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Finite-time chaos synchronization of the delay hyperchaotic Lü system with disturbance

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Abstract. In this paper, the dynamics and the finite-time synchronization of the delay hyperchaotic Lü system with disturbance are discussed. Based on the finite-time stability theory, a control law is put forward to realize finite-time chaos synchronization of the delay hyperchaotic Lü system with disturbance. Finally, numerical simulation results are provided to demonstrate the effectiveness and robustness of the proposed scheme.

Keywords: Chaos, delay system, disturbance, finite-time synchronization

1. Introduction

Chaos synchronization has attracted due attention of many researchers since the seminal work of Pecora and Carroll [1]. From then on, chaos synchronization has been developed in an extensive and intensive manner due to its potential application in varied fields, like secure communication [2, 3], complex networks [4-7], biotic science [8-13] and so on [14-26].

Nowadays, most of the major findings about chaos control and synchronization are derived based on the asymptotic stability of the chaotic systems. In fact, it is more valuable to control or synchronize chaotic systems as soon as possible. To obtain faster convergence, the finite-time control approach is an effective technique. In addition, the finite-time techniques have been demonstrated to show better robustness and disturbance rejection properties than those of asymptotic methods [27-37]. Therefore, the finite-time chaos control and synchronization have gained a great deal of attention over the past few decades. Mohammad et al. brought in an adaptive control scheme for chaos suppression of non-autonomous chaotic rotational machine systems with fully unknown parameters in finite time [38]. Gao et al. proposed a zero error system algorithm on the basis of automatic control theory and finite-time control principle [39]. Wang et al. employed a nonlinear controller to control chaos in a BLDCM system within the frameworks of the finite-time stability theory and the Lyapunov stability theory [40]. Several finite-time synchronization methods have been put forward in [41–43].

On the other hand, it is difficult to know the external disturbance always occurs in system. Thus, the chaos control and synchronization of chaotic system in the presence of external disturbance are effectively crucial in practical applications.

The present paper intends to present a controller with a view to realizing finite-time synchronization of delay hyperchaotic Lü system with disturbance. The controller is robust and simple to be constructed. Numerical simulations are presented to reveal the effectiveness and robustness of the proposed scheme.

The rest of the paper is organized as follows. Section 2 offers a brief account of the preliminary definitions and lemmas. Section 3 investigates the dynamics of delay hyperchaotic Lü system with disturbance and proposes the finite-time controllers. Simulation results are presented in Section 4 and the conclusion of the whole paper is drawn in Section 5.

2. Preliminary definitions and lemmas

By finite-time synchronization, it is meant that the state of the slave system can track that of the master system after a finite-time.

Definition 1. Consider the following two chaotic systems:

$$\begin{aligned} \dot{x}_t &= f(x_t), \\ \dot{x}_s &= h(x_t, x_s), \end{aligned} \tag{1}$$

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where x_t, x_s are two n -dimensional state vectors. The subscripts 't' and 's' stand for the master and slave systems, respectively. $f: R^n \to R^n$ and $h: R^n \to R^n$ are vector-valued functions. If there exists a constant $T > 0$, such that

$$\lim_{t \to T} \|x_t - x_s\| = 0,$$

and $\|x_t - x_s\| \equiv 0$, if $t \geq T$, then synchronization of the system (1) is achieved in a finite-time.

Lemma 1 [32]. Assume that a continuous, positive-definite function $V(t)$ satisfies differential inequality

$$\dot{V}(t) \leq -cV^{\eta}(t), \forall t \geq t_0, V(t_0) \geq 0, \tag{2}$$

where $c > 0, 0 < \eta < 1$ are constants, then, for any given t_0, $V(t)$ satisfies inequality

$$V^{1-\eta}(t) \leq V^{1-\eta}(t_0) - c(1-\eta)(t - t_0), t_0 \leq t \leq t_1, \tag{3}$$

and

$$V(t) \equiv 0, \forall t \geq t_1,$$

with t_1 given by

$$t_1 = t_0 + \frac{V^{1-\eta}(t_0)}{c(1-\eta)}. \tag{4}$$

Proof. Consider differential equation

$$\dot{X}(t) = -cX^{\eta}(t), X(t_0) = V(t_0), \tag{5}$$

although differential equation (6) does not satisfy the global Lipschitz condition, the unique solution of Eq.(6) can be found as

$$X^{1-\eta}(t) = X^{1-\eta}(t_0) - c(1-\eta)(t - t_0). \tag{6}$$

Therefore, from the comparison Lemma, one obtains

$$V^{1-n}(t) \leq V^{1-n}(t_0) - c(1-\eta)(t - t_0), t_0 \leq t \leq t_1, \tag{7}$$

and

$$V(t) \equiv 0, \forall t \geq t_1.$$

with t_1 given in (5).

Lemma 2 [34]. If $\alpha > \left(\frac{2}{3}\right)^{\frac{2}{3}}$, it can be gotten that

$$\left(\alpha|x_1| + \frac{1}{2}x_2^2\right)^{\frac{3}{2}} + x_1x_2 \geq 0, \tag{8}$$

where x_1 and x_2 are any real numbers .

Corollary 1 [34]. If $\alpha > \left(\frac{2}{3}\right)^{\frac{2}{3}}$, it can be obtained that

$$|x_1x_2| \leq \left(\alpha|x_1| + \frac{1}{2}x_2^2\right)^{\frac{3}{2}}, \tag{9}$$

where x_1 and x_2 are any real numbers .

Lemma 3 [44]. Let $0 < c < 1$. Then for positive real numbers a and b, the following inequality holds

$$(a + b)^c < a^c + b^c. \tag{10}$$

3. Main results

A chaotic system is of tremendous sensitivity to disturbance. In actual situation, the system is disturbed and cannot be exactly predicted. These uncertainties will in turn destroy the synchronization and even break it. Therefore, it is of great importance and necessity to study the synchronization of systems with disturbance. In this section, the dynamic behaviors of the delay hyperchaotic Lü system is to be explored, and the finite-time synchronization of the delay hyperchaotic Lü systems will be discussed as well.

3.1 Dynamics of delay hyperchaotic Lü system with disturbance

Delay hyperchaotic Lü system with disturbance is considered as

$$\begin{aligned} \dot{x}_1 &= a(x_2 - x_1), \\ \dot{x}_2 &= cx_2 - x_1x_3 + x_4(t-\tau) + Ax_2 \sin(\omega t), \\ \dot{x}_3 &= x_1x_2 - bx_3, \\ \dot{x}_4 &= -kx_1 - dx_2. \end{aligned} \tag{11}$$

where a, b, c, τ, k, ω, d,A are real positive constants. In this section, initial conditions of system (11) are chosen as $(-2,4,2,3)$ and the parameters of the system are selected as $a = 35$, $b = 1.3$,$c = 20$, $k = 1$, $d = 1$,$A = 0.01$,$\omega = 0.01$. Figs.1-5 depict the dynamics of system (11) for different values of τ. Fig.1 and Fig.5 indicate that the delay Lü system with disturbance is chaotic for $\tau = 0.3$and $\tau = 1.3612$.Fig.2, Fig.3 and Fig.4 show that the system has periodic solutions for $\tau = 0.4$, $\tau = 0.47$and $\tau = 1.3$. Fig.4 (c) indicates that the amplitude of the system is similar the same, but the amplitude of the system is gradually to zero in Fig.5 (c).

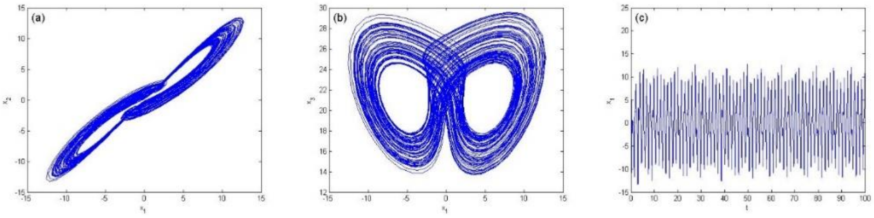

Fig.1. The phase portrait and time series of variables in system (11) for$\tau = 0.3$,
(a) phase portrait of x_1 andx_2, (b) phase portrait of x_1 andx_3, (c) time series ofx_1.

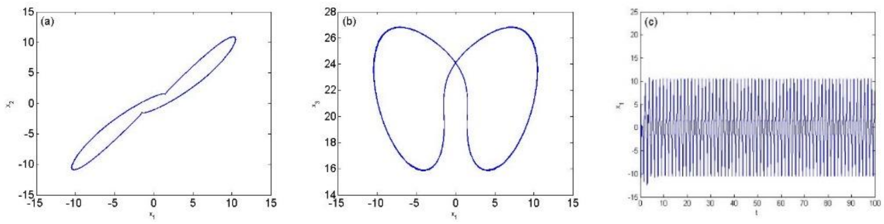

Fig.2. The phase portrait and time series of variables in system (11) for$\tau = 0.4$,
(a) phase portrait of x_1 andx_2, (b) phase portrait of x_1 andx_3, (c) time series ofx_1.

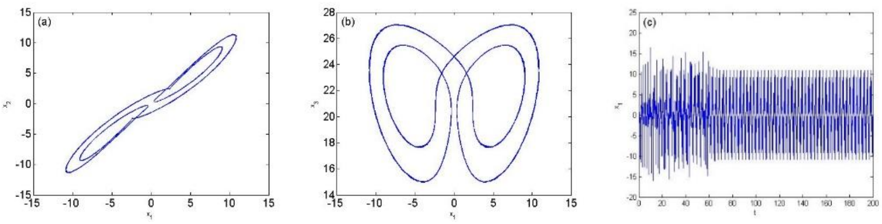

Fig.3. The phase portrait and time series of variables in system (11) for$\tau = 0.47$,
(a) phase portrait of x_1 andx_2, (b) phase portrait of x_1 andx_3, (c) time series ofx_1.

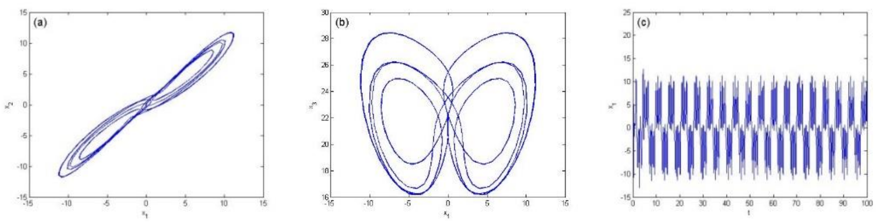

Fig.4. The phase portrait and time series of variables in system (11) for$\tau = 1.3$,
(a) phase portrait of x_1 andx_2, (b) phase portrait of x_1 andx_3, (c) time series ofx_1.

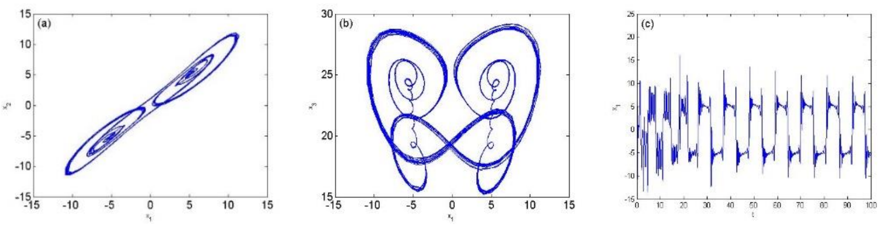

Fig.5. The phase portrait and time series of variables in system (11) for$\tau = 1.3612$,
(a) phase portrait of x_1 andx_2, (b) phase portrait of x_1 andx_3, (c) time series ofx_1.

3.2 Finite synchronization of delay Lü system with disturbance

System (11) is considered as the master system and the slave system is the controlled system as

$$\dot{y}_1 = a(y_2 - y_1),$$

$$\dot{y}_2 = cy_2 - y_1y_3 + y_4(t-\tau) + Ay_2\sin(\omega t) + u_1,$$
$$\dot{y}_3 = y_1y_2 - by_3 + u_2,$$
$$\dot{y}_4 = -ky_1 - dy_2 + u_3. \quad (12)$$

Let $e_1 = y_1 - x_1, e_2 = y_2 - x_2, e_3 = y_3 - x_3, e_4 = y_4 - x_4$ and subtract Eq.(11) from Eq.(12), the error system between systems (11) and (12) can be gotten as

$$\dot{e}_1 = a(e_2 - e_1),$$
$$\dot{e}_2 = ce_2 - y_1e_3 - e_1y_3 + e_1e_3 + e_4(t-\tau) + Ae_2\sin(\omega t) + u_1,$$
$$\dot{e}_3 = y_1e_2 + e_1y_2 - e_1e_2 - be_3 + u_2,$$
$$\dot{e}_4 = -ke_1 - de_2 + u_3. \quad (13)$$

Our aim is to design a controller that can achieve the finite-time synchronization of the delay Lorenz system (11) and the controlled system (12). The problem can be converted to design a controller to attain finite-time stable of the error system (13).

To achieve the finite-time stabilization, the controller is taken as

$$u_1 = -ce_2 + y_1e_3 + e_1y_3 - e_1e_3 - h_1 sign(e_1) - h_2 sign(e_2),$$
$$u_2 = -y_1e_2 - e_1y_2 + e_1e_2 + be_3 - l_1 sign(e_3) - l_2 sign(e_4),$$
$$u_3 = ke_1 + de_2 + m_1e_3... \quad (14)$$

where h_1, h_2, l_1, l_2, m_1 are positive parameters to be designed.

Substitute (14) into (13), we can get the closed-loop plant dynamics

$$\dot{e}_1 = a(e_2 - e_1),$$
$$\dot{e}_2 = e_4(t-\tau) + Ae_2\sin(\omega t) - h_1 sign(e_1) - h_2 sign(e_2),$$
$$\dot{e}_3 = -l_1 sign(e_3) - l_2 sign(e_4),$$
$$\dot{e}_4 = m_1e_3. \quad (15)$$

Choose a candidate Lyaupunov function for the system (15) as

$$V = (\alpha|e_1| + \tfrac{1}{2}{e_2}^2)^{\frac{3}{2}} + e_1e_2 + (\beta|e_4| + \tfrac{1}{2}{e_3}^2)^{\frac{3}{2}} + e_3e_4,$$

then the derivative of Valong the trajectory of (15) can be derived as

$$\dot{V} = \frac{3}{2}(\alpha|e_1| + \frac{1}{2}{e_2}^2)^{\frac{1}{2}}(\alpha sign(e_1)\dot{e}_1 + e_2\dot{e}_2) + \frac{3}{2}(\beta|e_4| + \frac{1}{2}{e_3}^2)^{\frac{1}{2}}(\beta sign(e_4)\dot{e}_4 + e_3\dot{e}_3)$$
$$+\dot{e}_1e_2 + e_1\dot{e}_2 + \dot{e}_3e_4 + e_3\dot{e}_4$$
$$= \tfrac{3}{2}(\alpha|e_1| + \tfrac{1}{2}{e_2}^2)^{\frac{1}{2}}[\alpha sign(e_1)a(e_2 - e_1) + e_2(e_4(t-\tau) + Ae_2\sin(\omega t) - h_1 sign(e_1) -$$
$$h_2 sign(e_2))] + a(e_2 - e_1)e_2 + e_1[e_4(t-\tau) + Ae_2\sin(\omega t) - h_1 sign(e_1) - h_2 sign(e_2)]$$
$$+\tfrac{3}{2}(\beta|e_4| + \tfrac{1}{2}{e_3}^2)^{\frac{1}{2}}[\beta m_1e_3 sign(e_4) + e_3(-l_1 sign(e_3) - l_2 sign(e_4))] + e_4[-l_1 sign(e_3)$$
$$-l_2 sign(e_4)] + m_1{e_3}^2$$
$$\le -\tfrac{3}{2}(\alpha|e_1| + \tfrac{1}{2}{e_2}^2)^{\frac{1}{2}}|e_2|[(h_1 - a\alpha)sign(e_1e_2) - e_4(t-\tau)sign(e_2) - A|e_2|\sin(\omega t) +$$
$$h_2] + a{e_2}^2 - |e_1|[h_1 + h_2 sign(e_1e_2) + ae_2 sign(e_1) - e_4(t-\tau)sign(e_1) -$$
$$Ae_2\sin(\omega t)sign(e_1)] - \tfrac{3}{2}(\beta|e_4| + \tfrac{1}{2}{e_3}^2)^{\frac{1}{2}}|e_3|[l_1 + (l_2 - m_1\beta)sign(e_3e_4)] + m_1{e_3}^2 -$$
$$|e_4|[l_2 + l_1 sign(e_3e_4)].$$

Let $h_1 - a\alpha \le 0, l_2 - m_1\beta \le 0$, $|e_2| \le M$, $|e_4| \le N$, $l_2 > l_1$, $l_1 + l_2 - m_1\beta > 0$, then we have

$$\dot{V} \le -\tfrac{3}{2}(\alpha|e_1| + \tfrac{1}{2}{e_2}^2)^{\frac{1}{2}}|e_2|[a\alpha - h_1 - AM + h_2 - N] + a{e_2}^2 - |e_1|[-AM + h_1 - h_2 - N - aM] -$$

$$\frac{3}{2}\ (\beta|e_4|+\frac{1}{2}{e_3}^2)^{\frac{1}{2}}|e_3|[l_1+l_2-m_1\beta]+m_1{e_3}^2-|e_4|[l_2-l_1].$$

Let

$$v_1=h_1-a\alpha-AM+h_2-N-\frac{2}{3}\sqrt{2}a>0,$$

$$v_2=h_1-h_2-aM-N-AM>0,$$

$$v_3=l_1+l_2-m_1\beta-\frac{2}{3}\sqrt{2}m_1>0,$$

$$v_4=l_2-l_1>0,$$

then we can arrive

$$\dot{V}\le-\frac{3}{2\sqrt{2}}v_1{e_2}^2-v_2|e_1|-\frac{3}{2\sqrt{2}}v_3{e_3}^2-v_4|e_4|\le-p[(\alpha|e_1|+\frac{1}{2}{e_2}^2)^{\frac{3}{2}}]^{\frac{2}{3}}-p((\beta|e_4|+\frac{1}{2}{e_3}^2)^{\frac{3}{2}})^{\frac{2}{3}}, \qquad (16)$$

where $p=min\left\{\frac{v_2}{\alpha},\frac{3v_1}{\sqrt{2}},\frac{v_4}{\beta},\frac{3v_3}{\sqrt{2}}\right\}$.

Based on Corollary 1, we have

$$e_1e_2+(\alpha|e_1|+\frac{1}{2}{e_2}^2)^{\frac{3}{2}}\le2(\alpha|e_1|+\frac{1}{2}{e_2}^2)^{\frac{3}{2}},$$

$$(\beta|e_4|+\frac{1}{2}{e_3}^2)^{\frac{3}{2}}+e_3e_4\le2(\beta|e_4|+\frac{1}{2}{e_3}^2)^{\frac{3}{2}}. \qquad (17)$$

Substituting (16) into (15) leads to the inequation

$$\dot{V}\le-p\frac{1}{2^{\frac{2}{3}}}\{[(\alpha|e_1|+\frac{1}{2}{e_2}^2)^{\frac{3}{2}}+e_1e_2)]^{\frac{2}{3}}+[(\beta|e_4|+\frac{1}{2}{e_3}^2)^{\frac{3}{2}}+e_3e_4)]^{\frac{2}{3}}\},$$

Based on Lemma 3, we can arrive

$$\dot{V}\le-p\frac{1}{2^{\frac{2}{3}}}[(\alpha|e_1|+\frac{1}{2}{e_2}^2)^{\frac{3}{2}}+e_1e_2+(\beta|e_4|+\frac{1}{2}{e_3}^2)^{\frac{3}{2}}+e_3e_4]^{\frac{2}{3}}\ =-\xi V^{\frac{2}{3}}$$

where $\xi=P\frac{1}{2^{\frac{2}{3}}}$.

By solving the above inequality, one gets

$$V(t)\le({V_0}^{\frac{1}{3}}-\frac{\xi t}{3})^3. \qquad (18)$$

Due to $V(t)\ge0$, it follows that $\frac{\xi t}{3}\le{V_0}^{\frac{1}{3}}$, which means that $t\le\frac{3}{\xi}{V_0}^{\frac{1}{3}}$. Therefore, there exists constant $T_1=\frac{3}{\xi}{V_0}^{\frac{1}{3}}$ such that $\lim_{t\to T_1}e_1=\lim_{t\to T_1}e_2=\lim_{t\to T_1}e_3=\lim_{t\to T_1}e_4=0$. From Lemma 1, the error system (15) is finite-time stable. That is to say $e_1\equiv0,e_2\equiv0,e_3\equiv0,e_4\equiv0$ after a finite-time T_1. Therefore, when $t>T_1$, $y_1\equiv x_1$,$y_2\equiv x_2$,$y_3\equiv x_3$,$y_4\equiv x_4$.

4. Simulation results

In this section, initial conditions of the master system and slave system are chosen as$(-2,4,2,3)$and$(-2.2,4.1,2.2,3.1)$, respectively. The system parameters of are taken as$a=35$,$b=1.3$,$c=20$,$k=1$,$d=1$,$A=0.01$,$\omega=0.01$,$h_1=1.7$,$h_2=1.5$,$l_1=1$,$l_2=1.9$,$m_1=1$. Fig.6 shows the dynamical behaviors of error systems of the delay hyperchaotic Lü system for$\tau=0.3$.

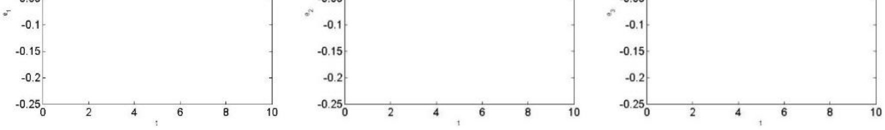

Fig.6. Synchronization errors of the delay hyperchaotic Lü system when$\tau=0.3$.

5.Conclusion

This paper is concerned with finite-time synchronization of the delay hyperchaotic Lü system with disturbance. The dynamics and the finite-time synchronization of the delay hyperchaotic Lü system with disturbance are discussed. Based on the finite-time stability theory, a control law is put forward to realize finite-time chaos synchronization of the delay hyperchaotic Lü system with disturbance. Finally, numerical simulations are given to demonstrate the effectiveness and robustness of the proposed scheme.

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