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Projective synchronization of fractional order hyperchaotic systems based on matrix decomposing

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Abstract: Dynamic behavior of the fractional order hyperchaotic Chen system is discussed. According to linear system stability judgment method, general projective synchronization method of hyperchaotic system with fractional order is introduced. By constructing constant full rank matrix, via designing response system, projective synchronization between it and the corresponding drive system can be achieved along with sufficient conditions being obtained. The proposed scheme is simple and easy to be implemented. To verify the effectiveness of the addressed method, numerical simulations are demonstrated.

Keywords: fractional order; projective synchronization; hyperchaotic system; matrix composing

1. Introduction

After Carroll and Pecora put forward the conception of chaotic synchronization [1], people have carried on much research on chaotic synchronization. Various synchronization methods have been proposed. For example, generalized synchronization [2,3], complete synchronization [4,5], lag synchronization [6,7], Q-S synchronization [8], phase synchronization [9,10], prediction and lag synchronization [11,12]. In 1999, Mainieri et al. observed a novel type of chaos synchronization, namely projective synchronization [13]. Since then, projective synchronization has attracted people's attention. Examples are given as follows. In Ref. [14], a scheme about secure communication on the basis of projective synchronization was mentioned. Projective synchronization relative to complicated factors was discussed in Ref. [15].

Recently, the research on fractional calculus has attracted great attention. So far, there have been many fractional order synchronization methods. Here are some examples. Fractional order synchronization conditions for two Lü systems were calculated [16]; By using linear control strategy, synchronization for fractional systems with time delayed was proposed [17]; Active control method was presented to realize fractional chaotic synchronization [18]; Qin et al. introduced an adaptive fuzzy controller and realized the synchronization of uncertain systems, which were fractional order systems with time delay [19]. Control and synchronization for unknown chaotic systems was proposed by utilizing adaptive back stepping tactics, in which fractional order system was used [20]. A nonlinear disturbance observer was explored on the basis of adaptive sliding mode control scheme [21]. Existing results suggested that fractional order nonlinear system can show richer dynamics and reflect more systematic engineering physics phenomena compared with integer order system. Therefore, it has wider range of applications.

In the last few years, various results regarding fractional order projective synchronization have been considered. Lag projective synchronization was obtained by utilizing compared principle for linear fractional order equation with time delayed [22]. Modified projective synchronization was described for chaotic systems in the presence of different dimensions [23]. Combination projective synchronization was discussed, which was divided into matrix form and inverse matrix form. When scaling factor was a constant full rank matrix, these two synchronization methods can be realized [24]. A robust control method for two different chaotic systems with external disturbances was designed to realize modified function projective

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synchronization [25]. Nowadays, Chaos synchronization has been used in many fields, such as image encryption [26-28], parameter estimation [29], secure communications [30,31], electrical circuits systems [32], physics and engineering sciences [33-35].

Other parts of this manuscript are arranged as follows. Definitions and predictor-corrector algorithm are introduced in section 2. In section 3, main results are depicted. Firstly, general projective synchronization scheme for hyperchaotic systems is presented. Secondly, dynamic behavior about fractional hyperchaotic Chen system is discussed. Thirdly, the proposed scheme was verified through numerical simulations. Conclusions are depicted in section 4.

2. Definitions and algorithm

In this part, definitions and algorithm are introduced.

2.1 Definitions

According to different research backgrounds, three definitions about fractional order derivative are introduced. They are Riemann-Liouville, Caputo and Grünwald-Letnikov definitions.

Definition 1([36]) Riemann-Liouville definition of function $f(t)$ is described as

$$\frac{d^\alpha f(t)}{dt^\alpha}=\frac{1}{\Gamma(m-\alpha)}\frac{d^m}{dt^m}\int_0^t\frac{f(\tau)}{(t-\tau)^{\alpha-m+1}}d\tau, \tag{1}$$

Where α is fractional number that satisfies $m-1<\alpha<m$ with $m\in N$ and $\Gamma(\cdot)$is the Gamma function.

Definition 2 ([36]) Caputo definition of function $f(t)$ is described as

$$\frac{d^\alpha f(t)}{dt^\alpha}=\frac{1}{\Gamma(m-\alpha)}\int_0^t\frac{f^m(\tau)}{(t-\tau)^{\alpha-m+1}}d\tau, \tag{2}$$

where α is fractional number satisfying $m-1<\alpha<m$ with $m\in N$. $\Gamma(\cdot)$is the Gamma function.

Definition 3([36]) Grünwald-Letnikov definition of function $f(t)$ is described as

$$\frac{d^\alpha f(t)}{dt^\alpha}=\lim_{h\to 0}\frac{1}{h^\alpha}\sum_{j=0}^{k}(-1)^j\binom{\alpha}{j}f(kh-jh), t=kh, \tag{3}$$

Where

$$\binom{\alpha}{0}=1,\binom{\alpha}{j}=\frac{\alpha(\alpha-1)\cdots(\alpha-j+1)}{j!}, j\geq 1. \tag{4}$$

Formula (4) can be rewritten as

$$\binom{\alpha}{j}=\frac{\Gamma(\alpha+1)}{\Gamma(j+1)\Gamma(\alpha-j+1)}, \tag{5}$$

Aforementioned definitions about fractional order derivatives have their own features. The Riemann-Liouville definition has good mathematical properties, but it is subject to many restrictions in engineering applications. Although the definition of Caputo has a clearer physical meaning in engineering applications, it is more difficult to perform discrete calculations on fractional calculus. The Grünwald-Letnikov definition is easy to discretize and is convenient for numerical operations. For some functions, above three different forms of fractional order derivative definitions are equivalent and can be mutually used. In this manuscript, the Caputo definition is utilized.

2.2 Algorithm

Predictor-corrector algorithm is a typical method of solving fractional differential equations and tonltiterm equations[37]. Differential equation can be discretized based on the following algorithm to get numerical solution.

Consider the following equation

$$D_t^q y(t)=f\big(t,y(t)\big), 0\leq t\leq T, \tag{6}$$

with initial value $y^{(k)}(0)=y_0^{(k)}, k=0,1\cdots,m-1, m=[q]$, corresponding Volterra integral form is

$$y(t)=\sum_{k=0}^{[q]-1}y_0^{(k)}\frac{t^k}{k!}+\frac{1}{\Gamma(q)}\int_0^t(t-s)^{q-1}f\big(s,y(s)\big)ds. \tag{7}$$

Let $h=\frac{T}{N}, t_n=nh, n=0,1\cdots,N\in Z^+$ performing Adams-Bashforth estimation on formula (7), we can obtain the estimation formula as

$$y_h(t_{n+1})=\sum_{k=0}^{[q]-1}y_0^{(k)}\frac{t_{n+1}^k}{k!}+\frac{h^q}{\Gamma(q+2)}f\left(t_{n+1},y_h^p(t_{n+1})\right)+\frac{h^q}{\Gamma(q+2)}a_{j,n+1}f\left(t_j,y_h(t_j)\right), \tag{8}$$

Where

$$a_{j,n+1}=\begin{cases}n^{q+1}-(n-q)(n+1)^{q}, j=0,\\(n-j+2)^{q+1}+(n-j)^{q+1}-2(n-j+1)^{q+1}, 1\le j\le n,\\1, j=n+1\end{cases}\tag{9}$$

$$y_h^p(t_{n+1})=\sum_{k=0}^{[q]-1}y_0^{(k)}\frac{t_{n+1}^k}{k!}+\frac{1}{\Gamma(q)}\sum_{j=0}^{n}b_{j,n+1}f\left(t_j,y_h(t_j)\right),\tag{10}$$

$$\beta_{j,n+1}=\frac{h^q}{q}((n+1-j)^q-(n-j)^q).\tag{11}$$

Suppose $q>0$ satisfying $D_t^q y(t)\in C^2$, and there is error estimate.

$$\max_{0\le j\le N}|y(t_j)-y_h(t_j)|=0(h^q),\tag{12}$$

where $p=min(2,1+q)$.

3 Main results

3.1 Projective synchronization scheme

Theorem 1 For fractional order hyperchaotic system, we can construct corresponding response system according to the given scaling factor, and make the two systems obtain projective synchronization.

Proof. To prove the result of Theorem 1, two steps are taken as following. Firstly, the linear part is separated from fractional order hyperchaotic system. Secondly, constant full rank matrix B is constructed and the real part of all characteristic roots of matrix B is required to be negative.

Consider fractional order hyperchaotic system

$$\frac{d^q x}{dt^q}=f\left(x(t)\right),\tag{13}$$

where $x=(x_1,x_2,\dots,x_n)^T\in R^n$ is state vector， and $f: R^n\to R^n$ is non-linear vector function.

We can decompose function $f\left(x(t)\right)$ as

$$f\left(x(t)\right)=Ax(t)+h(x(t)).\tag{14}$$

Where $Ax(t)$ is linear part, and $h\left(x(t)\right)$ is non-linear part.

Furthermore, it is well known that $Ax(t)$ can be depicted as the plus of a matrix of constant full rank B with all its eigenvalues being negative and another matrix C, namely, $Ax(t)=Bx(t)+Cx(t)$. Accordingly, it can be obtained that

$$f\left(x(t)\right)=Bx(t)+Cx(t)+h(x(t)).\tag{15}$$

Let

$$H\left(x(t)\right)=Cx(t)+h(x(t)),\tag{16}$$

then system (13) is written as

$$\frac{d^q x}{dt^q}=Bx(t)+H\left(x(t)\right).\tag{17}$$

To gain the projective synchronization, system (13) is regarded as drive system and the corresponding response system is expressed as

$$\frac{d^q y}{dt^q}=By(t)+\frac{H\left(x(t)\right)}{\alpha},\tag{18}$$

where $y=(y_1,y_2,\cdots,y_n)^T\in R^n$ is state vector, α is specified projective factor.

Define synchronization error as

$$e(t)=x(t)-ay(t),\tag{19}$$

and then error system is obtained as

$$\frac{d^q e(t)}{dt^q}=\frac{d^q x(t)}{dt^q}-\alpha\frac{d^q y(t)}{dt^q}=Bx(t)-\alpha By(t)=B\left(x(t)-ay(t)\right)=Be(t).\tag{20}$$

All eigenvalues of matrix B can be calculated. Since its real parts are non-positive, system (20) gradually stabilizes to zero on the basis of stability judgment theory of linear system. That is, the state vector $x(t)$ of system (13) and the state vector $y(t)$ of system (18) can achieve projection synchronization.

3.2 System description

In this part, to verify main result in section 3.1, we take fractional hyperchaotic Chen system as an example, which can be given as

$$\begin{cases} \frac{d^q x_1}{dt^q} = a(x_2 - x_1) + x_4, \\ \frac{d^q x_2}{dt^q} = dx_1 - x_1 x_3 + cx_2, \\ \frac{d^q x_3}{dt^q} = x_1 x_2 - bx_3, \\ \frac{d^q x_4}{dt^q} = x_2 x_3 + rx_4. \end{cases} \tag{21}$$

where $(x_1, x_2, x_3, x_4) \in R^4$ and $a, b, c, d, r \in R^1$ are system parameters, q is fractional order. Predictor-corrector algorithm is adapted in the simulations. We choose system parameters as $q = 0.95, r = 0.5, a = 35, b = 3, c = 12$ and $d = 7$, with which system (21) exhibits hyperchaotic behavior (see Fig.1 and Fig.2).

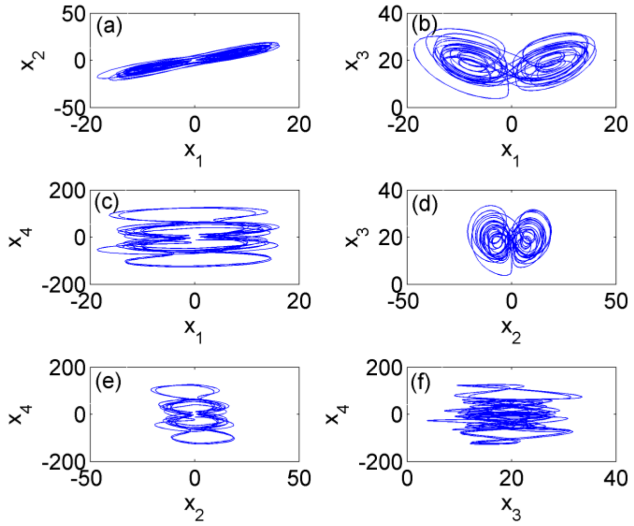

Fig.1 Chaotic attractors of system (21) in 2D space. (a) $x_1 - x_2$ plane; (b) $x_1 - x_3$ plane; (c) $x_1 - x_4$ plane; (d) $x_2 - x_3$ plane; (e) $x_2 - x_4$ plane; (f) $x_3 - x_4$ plane.

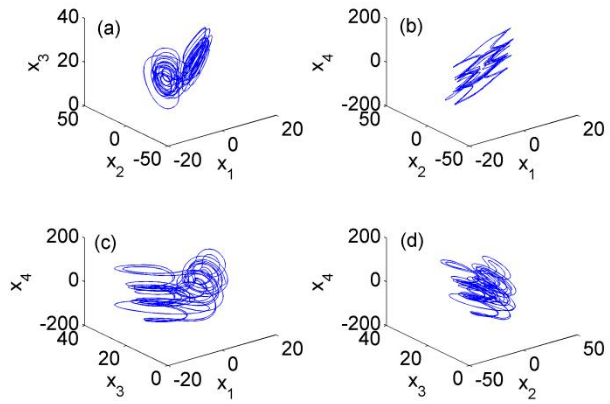

Fig.2 Chaotic attractors of system (21) in 3D space. (a) $x_1 - x_2 - x_3$ space; (b) $x_1 - x_2 - x_4$ space; (c) $x_1 - x_3 - x_4$ space; (d) $x_2 - x_3 - x_4$ space.

Fig.1 shows two-dimensional phase diagram of system (21) on different plane, and Fig.2 depicts attractors of system (21) in three-dimensional space. From these two figures, we can see that fractional order hyperchaotic Chen system is hyperchaotic.

3.3 Implementation of projective synchronization scheme

In this section, a constant full-rank matrix B and response system is constructed according to system (21) for given scaling factor.

Firstly, system (21) is decomposed into two parts $Ax(t)$ and $h(x(t))$ with

$$Ax(t)=\begin{pmatrix}-a & a & 0 & 1\\ d & c & 0 & 0\\ 0 & 0 & -b & 0\\ 0 & 0 & 0 & r\end{pmatrix}\begin{pmatrix}x_1\\ x_2\\ x_3\\ x_4\end{pmatrix}, \quad (22)$$

and

$$h(x(t))=\begin{pmatrix}0\\ -x_1x_3\\ x_1x_2\\ x_2x_3\end{pmatrix}. \quad (23)$$

Secondly, $Ax(t)$ can be decomposed into $Bx(t)$ and $Cx(t)$ with

$$Bx(t)=\begin{pmatrix}-a & a & 0 & 1\\ d & u & 0 & 0\\ 0 & 0 & -b & 0\\ 0 & 0 & 0 & v\end{pmatrix}\begin{pmatrix}x_1\\ x_2\\ x_3\\ x_4\end{pmatrix}, \quad (24)$$

$$Cx(t)=\begin{pmatrix}0 & 0 & 0 & 0\\ 0 & c-u & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & r-v\end{pmatrix}\begin{pmatrix}x_1\\ x_2\\ x_3\\ x_4\end{pmatrix}, \quad (25)$$

Let $H(x(t))=Cx(t)+h(x(t))$, then

$$H(x(t))=\begin{pmatrix}0 & 0 & 0 & 0\\ 0 & c-u & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & r-v\end{pmatrix}\begin{pmatrix}x_1\\ x_2\\ x_3\\ x_4\end{pmatrix}+\begin{pmatrix}0\\ -x_1x_3\\ x_1x_2\\ x_2x_3\end{pmatrix}=\begin{pmatrix}0\\ (c-u)x_2-x_1x_3\\ x_1x_2\\ (r-v)x_4+x_2x_3\end{pmatrix}. \quad (26)$$

Therefore, system (21) is regarded as drive system. Which can be rewritten as

$$Bx(t)+H(x(t))=\begin{pmatrix}-a & a & 0 & 1\\ d & u & 0 & 0\\ 0 & 0 & -b & 0\\ 0 & 0 & 0 & v\end{pmatrix}\begin{pmatrix}x_1\\ x_2\\ x_3\\ x_4\end{pmatrix}+\begin{pmatrix}0\\ (c-u)x_2-x_1x_3\\ x_1x_2\\ (r-v)x_4+x_2x_3\end{pmatrix}. \quad (27)$$

Then response system can be constructed as

$$By(t)+\frac{H(x(t))}{\alpha}=\begin{pmatrix}-a & a & 0 & 1\\ d & u & 0 & 0\\ 0 & 0 & -b & 0\\ 0 & 0 & 0 & v\end{pmatrix}\begin{pmatrix}y_1\\ y_2\\ y_3\\ y_4\end{pmatrix}+\frac{1}{\alpha}\begin{pmatrix}0\\ (c-u)x_2-x_1x_3\\ x_1x_2\\ (r-v)x_4+x_2x_3\end{pmatrix}, \quad (28)$$

which is

$$\begin{cases}\frac{d^qy_1}{dt^q}=a(y_2-y_1)+y_4,\\ \frac{d^qy_2}{dt^q}=dy_1+uy_2+\frac{1}{\alpha}\left((c-u)x_2-x_1x_3\right),\\ \frac{d^qy_3}{dt^q}=-by_3+\frac{1}{\alpha}x_1x_2,\\ \frac{d^qy_4}{dt^q}=vy_4+\frac{1}{\alpha}\left((r-v)x_4+x_2x_3\right).\end{cases} \quad (29)$$

3.4 Numerical simulation

In numerical simulations, system parameters are set to $a=35, b=3, c=12, d=7$ and $r=0.5$. Then the matrix B can be gained as

$$B=\begin{pmatrix}-a & a & 0 & 1\\ d & u & 0 & 0\\ 0 & 0 & -b & 0\\ 0 & 0 & 0 & v\end{pmatrix}=\begin{pmatrix}-35 & 35 & 0 & 1\\ 7 & u & 0 & 0\\ 0 & 0 & -3 & 0\\ 0 & 0 & 0 & v\end{pmatrix}. \quad (30)$$

When we choose $u=-10$ and $v=-2$, characteristic root of matrix B is calculated as $\lambda_1=-42.5312, \lambda_2=-2.4688, \lambda_3=-3$ and $\lambda_4=-2$, which are all negative.

We Set the initial value to $x_1(0)=6, x_2(0)=8, x_3(0)=10$ and $x_4(0)=15$, while those of system (29) are set to $y_1(0)=1, y_2(0)=10, y_3(0)=15$ and $y_4(0)=-5$. For scaling factor $\alpha=2$, attractors of two systems are displayed in Fig.3. It can be shown from Fig.3 that projection synchronization can be reached.

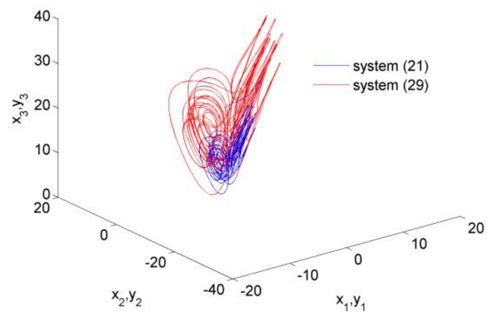

Fig.3 When $u = -10, v = -2$ and $\alpha = 2$, attractors of system (21) and system (29), where blue diagram represents attractor of drive system (21) and red one represents that of response system (29).

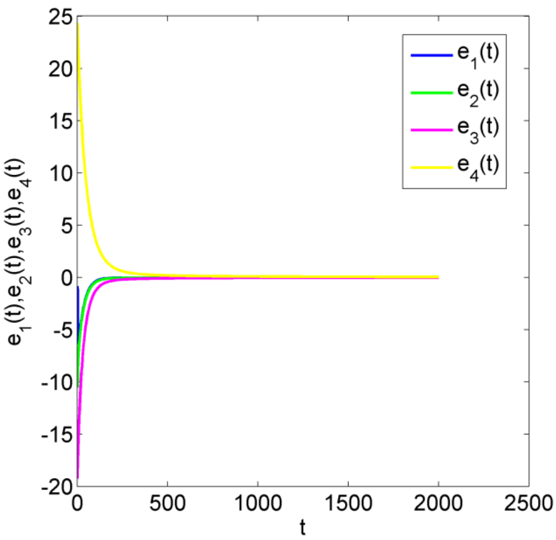

Fig.4 When $u = -10, v = -2$ and $\alpha = 2$, error curves of system (21) and system (29).

When $\alpha = 2$, synchronization error curve of two systems are displayed in Fig.4, from which it can be known that the errors of system (21) and system (29) are asymptotically stable at zero, that is to say, system (21) and system (29) are projective synchronized.

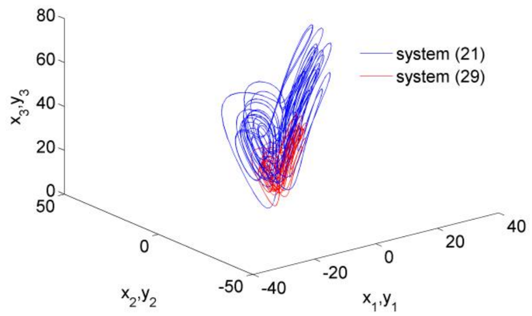

Fig.5 When $u = -10, v = -2$ and $\alpha = 0.5$, attractor diagrams of system (21) and system (29), where blue diagram represents attractor of drive system (21) and red one represents that of response system (29).

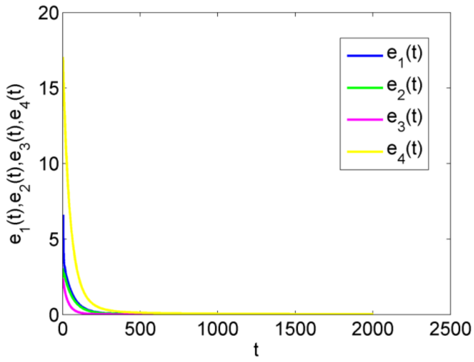

Fig.6 When $u = -10, v = -2$ and $\alpha = 0.5$, error curves of system (21) and system (29).

When $u = -10, v = -2$ and $\alpha = 0.5$, attractors of system (21) and system (29) are shown in Fig.5, and error curves of projection synchronization of system (21) and system (29) are shown in Fig.6.

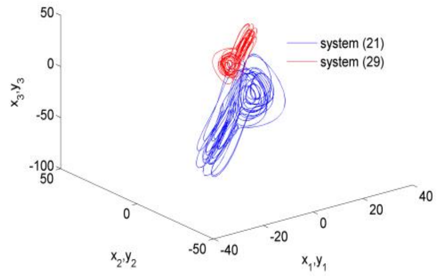

Fig.7 When $u = -10, v = -2$ and $\alpha = -0.5$, attractors of system (21) and system (29), where blue diagram represents attractor of drive system (21) and red one represents that of response system (29).

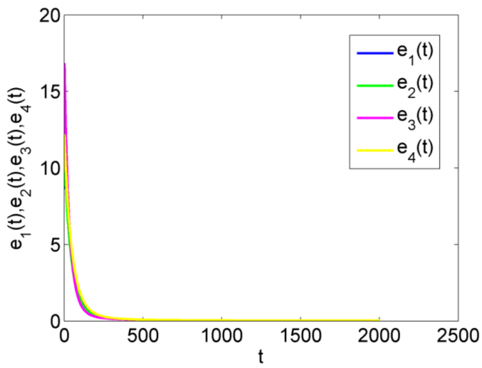

Fig.8 When $u = -10, v = -2$ and $\alpha = -0.5$, error curves of system (21) and system (29).

For scaling factor $\alpha = -0.5$, attractors of system (21) and system (29) are expressed in Fig.7. Projection synchronization error are pictured in Fig.8. Figs.3-8 indicates that, for different scaling factors, projection synchronization between systems (21) and (29) can be obtained.

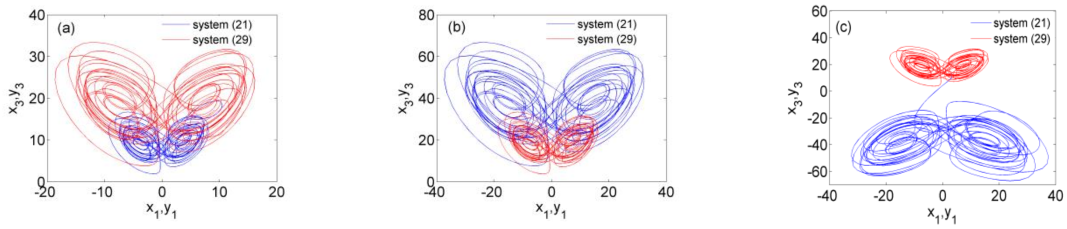

Fig.9 For different values of α, the attractors of system (21) and system (29),

(a) $\alpha = 2$; (b) $\alpha = 0.5$; (c) $\alpha = -0.5$

Fig.9 shows the attractors when system (21) and system (29) reach the projection synchronization for scale factors 2, 0.5, and -0.5, respectively. It can be known from Fig.9 that, for different scale factors, two systems can realize projection synchronization.

4. Conclusions

Dynamic behavior is investigated for fractional order hyperchaotic Chen system, and a general projective synchronization scheme is designed. A response system is designed on the basis of the linear system stability judgment theory. By constructing a constant full-rank matrix, a general projective synchronization scheme is presented. The correctness of the presented scheme is theoretically proved, and then its effectiveness is verified. Results suggest that, when scaling factor α is chosen as different values, such as $\alpha = 2$, $\alpha = 0.5$ and $\alpha = -0.5$, the projection synchronizations for system (21) and the constructed system have been realized.

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