilde{c} \\ \dot{\hat{d}} = -u_2e_4 + e_4^2 - \tilde{d} \\ \dot{\hat{e}} = u_2e_4 - \tilde{e} \\ \dot{\hat{f}} = y_2e_1 - \tilde{f} \\ \dot{\hat{g}} = x_2e_1 - \tilde{g} \end{cases} \quad (12)$$

With the control rule (11) and the update law (12), the time derivation of the Lyapunov function V becomes

$$\dot{V} = -ae_1^2 - 5e_2^2 - be_3^2 - e_4^2 - \tilde{a}^2 - \tilde{b}^2 - \tilde{c}^2 - \tilde{d}^2 - \tilde{e}^2 - \tilde{f}^2 - \tilde{g}^2 < 0$$

Thus, in terms of the Lyapunov stability theory, the control rule (11) and the update law (12) can stabilize the error dynamical system (10) to the origin asymptotically. Hence, synchronization between two hyperchaotic systems (8) and (9) with uncertain parameters is achieved.

To illustrate the control effect of the controller (11) and the parameter update law (12), numerical simulation of synchronization between the two different hyperchaotic systems with unknown parameters is given.

Choose parameters $a=10$; $b=8/3$; $c=28$; $d=1.3$; $e=0.25$; $f=-0.5$ and $g=0.05$, the two systems are hyperchaotic. Runge-Kutta method of order 4 is employed with time step $\Delta t = 0.001$. The initial conditions of the system (8) and the controlled system (9) are $(-2, 8, 2, -8)$ and $(14, 2, -17, 7)$, respectively. And the initial condition of the parameter update law is $(6, 7, 12, -7, -6, 5, -5)$. From fig.4 and fig.5, we can see that synchronization of the two systems is quickly achieved, and the parameter estimate errors vanish as the time evolves.

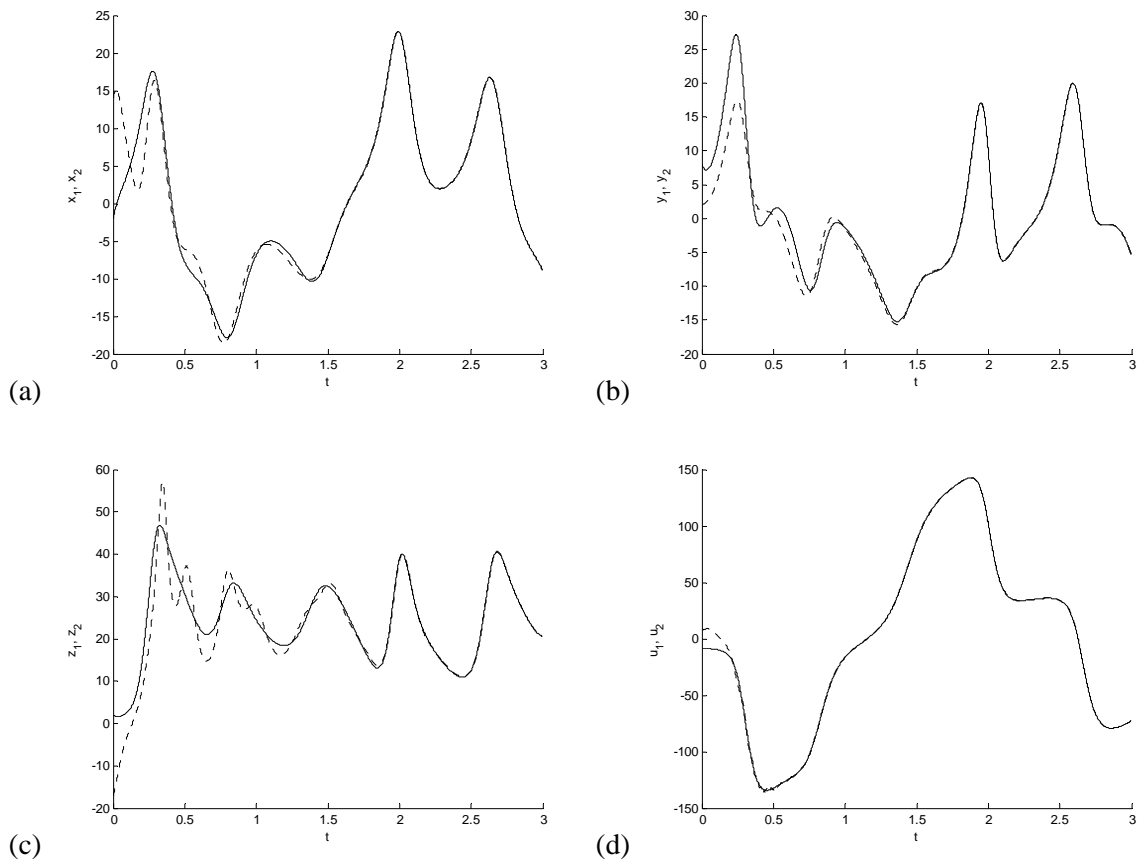


Figure 4: time responses of system (8) (the solid line) and controlled system (9) (the dotted line).

(a) $x_1 - x_2$, (b) $y_1 - y_2$, (c) $z_1 - z_2$ and (d) $u_1 - u_2$.

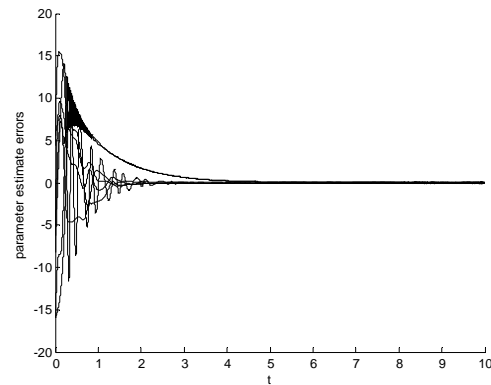


Figure 5: time evolutions of the seven parameter estimation errors.

5. Conclusion

In this paper, issues of synchronization between two different hyperchaotic systems are studied. The hyperchaotic Rössler system can be synchronous asymptotically with the hyperchaotic Lorenz system using an effective active controller. Based on the Lyapunov stability theory and adaptive control theory, synchronization of the two hyperchaotic system with unknown parameters is also realized. The feasibility and effectiveness of the controllers is illustrated and verified via the simulation given in the context.

6. References

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