

Linear retrieval of microwave Land Surface Emissivity in Taklimakan Desert

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ABSTRACT: Land Surface Emissivity is an inherent property of land surface, an important condition for retrieving surface and atmospheric parameters, and a key element in land surface data assimilation system. In order to obtain more accurate and physically meaningful microwave Land Surface Emissivity, the linear retrieval model is constructed for estimating the microwave Land Surface Emissivity in the Taklimakan Desert. Firstly, the function relationship between the microwave Land Surface Emissivity and two factors is deduced by using a Taylor expansion of a multivariate function. Secondly, according to the optimal control theory and the principle of atmospheric radiation transfer, a cost function is established by combining the observational brightness temperature with simulated brightness temperature. At last, the optimal solution is obtained with the Newton iteration method. The result shows that the optimal microwave Land Surface Emissivity improves the simulated value of brightness temperature. In addition, an independent test of the retrieval model in different area demonstrates the effectiveness and feasibility of this proposed model. The optimal control principle and Newton iteration method can be applied to the linear retrieval of LSE.

Keywords: Optimal control theory; Linear retrieval; Newton iteration method; microwave Land Surface Emissivity

1. Introduction

With the development of the Feng-Yun (FY) series satellites and the maturity of the theory and method of Satellite Meteorological remote sensing, the satellite-borne remote sensor plays an irreplaceable role in numerical weather prediction, climate monitoring and prediction, tropical cyclone and so on [1]. The radiation that the remote sensing receives includes the contribution of atmospheric radiation and surface radiation. Surface radiation is mainly affected by the microwave Land Surface Emissivity (LSE), so it is crucial to obtain accurate LSE.

LSE is a variable related to many factors. The LSE of bare soil generally decreases with the increase of surface temperature and soil moisture content, but increases with the increase of surface roughness. In addition, the LSE of bare soil is also affected by detection frequency and polarization mode, with the low-frequency LSE being more sensitive to the change of soil moisture content [2]. The LSE of vegetation-covered land surface is higher than that of bare soil [3], the LSE of vertical polarization increases with the incident angle, while that of horizontal polarization shows obvious seasonal variation [4]. At present, there are two main common algorithms for the calculation of LSE, namely, the statistical method and the inversion calculation method based on the satellite data. The statistical method includes genetic algorithm, Monte Carlo method, simulated annealing method, neural network method, etc. Aires et al. [5] used neural network method to calculate LSE in day time, and found that the convergence speed of the method is very slow, the calculation is complex and it is very difficult to obtain some surface parameters. The inversion calculation method based on the satellite data usually needs lots of input data such as observational brightness temperature, surface temperature, the upward and downward radiation, the atmospheric transmittance, channel frequency, soil humidity, vegetation coverage percentage and so on. Although the physics of the inversion calculation method is clear, there are too many parameters for inputting. Moreover,

the uncertainty of parameters will also affect the precision of derived LSE. Therefore, it is imperative to develop a new retrieval method of LSE.

The optimal control theory and method is about how to find the optimal and the best scheme among many schemes by minimizing a prescribed cost function. Due to the continuous expansion of computer application, the optimization control theory is widely used in various fields, such as economics, engineering, and especially in meteorology in recent years[6-8]. In order to derive a more accurate LSE with physical meaning, and improve the utilization rate of microwave Radiation Imager (MWRI) data on FY-3C satellite, we will take the optimal control theory for the calculation of LSE in Taklimakan Desert area. The linear retrieval model of LSE is constructed by considering the influence of surface temperature and specific humidity, then the formulas of calculating LSE is obtained.

This paper is organized as follows. Preliminaries are briefly described in the next section. The construction of the Linear retrieval model and the numerical result analysis are provided in Sect. 3. The model test is implemented in Sect. 4. Summary and discussion are given in the final section.

2. Preliminaries

In this section, we introduce the research area and the algorithm principle of Community Radiative Transfer Model (CRTM),

2.1. Research area

Desert accounts for 15% of the earth's land area, which, with scarce vegetation, relatively flat surface and small roughness. Due to the special geographical environment in desert area, the LSE retrieval from satellite observations is always different from other surface types in space and time [9-11]. In addition, since its associated frequent sandstorms and other disastrous weather seriously affect the surrounding areas, it is of great meteorological significance to monitor and study LSE in the desert area. In this study, we choose the Taklimakan desert ($37^{\circ}\sim 41^{\circ}\text{N}$, $78^{\circ}\sim 88^{\circ}\text{E}$) for investigation. Figure 1 shows the observational brightness temperature in the Taklimakan Desert on November 3, 2014. The blue box is selected retrieval area ($38^{\circ}\sim 40^{\circ}\text{N}$, $81^{\circ}\sim 85^{\circ}\text{E}$), containing 998 scanning points.

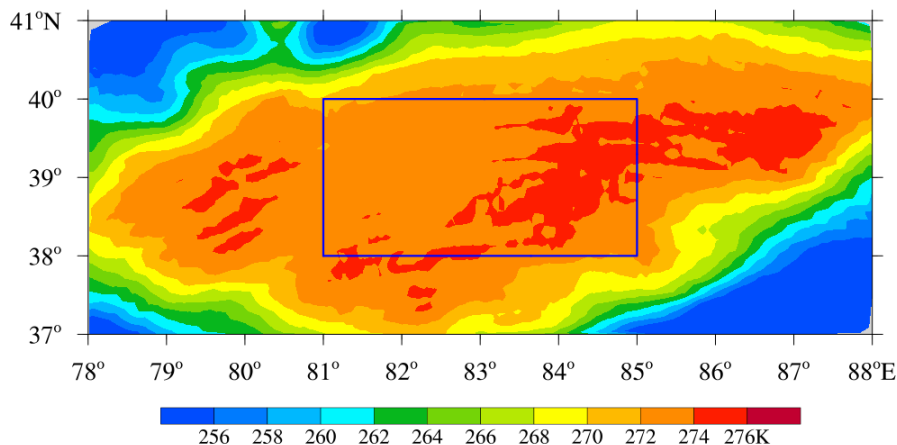


Figure 1. The observational brightness temperature in the Taklimakan Desert on November 3, 2014. (The blue box is retrieval area($38^{\circ}\sim 40^{\circ}\text{N}$, $81^{\circ}\sim 85^{\circ}\text{E}$))

2.2. Model

CRTM is a fast radiation transmission model for satellite visible light, infrared, ultraviolet or microwave channel radiation transmission [12], developed by the United States Satellite Data Assimilation Joint Center. CRTM also computes radiance sensitivities such as the radiance derivatives (Jacobians) with respect to the state variables. We input T639 model forecast data, ERA-Interim reanalysis data, and Satellite (FY-3C, Microwave Radiation Imager (MWRI), FY-3C/MWRI) observation data into CRTM model to obtain the simulated brightness temperature of MWRI. Here we introduce the principle of CRTM simulation of bright temperature.

Assuming a vertically-stratified, plane-parallel and non-polarized atmosphere, the monochromatic radiative transfer equation will be written as

$$\begin{aligned} \mu \frac{dI(\tau; u, \phi)}{d\tau} &= I(\tau; u, \phi) - \frac{\varpi}{4\pi} \int P(\tau; u, \phi; u', \phi') I(\tau; u', \phi') du' d\phi' \\ &\quad - \frac{\varpi}{4\pi} P(\tau; u, \phi; -u_{\otimes}, \phi_{\otimes}) F_{\otimes} e^{-\tau/\mu_{\otimes}} - (1 - \varpi) B(T), \end{aligned} \tag{1}$$

where I is the intensity, ϖ is the single-scattering albedo; τ denotes the optical depth, B denotes the Planck function, and P represents the phase function. The directions of the incoming and outgoing light beams are represented by (μ', ϕ') and (μ, ϕ) , where $\mu' = \cos(\theta')$ and $\mu = \cos(\theta)$, ϕ' and ϕ are the azimuthal angles, while θ' and θ are the zenith angles. In the third term on the right side of the equation (1), F_{\otimes} denotes the solar irradiance incident at the direction $(-\mu_{\otimes}, \phi_{\otimes})$.

When the sky is clear, then $\varpi \approx 0$, that is, the scattering terms in equation (1) are neglected in the microwave and infrared regions. The solution to the monochromatic intensity can then be written as:

$$\begin{aligned} I(\mu) &= \left[r \int_0^{\tau_N} B(T) dT_d(\tau', \mu_d) + r_{\otimes} \frac{F_{\otimes}}{\pi} T_d(0, \mu_{\otimes}) + \varepsilon B(T_s) \right] T_u(\tau_N, \mu) \\ &\quad - \int_0^{\tau_N} B(T) dT_u(\tau', \mu), \end{aligned} \tag{2}$$

among which, r is surface reflectance; τ_N is the atmospheric optical thickness from top to bottom, T_u and T_d are upward and downward transmission respectively, ε is LSE. The radiative transfer equation (2) consists of four terms. The first term on the right side of (2) is the atmospheric downwelling radiation reflected by the Earth's surface. The second term is the surface reflected solar radiation. The third term is the contribution of the surface emission at the surface temperature T_s and the fourth term is the contribution of atmospheric upwelling radiation.

Radiation intensity I is obtained by using the equation (2) in the CRTM model, then we can calculate brightness temperature by the following formula,

$$T_b = \frac{P_1}{B_2 \cdot \ln\left(\frac{P_2+1}{I}\right)} - \frac{B_1}{B_2}, \tag{3}$$

where P_1, P_2, B_1, B_2 represent Planck constant, T_b is simulated brightness temperature. According to the optimal control theory [13]. We can see that LSE calculation module (ε) is a very important part.

3. Linear retrieval model

In this section, the linear function relation of LSE is deduced, and a cost function is constructed by using the simulated and observational brightness temperature, then the function relationship is obtained with Newton's iteration method. Finally, we analyze the numerical results.

3.1. Derivation of LSE

According to the previous analysis, the LSE is a function of many factors, that is

$$\varepsilon = g(T_s, Q_s, \lambda, \alpha, \theta, \dots), \tag{4}$$

where $T_s, Q_s, \lambda, \alpha$, and θ stand for surface temperature, surface humidity, wavelength, surface state and observation angle, respectively. In view of the simplicity of the surface in desert area and the conical scanning characteristics of FY3C/MWRI, and there is also a significant correlation between LSE and surface temperature and humidity in desert areas, so LSE can be regarded as a function of surface temperature and humidity here, that is

$$\varepsilon_{desert} = f(T_s, Q_s), \tag{5}$$

where T_s represents the surface temperature, Q_s is the surface humidity, respectively. Because the exact function relationship between LSE and them is unclear, the function (5) is expanded as follows according to Taylor formula for multivariate functions [13],

$$\begin{aligned} f(T_s, Q_s) &= f(0,0) + f_{T_s}'(0,0)T_s + f_{Q_s}'(0,0)Q_s + \frac{1}{2!} f_{T_s Q_s}''(0,0)T_s Q_s \\ &\quad + \frac{1}{2!} f_{T_s}''(0,0)T_s^2 + \frac{1}{2!} f_{Q_s}''(0,0)Q_s^2 + \frac{1}{2!} f_{Q_s T_s}''(0,0)Q_s T_s + \dots + o^n \end{aligned} \tag{6}$$

When we take a linear approximation, the linear function of LSE about surface temperature humidity is obtained, that is

$$\varepsilon_{lin} = a_{lin} T_s + b_{lin} Q_s + c_{lin} \tag{7}$$

ε_{lin} is the LSE under the linear function relationship; $a_{lin}, b_{lin}, c_{lin}$ and is the undetermined coefficients in the linear function relationship respectively.

3.2. Cost function

In order to obtain the constant in the linear function relationship, a cost function, which combines the simulated brightness temperature obtain by CRTM model and its observational counterpart of the 998 scanning points from FY3C/MWRI , may be formulated in the following way :

$$J(X) = \|T_b(X) - T_o\|_2^2, \quad (8)$$

In the linear retrieval model, $X = [a_{lin}, b_{lin}, c_{lin}]^T$; $T_b(X)$ is the simulated brightness temperature with respect to X by CRTM model, T_o represents the observational brightness temperature from FY3C/MWRI.

3.3. Algorithm

The coefficients in the function (7) can be derived by minimizing the cost function (8) [14]. The minimization algorithm used here is Newton iteration method, which is a method used to find the minimum of the cost function by means of partial derivative information [15]. Taking the first three terms of Taylor's expansion, the function (8) is as follows:

$$J(X) = J(X_k) + \left(\frac{\partial J(X_k)}{\partial X_k}\right)^T (X - X_k) + \frac{1}{2}(X - X_k)^T \left(\frac{\partial^2 J(X_k)}{\partial X_k^2}\right) (X - X_k). \quad (9)$$

Let X_k be the minimum value, minimizing the right side of the equation (9) ,

$$\frac{\partial J(X_k)}{\partial X_k} + \left(\frac{\partial^2 J(X_k)}{\partial X_k^2}\right) X - \left(\frac{\partial^2 J(X_k)}{\partial X_k^2}\right) X_k = 0, \quad (10)$$

then we obtain the iterative formula:

$$X_{k+1} = X_k - \left(\frac{\partial^2 J(X_k)}{\partial X_k^2}\right)^{-1} \cdot \frac{\partial J(X_k)}{\partial X_k}, \quad (11)$$

among them

$$\begin{aligned} \frac{\partial J(X_k)}{\partial X_k} &= \nabla J(X_k)^T J(X_k) = 2 \sum_{i=1}^{998} \nabla(T_b^i(X_k) - T_o^i) (T_b^i(X_k) - T_o^i), \\ \frac{\partial^2 J(X_k)}{\partial X_k^2} &= 2 \sum_{i=1}^{998} \nabla(T_b^i(X_k) - T_o^i) \nabla(T_b^i(X_k) - T_o^i) \\ &\quad + \sum_{i=1}^{998} (T_b^i(X_k) - T_o^i) \nabla^2(T_b^i(X_k) - T_o^i). \end{aligned} \quad (12)$$

The main problem of Newton iteration method is that the second-order partial derivatives of Heisen matrix are usually difficult to calculate in the process of iteration. To simplify the calculation and make the algorithm more effective, a new iterative formula is obtained by ignoring the second-order partial derivatives[16]:

$$X_{k+1} = X_k - (\nabla J(X_k)^T \nabla J(X_k))^{-1} \nabla J(X_k) J(X_k), \quad (14)$$

the second part on the right of the iterative formula (14) is also called the direction of iteration (d_k). However, there isn't step-length factor in the iterative formula (14). As a result, the value of the cost function sometimes increases rather than decreases. Thus, it is necessary for each iteration to introduce a step-length factor to limit the decline in this direction. Based on the Armijo-Goldstein criterion [17], the step-length in one-dimensional linear search can be written as:

$$\lambda_k = \arg \min_{\lambda \in R} J(X_k + \lambda d_k). \quad (15)$$

The iteration will stop until the gradient of the cost function satisfies a certain precision ($\|\nabla J(X)\| < 10^{-4}$), with the optimal solution achieved in the last step. After this, the coefficients ($a_{lin}, b_{lin}, c_{lin}$) in the function can be obtained. Then, the function relationship of the LSE associated with the surface temperature and humidity can be determined.

3.4. Numerical results

998 scanning points in the Taklimakan desert are chosen for the retrieval experiment. For the cost function of the linear retrieval model, we use Newton iteration method to find its minimum value. Figure 2 shows the evolution of the cost function with iteration number. It can be seen that the cost function of the linear retrieval model tends to be stable when the iteration is 12 times, according to the accuracy requirement, the iteration value of thirtieth times is taken as the optimal value of the cost function. At this time, the cost function decreases from 14784.162K2 to 1010.996K2, and the average error of each brightening temperature is about 1.007K.

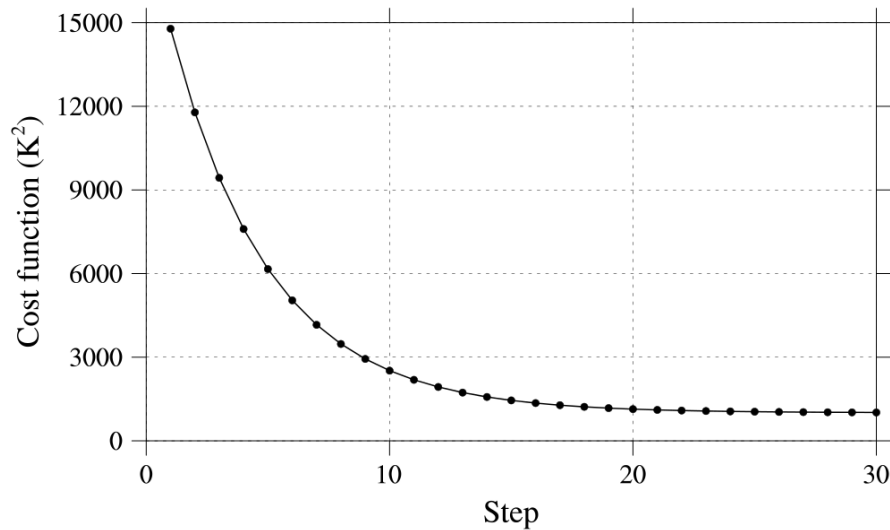


Figure 2. The evolution of the cost function of the linear retrieval model with the number of iterations

The LSE function relationship obtained from the linear retrieval model is:

$$\epsilon_{lin} = -0.00287T_s - 0.41238Q_s + 1.06526 \tag{16}$$

From the function (16), it is obvious that the LSE in the desert area decreases with the increase of surface temperature and humidity.

In order to compare the old model with the linear retrieval model more intuitively, the calculation results under the two models are given in Table 1. From the range of the deviations, both models have greatly improved the original results. It can be seen that the LSE obtained by the retrieval model not only improves the simulation accuracy of brightness temperature, but also improves the trend of brightness temperature simulation.

Table1. Result comparison

Model	Old	Linear
Average error (K)	3.938	1.008
Standard deviation of deviation(K)	0.911	1.006
Percentage improvement(%)	/	74.403%

To further verify the reasonability of the LSE retrieval model, we use the LSE obtained by the linear retrieval model to simulate brightness temperature with CRTM model, which is called as T_{lin} . T_{old} is the simulated brightness temperature by using the old LSE provided by CRTM model. T_{lin} and T_{old} are compared with the observational brightness temperature (T_{obs}). Figure 3 is the sequence diagram of T_{obs} , T_{old} and T_{lin} . It can be seen that the change range of the first 500 scanning points is very small, which makes T_{old} more difficult to reflect the surface information accurately and T_{old} is obviously lower and less dispersed than T_{obs} , that may be due to the low calculation value of the original surface emissivity in CRTM model. On the contrary, T_{lin} is closer to T_{obs} in the numerical value, besides, the trend change of T_{lin} is more consistent with T_{obs} than that of T_{old} .

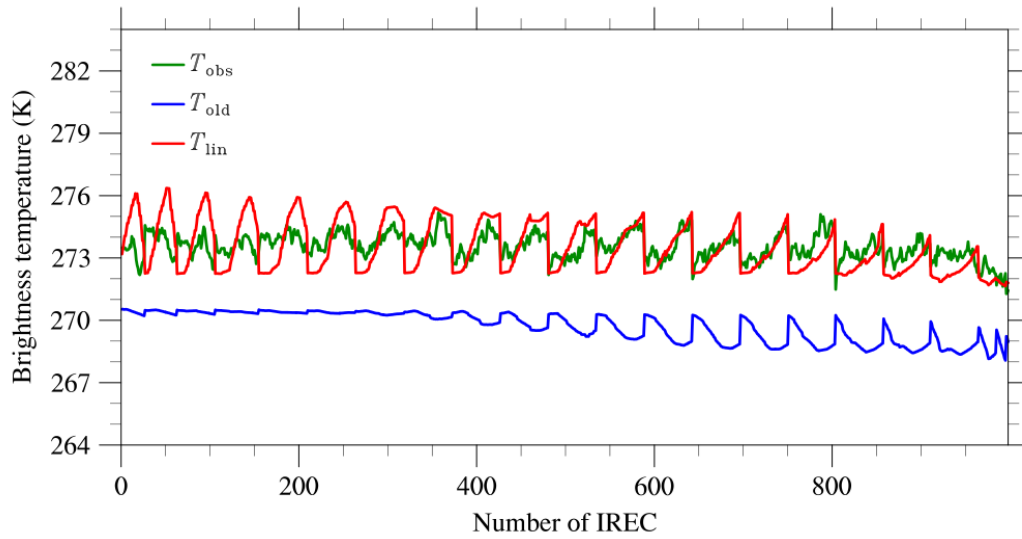


Figure 3. The Sequence diagram of brightness temperature in the retrieval area (T_{obs} : FY3C/MWRI observational brightness temperature; T_{old} : Simulated brightness temperature with the old LSE; T_{lin} : simulated brightness temperature with the LSE obtained by the linear retrieval model.)

Figure 4 demonstrates a statistical histogram of the deviation of T_{lin} and T_{old} from T_{obs} . The abscissa represents the deviation interval while the ordinate represents the number of deviations within the interval. It can be seen that the deviations between T_{obs} and T_{old} are biased positive, with most of them ranging from 3 to 4K, but the deviations in mostly range from -1 to 1K. The deviations between T_{obs} and T_{lin} is obviously smaller than those between T_{obs} and T_{old} . Besides, the deviations between T_{obs} and T_{lin} , is closer to the normal distribution with the mean of 0 than those between T_{obs} and T_{old} . From the results of numerical results, the LSE obtained by the linear retrieval model is better than the old one provided by CRTM model.

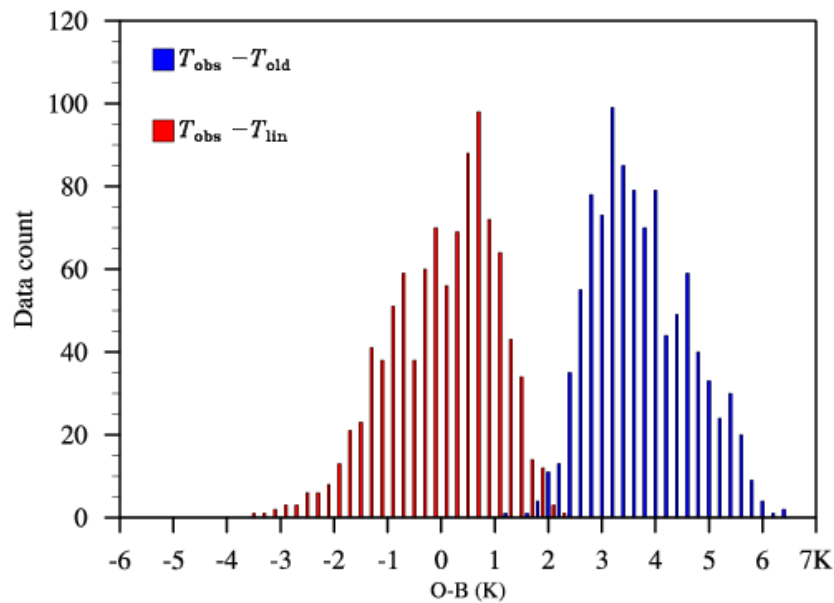
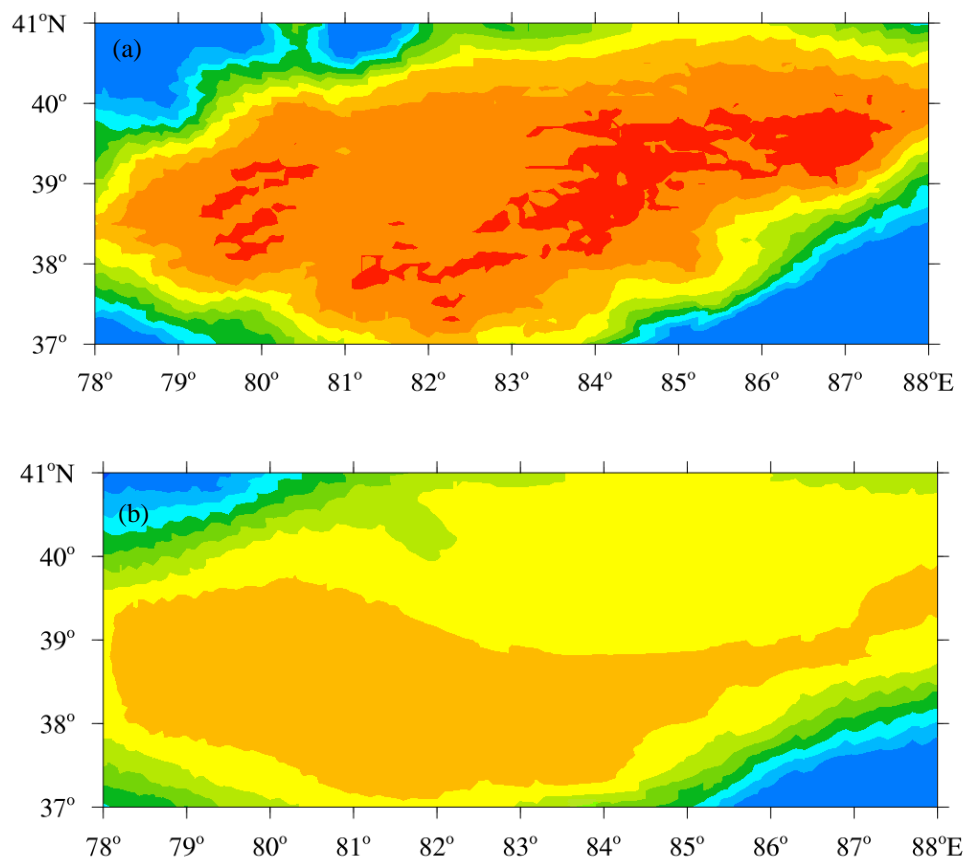


Figure 4. Statistical histogram of deviation of T_{old} , T_{lin} and T_{obs} in retrieval area (The abscissa represents the deviation, the ordinate represents the number of deviations within the interval, and the total number of red and blue deviations is the same, which is 998 equal to the scanning points in retrieval area. The red bar is T_{obs} minus T_{old} , and the blue bar is T_{obs} minus T_{lin})

4. Model test

The linear retrieval model is tested in this section to verify the reliability and stability of it. We use the equation (16) obtained by the retrieval area to calculate LSE in the whole Taklimakan desert area (37°~41°N, 78°~88°E), and the LSE are further provided to the CRTM model to simulate the brightness temperature which here are called T_{lin}^1 . The original simulated brightness temperature with the old LSE is denoted as T_{old}^1 in the whole Taklimakan desert. Figure 5 is the spatial distribution of observational brightness temperature and simulated brightness temperature with different LSEs in the whole Taklimakan desert area. Figure 5(a) is the spatial distribution of observational brightness temperature (T_{obs}^1), it can be seen from the map that the outline of the desert area is obvious, that is, the brightness temperature of the desert edge area is lower than that of the desert hinterland, which coincides with the characteristics of surface temperature in the desert area. Compared with T_{obs}^1 , the simulated brightness temperature of Taklimakan Desert under the LSE provided by the CRTM model (T_{old}^1 , figure 5(b)), there are great differences between them, not only in the numerical value, but also in the trend of change, so it is difficult to accurately describe the geographical characteristics of desert areas. Figure 6 (c) shows the simulated brightness temperature under the LSE obtained by the linear retrieval model. It is obvious that the values of T_{lin}^1 and T_{obs}^1 are closer in both numerical value and changing trends. The brightness temperature of T_{lin}^1 is about 273 K in most areas, and it is higher in some parts of the central and Eastern regions, reaching about 275 K, which is very similar to that of T_{obs}^1 .



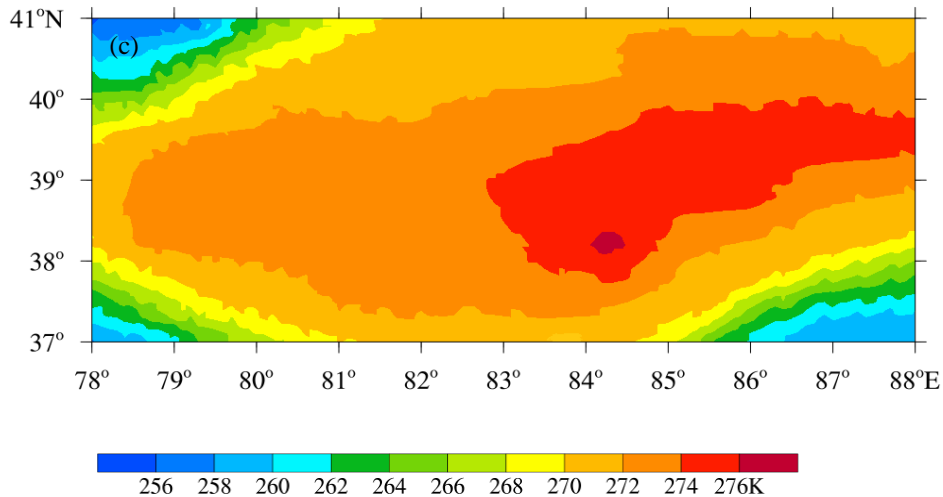


Figure 5. Spatial distribution of observational brightness temperature and simulated brightness temperature under two different LSEs in Taklimakan desert (a: T_{obs}^1 ; b: T_{old}^1 ; c: T_{lin}^1).

5. Summary and discussion

In this study, the optimal control theory is applied for the calculation of Microwave Land Surface Emissivity in the desert area. By combining the observations from FY3C/MWRI and the simulated brightness temperature, the linear retrieval model of LSE is constructed for the desert area. The minimum value of the cost functions are obtained by using the Newton iteration method. Then, the specific linear function relationship between the LSE and surface temperature, surface humidity is found. Using the new LSE provided by the linear retrieval model, the simulated brightness temperature in the retrieval area is obviously improved compared to that with the old LSE. At last, an independence test by applying the linear retrieval model in the whole Taklimakan Desert verifies the validity of the new LSE scheme. Therefore, the optimal control theory can be effectively applied to the calculation of LSE. In future, more influence factors of LSE will be introduced to build the model, and the model can also be studied in a wider space and time range.

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