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Research on the rumor spreading model with noise inference in the homogeneous network

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Abstract. In this paper, a dynamic I2SR model based on rumor spreading with random noise interference is proposed. Active state of the spreaders and the random noise interference in the homogeneous network can be considered. This paper divided spreaders into high probability spreaders and low probability spreaders that can be transformed. The existence and uniqueness of global positive solutions, extinction , and the spreading range of rumors be proved. Through numerical simulation, we have three conclusions as follows: (1)The spreaders can quickly reach to the peak in the shorter time with the bigger of the transmissions rate;(2)The high probability spreaders not only pay more attention to rumors but also more likely to spread;(3)There have a negative correlation between the spreading range of rumors and noise intensity.

Keywords: rumor spreading，active state，noise interference，complex network

1. Introduction

Rumors are often defined as the spreading of things that are of interest to the public or the interpretation of unconfirmed events through various ways. Rumor as an important method of social topics, its spreading pay more attention to people's lives. Rumor can lead to people reputation damage to some extent[1], the community or the company has an immeasurable consequences[2,3]. Because of its important role in society, many scholars began to study the spreading of rumors[4,5].

Because of the importance of the infectious disease model, it is widely used in various fields, especially the rumor spreading model [6], Delay and Kendall introduced the D-K model based on the classical epidemic disease model in 1965, and the difference between the spreading of diseases and rumors was analyzed. In this model, the total population is divided into three groups: one kind of people who has never heard rumors; one who has heard and spread it; one is a person who knows but never spread it. These groups are called "ignorant", "spreaders", "stiflers". Since then, many scholars have begun to study the relationship between rumor spreading and epidemic disease models. Some scholars[7-9] have studied the methods of disease transmission and the spread of rumor on complex networks based on the theory of mean field theory and infiltration. A number of people [10-13] have studied the rumor of the social network topology .In reality, the spreaders shows different interests and motivations due to the different characteristics of rumors and their connection with the individual. Subsequently, Huo and Wang et al. considered the effect of spreaders with different infection rates on rumor spreading in the homogeneous network [14]. However, epidemic disease models are unavoidably disturbed by environmental noise [15-18]. The stochastic model can be used to predict the future dynamic changes of the system. The stochastic differential equation model has also been widely used in the model of noise interference. Cai et al. describes the influence of environmental fluctuation on the infection rate coefficient about SIRS model, it is described as Gaussian white noise. Because of the similarity between rumor spreading and the spreading of epidemic diseases, it is also disturbed by the noise during the rumor spreading [19].

In this paper, we study the spreading of rumors from different perspectives and carry out a system analysis. In reality, spreaders will show different interests and motivations due to the different characteristics of rumor and their connection with the individual. Some people take the initiative to participate in the rumor spreading, others are showing a lower interest. In this paper, we assume that the average degree of the system in the homogeneous network is disturbed by random noise, and that the spreader's active state in the homogeneous network is considered in the process of the rumor spreading, and the spreaders with high activity is active and spread rumors with greater probability; And low active spreaders will show negative reactions, and with a lower probability to spread rumors, under certain conditions they can be transformed into each other. Based on these two factors, we put forward a dynamic I2SR model based on the rumor spreading that is provided with random noise interference at different infection rates. In this paper, we establish the model of stochastic differential equation system on complex networks, and prove the existence and uniqueness of global positive solutions, extinction and the spreading range of rumors. In the numerical simulation section, we take into account the impact of each parameter on rumor spreading, and the results are analyzed and summarized.

2. Model establishment and theoretical analysis

In this paper, we consider that the noise interference is $\langle k\rangle+\theta\dot{B}(t)$ .where $\langle k\rangle$ is the average degree and $\dot{B}(t)$ is the standard Brownian motion on the complete probability space, θ indicates the noise intensity. We analyze the following I2SR model with random noise interference

$$\begin{cases}\frac{dI(t)}{dt}=-\lambda_1 I(t)S_1(t)\langle k\rangle-\lambda_2 I(t)S_2(t)\langle k\rangle-\lambda_1 I(t)S_1(t)\theta\dot{B}(t)-\lambda_2 I(t)S_2(t)\theta\dot{B}(t)\\ \frac{dS_1(t)}{dt}=\lambda_1 I(t)S_1(t)\langle k\rangle+\alpha S_2(t)-\delta_1 S_1(t)-\beta S_1(t)+\lambda_1 I(t)S_1(t)\theta\dot{B}(t)\\ \frac{dS_2(t)}{dt}=\lambda_2 I(t)S_2(t)\langle k\rangle-\alpha S_2(t)-\delta_2 S_2(t)+\beta S_1(t)+\lambda_2 I(t)S_2(t)\theta\dot{B}(t)\\ \frac{dR(t)}{dt}=\delta_1 S_1(t)+\delta_2 S_2(t)\end{cases}\quad(1)$$

where $I(t), S_1(t), S_2(t), R(t)$ represent the density of the ignorants, the low probability spreaders, the high probability of spreaders, and the stiflers at time t respectively. When the ignorant nodes is touch with the spreader nodes, some of the ignorant nodes becomes the low probability spreader nodes with the probability λ_1, and some of the ignorant nodes becomes the high probability spreader nodes with the probability λ_2, we suppose $\lambda_2>\lambda_1$. When the low probability spreader nodes $S_1(t)$ is contact with the stifler nodes, with the probability δ_1 becomes the stiflers, and when the high probability spreader nodes $S_2(t)$ gets contact with the stifler nodes, with the probability δ_2 becomes the stiflers. And with the shrinking of rumor and the increasing of rumor's heat, we consider the probability of low probability and high probability spreaders can be transform. That is, we use the decay rate α to express high probability spreaders to low probability spreaders, the low probability spreaders is transformed into the high probability spreaders with probability β.

2.1. Existence and Uniqueness of Global Positive Solutions

Theorem 2.2.1 For any given initial value $(I(t), S_1(t), S_2(t), R(t))\in R_+^4$, Then system (1) has a unique positive solution when $t>0$, and the solution will stay in R_+^4 with probability 1. where

$R_+^4=\{(I(t), S_1(t), S_2(t), R(t))\,|\,0<I(t), S_1(t), S_2(t), R(t)<N, I(t)+S_1(t)+S_2(t)+R(t)=N\}$.

Proof Because the coefficients of the system satisfy the local Lipschitz condition, then for any given initial value $(I(0), S_1(0), S_2(0), R(0))\in R_+^4$ .System (1) has a unique local solution $(I(t), S_1(t), S_2(t), R(t))\in R_+^4$ on $t\in[0,\tau_e)$, where τ_e is explosion time.

To prove that this solution is global, we just have to prove that $\tau_e=\infty$ a.s is true. To achieve this objective, we assume that $k_0>0$ is large enough that $I(0), S_1(0), S_2(0), R(0)$ falls entirely on interval $[1/k_0, k_0]$, and for each integer $k>k_0$, we define the following stopping time

$$\tau_k = \inf\left\{t \in [0, \tau_e) : \Psi_i(t) \notin \left(\frac{1}{k}, k\right), i = 1,2,3,4\right\}. \quad (2)$$

where $(\Psi_1(t), \Psi_2(t), \Psi_3(t), \Psi_4(t)) = (I(t), S_1(t), S_2(t), R(t))$, in this paper we set $\inf \varnothing = \infty$ ($\varnothing$ *is an empty set*).Obviously, τ_k is increasing with $k \to \infty$, letting $\tau_\infty = \lim_{k\to\infty} \tau_k$, and then the $\tau_\infty \le \tau_e$ a.s.

If we assume that $\tau_\infty = \infty$ a.s.is true, then $\tau_e = \infty$ is true. In other words, we just have to prove that $\tau_\infty = \infty$ a.s. is true. If this statement is wrong, there must be a pair of constants $T > 0$ and $\varepsilon \in (0,1)$, such that $P\{\tau_k \le T\} > \varepsilon$. Therefore, there exists an integer $k_1 \ge k_0$ such that

$$P\{\tau_k \le T\} \ge \varepsilon, \forall k \ge k_1. \quad (3)$$

Let's consider function $V(\chi): R_+^4 \to R_+ \left(R_+ = \{V \mid V \ge 0, V \in R\}\right)$, and we define the following function

$$V\left(I(t), S_1(t), S_2(t), R(t)\right) = \left(I - 1 - \ln I\right) + (S_1 - 1 - \ln S_1) + \left(S_2 - 1 - \ln S_2\right) + (R - 1 - \ln R). \quad (4)$$

The non-negative properties of this function can be obtained from the following inequalities

$$u - 1 - \ln u \ge 0, \forall u \ge 0,$$

So for any $\left(I(t), S_1(t), S_2(t), R(t)\right) \in R_+^4$, because V is non-negative, then the application of *Itô* formula can be obtained

$$\begin{aligned}
\mathrm{d}V &= (1-\frac{1}{I})\mathrm{d}I + \frac{1}{2}\frac{1}{I^2}\left(\mathrm{d}I\right)^2 + (1-\frac{1}{S_1})\mathrm{d}S_1 + \frac{1}{2}\frac{1}{S_1^2}\left(\mathrm{d}S_1\right)^2 \\
&\quad + (1-\frac{1}{S_2})\mathrm{d}S_2 + \frac{1}{2}\frac{1}{S_2^2}\left(\mathrm{d}S_2\right)^2 + (1-\frac{1}{R})\mathrm{d}R + \frac{1}{2}\frac{1}{R^2}\left(\mathrm{d}R\right)^2 \\
&= \{(1-\frac{1}{I_1})(-\lambda_1 I S_1\langle k\rangle - \lambda_2 I S_2\langle k\rangle)\mathrm{d}t \\
&\quad + (1-\frac{1}{S_1})(\lambda_1 I S_1\langle k\rangle + \alpha S_2 - \delta_1 S_1 - \beta S_1)\mathrm{d}t \\
&\quad + (1-\frac{1}{S_2})(\lambda_2 I S_2\langle k\rangle + \beta S_1 - \delta_2 S_2 - \alpha S_2)\mathrm{d}t \\
&\quad + (1-\frac{1}{R})(\delta_1 S_1 + \delta_2 S_2)\mathrm{d}t + \frac{1}{2}(\lambda_1 S_1\theta + \lambda_2 S_2\theta)^2\mathrm{d}t \\
&\quad + \frac{1}{2}\lambda_1^2 I^2\theta^2\mathrm{d}t + \frac{1}{2}\lambda_2^2 I^2\theta^2\mathrm{d}t\} + \frac{1}{I}(\lambda_1 I S_1\theta\mathrm{dB}(t) \\
&\quad + \lambda_2 I S_2\theta\mathrm{dB}(t)) - \frac{1}{S_1}\lambda_1 I S_1\theta\mathrm{dB}(t) - \frac{1}{S_2}\lambda_2 I S_2\theta\mathrm{dB}(t).
\end{aligned}$$

The above formula can be written as follows

$$\mathrm{d}V = A(\mathrm{t})\mathrm{d}t + (\frac{1}{I} - \frac{1}{S_1})\lambda_1 I S_1\theta\mathrm{dB}(t) + (\frac{1}{I} - \frac{1}{S_2})\lambda_2 I S_2\theta\mathrm{dB}(t), \quad (5)$$

where

$$A(t)=(1-\frac{1}{I})(-\lambda_1 IS_1\langle k\rangle-\lambda_2 IS_2\langle k\rangle)+(1-\frac{1}{S_1})(\lambda_1 IS_1\langle k\rangle+\alpha S_2-\delta_1 S_1-\beta S_1)$$
$$+(1-\frac{1}{S_2})(\lambda_2 IS_2\langle k\rangle+\beta S_1-\delta_2 S_2-\alpha S_2)+(1-\frac{1}{R})(\delta_1 S_1+\delta_2 S_2)$$
$$+\frac{1}{2}(\lambda_1 S_1\theta+\lambda_2 S_2\theta)^2+\frac{1}{2}\lambda_1^2 I^2\theta^2+\frac{1}{2}\lambda_2^2 I^2\theta^2.$$

where $A(t)$ is bounded, we assume $A(t)\le K, K\in R_+$, so we have

$$\mathrm{d}V\le K\mathrm{d}t+(\frac{1}{I}-\frac{1}{S_1})\lambda_1 IS_1\theta\mathrm{dB}(t)+(\frac{1}{I}-\frac{1}{S_2})\lambda_2 IS_2\theta\mathrm{dB}(t). \tag{6}$$

For inequality (6) integral from 0 to $\tau_k\wedge T$, we can get that

$$\int_0^{\tau_k\wedge T}\mathrm{d}V\le\int_0^{\tau_k\wedge T}K\mathrm{d}t+\int_0^{\tau_k\wedge T}\left|(\frac{1}{I}-\frac{1}{S_1})\lambda_1 IS_1\theta+(\frac{1}{I}-\frac{1}{S_2})\lambda_2 IS_2\theta\right|\mathrm{dB(t)}. \tag{7}$$

where $\tau_k\wedge T=\min\{\tau_k,T\}$.

And then we can take the expectation of both sides of the equation (7)

$$\mathrm{E}V(I(\tau_k\wedge T),S_1(\tau_k\wedge T),S_2(\tau_k\wedge T),R(\tau_k\wedge T))\le V(I(0),S_1(0),S_2(0),R(0))+KE(\tau_k\wedge T)$$
$$\le V(I(0),S_1(0),S_2(0),R(0))+KT. \tag{8}$$

Set $\Omega_k=\{\tau_k\le T\}$ be true for $k\ge k_1$, and take into account (3) can have $P(\Omega_k)\ge\varepsilon$. Observing the definition of stopping time .we can know that there is at least one of $I(\omega\wedge\tau_k)$, $S_1(\omega\wedge\tau_k)$, $S_2(\omega\wedge\tau_k)$, $R(\omega\wedge\tau_k)$ equal to k or $1/k$ for each of $\omega\in\Omega_k$. Hence, we can get the following form

$$V(I(0),S_1(0),S_2(0),R(0))+KT\ge\mathrm{E}[I_{\Omega_k}(\omega)V(I(\tau_k\wedge\omega),S_1(\tau_k\wedge\omega),S_2(\tau_k\wedge\omega),R(\tau_k\wedge\omega))]$$
$$\ge\varepsilon((k-1-\ln k)\wedge(1/k+1+\ln k)), \tag{9}$$

where I_{Ω_k} indicates the indicator function of Ω_k. Setting $k\to\infty$, then we can obtain that

$$V((0),S_1(0),S_2(0),R(0))+KT=\infty. \tag{10}$$

Because both $V(I(0),S_1(0),S_2(0),R(0))$ and K have a boundary, therefore, we come to the contradiction, and $\tau_\infty=\infty$ is established. This means that in a short time $I(t)$ and $S_1(t)$, $S_2(t)$ will not be the outbreak of the situation, the proof come to an end.

2.2. The extinction of rumor diffusion

Theorem 2.2.1 For any given initial value $(I(t),\mathrm{S}_1(t),\mathrm{S}_2(t),R(t))\in R_+^4$, The system (1) has the following properties

$$\lim_{t\to\infty}\sup\frac{\ln S_1(t)}{t}+\lim_{t\to\infty}\sup\frac{\ln S_2(t)}{t}\le\frac{\langle k\rangle^2}{\theta}-(\delta_1+\delta_2)+(\sqrt{\alpha}+\sqrt{\beta})^2.$$

Where $\{(I(t),S_1(t),S_2(t),R(t)\mid 0<I(t),S_1(t),S_2(t),R(t)\le N, I(t)+S_1(t)+S_2(t)+R(t)=N\}$.

Proof We have related operations about the second and third equations of the system (1), then

$$\ln S_1+\ln S_2\le[(\lambda_1+\lambda_2)I\langle k\rangle-(\delta_1+\delta_2)+(\alpha+\beta)+2\sqrt{\alpha\beta}]dt+\lambda_1 I\theta d\mathrm{B}(t)+\lambda_2 I\theta d\mathrm{B}(t).$$

Set $\mathrm{B}_\tau=\mathrm{B}(t)$ in the following operation, define the function

$$V_1(t)=(\lambda_1 I\langle k\rangle-\delta_1+\beta+\sqrt{\alpha\beta})dt+\lambda_1 I\theta d\mathrm{B}_\tau,$$
$$V_2(t)=(\lambda_2 I\langle k\rangle-\delta_2+\alpha+\sqrt{\alpha\beta})dt+\lambda_2 I\theta d\mathrm{B}_\tau.$$

It can be seen that $V_i(t)(i=1,2)$ is quadratic differential about I and continuous differentiable about t, then can be derived from $It\hat{o}$ formula

$$\begin{aligned}\mathrm{d}V_1&=[\lambda_1 I\langle k\rangle-\delta_1+\beta+\sqrt{\alpha\beta}-\frac{1}{2}\lambda_1^2\theta^2 I^2]\mathrm{d}t+\lambda_1 I\theta\mathrm{dB}_\tau\\&\le[\frac{\theta^2}{2}(\frac{\langle k\rangle}{\theta^2})^2-\delta_1+\beta+\sqrt{\alpha\beta}]\mathrm{d}t+\lambda_1 I\theta\mathrm{dB}_\tau\\&=(\frac{\langle k\rangle^2}{2\theta^2}-\delta_1+\beta+\sqrt{\alpha\beta})\mathrm{d}t+\lambda_1 I\theta\mathrm{dB}_\tau.\end{aligned}$$

$$\begin{aligned}\mathrm{d}V_2&=[\lambda_2 I\langle k\rangle-\delta_2+\alpha+\sqrt{\alpha\beta}-\frac{1}{2}\lambda_2^2\theta^2 I^2]\mathrm{d}t+\lambda_2\theta I\mathrm{dB}_\tau\\&\le[\frac{\theta^2}{2}(\frac{\langle k\rangle}{\theta^2})^2-\delta_2+\alpha+\sqrt{\alpha\beta}]\mathrm{d}t+\lambda_2\theta I\mathrm{dB}_\tau\\&=(\frac{\langle k\rangle^2}{2\theta^2}-\delta_2+\alpha+\sqrt{\alpha\beta})\mathrm{d}t+\lambda_2\theta I\mathrm{dB}_\tau.\end{aligned}$$

Thereby, $\mathrm{d}V_1+\mathrm{d}V_2\le[\frac{\langle k\rangle^2}{\theta^2}-(\delta_1+\delta_2)+(\sqrt{\alpha}+\sqrt{\beta})^2]\mathrm{d}t+\lambda_1\theta I\mathrm{dB}_\tau+\lambda_2\theta I\mathrm{dB}_\tau$.

We take the integral and limit about the above inequality, then we have

$$\begin{aligned}&\ln S_1(t)+\ln S_2(t)-\ln S_1(0)-\ln S_2(0)\\&\le\int_0^t[\frac{\langle k\rangle^2}{\theta^2}-(\delta_1+\delta_2)+(\sqrt{\alpha}+\sqrt{\beta})^2]\mathrm{d}\tau+\int_0^t(\lambda_1+\lambda_2)\theta I\mathrm{dB}_\tau,\\&\lim_{t\to\infty}\frac{\ln S_1(t)}{t}+\lim_{t\to\infty}\frac{\ln S_2(t)}{t}\\&\le\lim_{t\to\infty}\frac{\ln S_1(0)}{t}+\lim_{t\to\infty}\frac{\ln S_2(0)}{t}+\frac{\langle k\rangle^2}{\theta^2}\\&\quad-(\delta_1+\delta_2)+(\sqrt{\alpha}+\sqrt{\beta})^2+\lim_{t\to\infty}\frac{1}{t}\int_0^t(\lambda_1+\lambda_2)\theta I\mathrm{dB}_\tau.\end{aligned}$$

since B_τ is standard Brownian motion and $\mathrm{B}_\tau\subset N(0,1)$, then we can get

$$\lim_{t\to\infty}\frac{1}{t}\int_0^t\theta I(t)\mathrm{dB}_\tau=0.$$

Therefore,

$$\lim_{t\to\infty}\sup\frac{\ln S_1(t)}{t}+\lim_{t\to\infty}\sup\frac{\ln S_2(t)}{t}\le\frac{\langle k\rangle^2}{\theta}-(\delta_1+\delta_2)+(\sqrt{\alpha}+\sqrt{\beta})^2.$$

In particular, $\lim_{t\to\infty}S_1(t)+\lim_{t\to\infty}S_2(t)=0$ can be established when $\theta\ge\theta_0=$

$\sqrt{\frac{\langle k\rangle^2}{(\delta_1+\delta_2)-(\sqrt{\alpha}+\sqrt{\beta})^2}}$ .That means rumor will disappear when the time tends to infinity.

2.3. The spreading range of rumors

Based on the above theory, the non-zero rumor spread threshold λ_c is given.[20] if $\lambda_{\min}$ is higher than the threshold λ_c, then the rumor will spread on the network and infect a certain number of people; and $\lambda_{\min}$ below the threshold λ_c, the number is infinitesimal in a very large population. that is, compared with a large number of people, rumor spread on the network will only infect infinitely small crowd, the number tend to zero. From theorem 2.2.1 we can see that the above conclusion can be established, that is, when the noise intensity is large enough to ensure that $\lim_{t\to\infty} S_1(t)+\lim_{t\to\infty} S_2(t)=0$. But we take into account the addition of random variables, rumor spread the number of individuals infected does not necessarily tend to 0 when $t\to\infty$ at any noise intensity, at the same time, $R(t)$ is a random variable, rather than a value, it is determined by the distribution of $\mathrm{B}(t)$ at every moment. So, we consider statistical average of $R(t)$ to study the threshold of rumors.

For convenience, we define $I(t), S_1(t), S_2(t), R(t)$ to be the density of an individual with three different states in the following derivation. That is

$$I(t)+S_1(t)+S_2(t)+R(t)=1. \tag{11}$$

We analyze the stochastic differential equations by taking integral of $I(t)$, considering that the initial proportion of the initial conditions $I(0)\approx 1, R(0)=0$ and $S_1(0), S_2(0)$ is too small. Therefore

$$I(t)=I(0)e^{-\lambda_1\langle k\rangle S_1(t)-\lambda_2\langle k\rangle S_2(t)-(\lambda_1+\lambda_2)\theta\int_0^t (S_1(t)+S_2(t))\mathrm{dB}_\tau}. \tag{12}$$

The statistical average of the total number of infections can be obtained by satisfying the following equation (11) and theorem 2.2.1

$$\mathrm{E}R(\infty)=1-\mathrm{E}I(\infty), \tag{13}$$

where

$$I(t)=e^{-\lambda_1\langle k\rangle\frac{R-\delta_2 S_2(t)}{\delta_1}-\lambda_2\langle k\rangle S_2(t)+(\lambda_1+\lambda_2)\theta\int_0^t (S_1(t)+S_2(t))\mathrm{dB}_\tau}. \tag{14}$$

Hence, we have

$$\mathrm{E}R(\infty)\le 1-\mathrm{E}(e^{-\frac{\lambda_1}{\delta_1}\langle k\rangle R(\infty)-(\lambda_1+\lambda_2)\int_0^\infty (S_1(t)+S_2(t))\mathrm{dB}_\tau}) \quad \le 1-e^{-\frac{\lambda_1}{\delta_1}\langle k\rangle \mathrm{E}R(\infty)-\mathrm{E}[(\lambda_1+\lambda_2)\int_0^\infty (S_1(t)+S_2(t))\mathrm{dB}_\tau]}, \tag{15}$$

For the stochastic integral of $\int_0^\infty (S_1(t)+S_2(t))\,\mathrm{d}\,\mathrm{B}_\tau$, we get

$$\mathrm{E}(\int_0^\infty (S_1(t)+S_2(t))\mathrm{dB}_\tau)=0.$$

Meanwhile, the above inequality is established if and only if $\mathrm{E}R(\infty)=0$. So the above inequality can be written as follows

$$\mathrm{E}R(\infty)\le 1-e^{-\frac{\lambda_1}{\delta_1}\langle k\rangle ER(\infty)}. \tag{16}$$

Define the function $f(x)=1-e^{-\gamma x}$, where $\gamma=\frac{\lambda_1}{\delta_1}\langle k\rangle, \mathrm{E}R(\infty)=x$. Then (16) can be written as $\mathrm{E}R(\infty)=f(\mathrm{E}R(\infty))$. Where $f(0)=0$ and $f(1)<1$. Since $f(x)$ is monotonically increasing, the positive solution exists if and only if $f^{'}(0)>1$, we have

$$\frac{d}{dER(\infty)}(1-e^{-\frac{\lambda_1}{\delta_1}\langle k\rangle ER(\infty)})|_{ER(\infty)=0}>1.$$

The equivalent condition for the above inequality is $\frac{\lambda_1}{\delta_1}>\frac{1}{\langle k\rangle}$. Similarly, we also can get $\frac{\lambda_2}{\delta_2}>\frac{1}{\langle k\rangle}$, and $\lambda_{\min}=\min\{\frac{\lambda_1}{\delta_1},\frac{\lambda_2}{\delta_2}\}$. Therefore, we can get rumor threshold $\lambda_c=\frac{1}{\langle k\rangle}$, when $\lambda_{\min}$ below the threshold λ_c, rumor gradually disappear, when $\lambda_{\min}$ higher than the threshold λ_c, rumor spread on the network and infected a certain number of people.

3. Numerical simulation

In this section, we discuss the theoretical model of rumor with noise interference on complex networks through numerical simulation. In order to carry out the simulation model better, we construct a complex network with a population number of $N=10^6$.

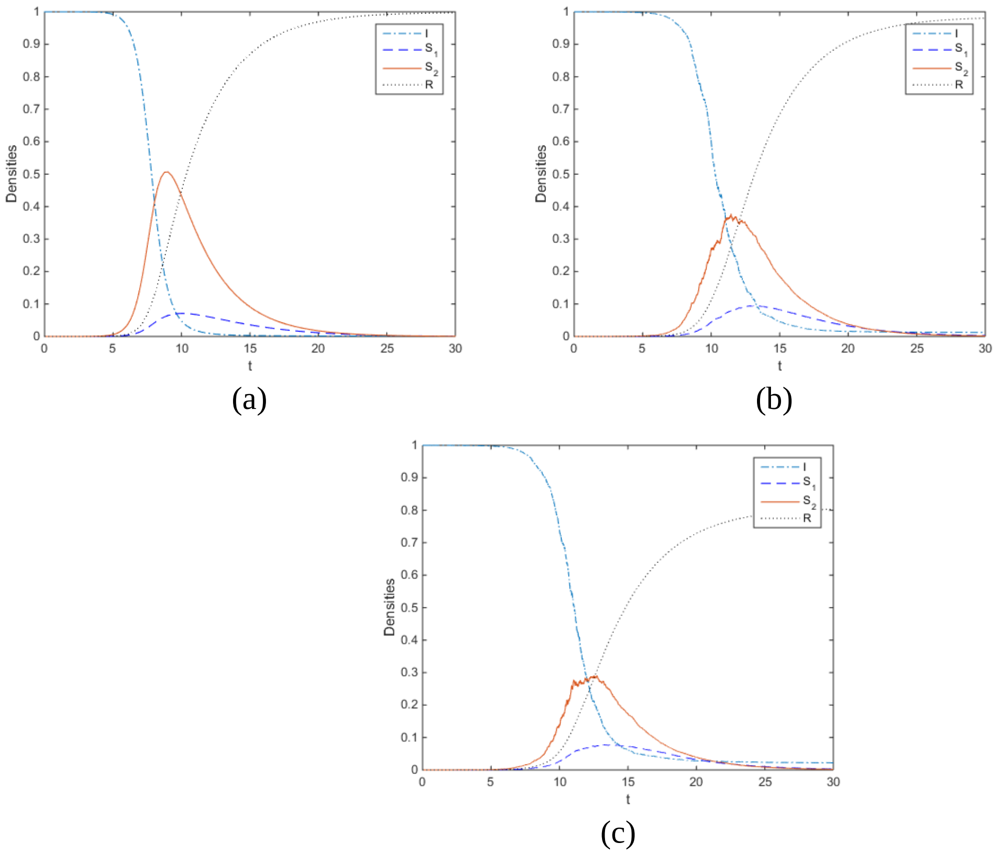


Fig.1. Theoretical model of Stochastic Differential Equations with different random noise intensity (a) $\theta=0$. (b) $\theta=100$. (c) $\theta=150$.

Fig.1 depicts the change in the stochastic differential equation system by adding different random noise disturbances, which proves its theoretical value. Here, we take $\lambda_1=0.03$, $\lambda_2=0.05$, $\delta_1=\delta_2=0.3$, $\alpha=0.07$, $\beta=0.02$, $\langle k\rangle=30$. From the figure we can find: the number of infected individuals increases, reaches the peak, and then decreasing until it disappears. At the same time, in the graphs (b) and (c) of the fig.1, we set the parameters to satisfy the condition of theorem 2.2.1, that is $\theta\geq\theta_0=\sqrt{\frac{\langle k\rangle^2}{(\delta_1+\delta_2)-(\sqrt{\alpha}+\sqrt{\beta})^2}}$. Only susceptible and infected individuals are still in a steady state, and rumor disappear eventually, which is consistent with the conclusion of theorem 2.2.1. Compared with the addition of noise, we can find that the spreading range is reduced relatively, the peak of the system is less than no noise interference system on the complex network, and the life cycle of rumor is shorter than before. Compared with the graph (b) and (c), we can observe that the longer the noise intensity, the longer the time required for the infected individuals to reach the peak, and the smaller the peak value. In addition, we also receive that the greater of the noise intensity, its spreading range is smaller, indicating that noise affects the spreading of rumor on the network.

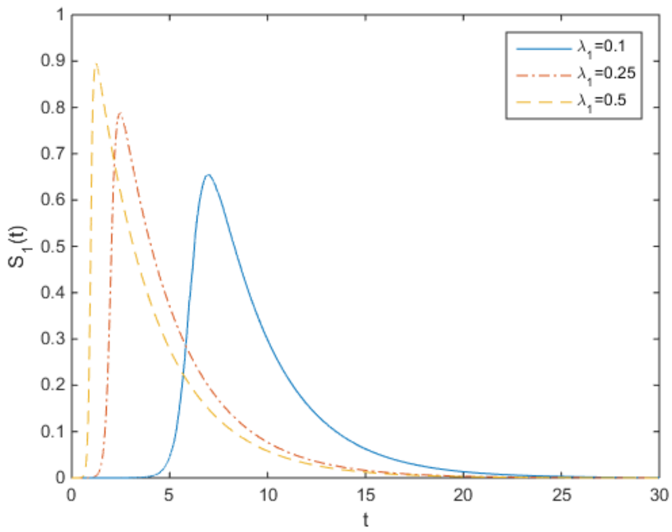


Fig.2. The density of $S_1(t)$ under different λ_1

Fig. 2 shows the density of $S_1(t)$ at different rates of infection from ignorants to low probability spreaders with time changes. And we take $\lambda_1 = 0.1, 0.25, 0.5$.It can be observed that with the infection rate increases, the time of $S_1(t)$ to reach the peak is shorter and the greater the infection rate, the greater the peak.

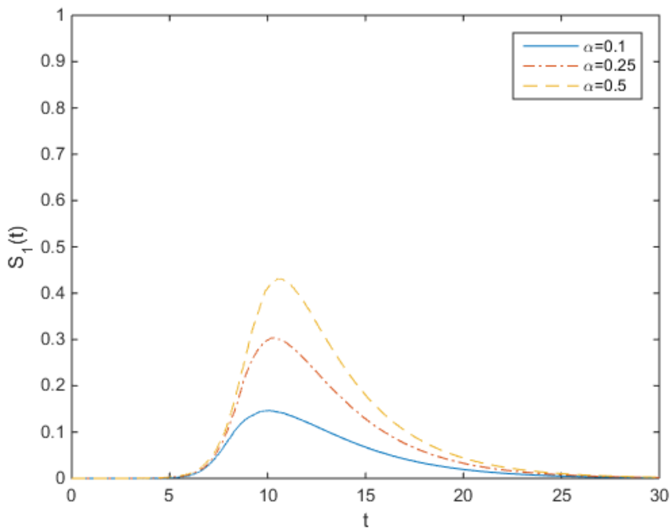
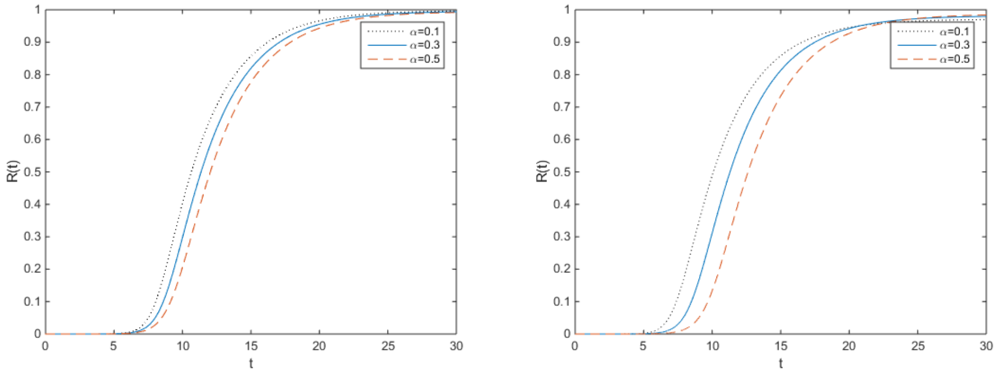


Fig.3. The density of $S_1(t)$ under different α

Fig.3 describes the density of $S_1(t)$ at different decay rates α over time. In the whole process, at first $S_1(t)$ increases, reaches the peak, then falls, and finally reduces to zero. In addition, when the value of α is getting bigger, its peak value is growing at the same point, which we can see from the figure. With the spreader's interest in rumor diminishes and rumor diminish, more and more spreaders from high active to low active state. According to the different characteristics of rumors, we combine these problems with the individual, and we receive that the decay rate has some effect on the spreaders.



(a) (b)

Fig.4. The changes of $R(t)$ over time in different α when (a) $\theta=0$. (b) $\theta=50$.

The graphs (a) and (b) of figure 4 depicts the changes of the spreading range over time with different α before and after the addition of noise. α represents the decay rate of the high probability spreaders to the low probability spreaders, the greater the value of α, the higher of the number of high probability spreaders to the low probability spreaders, and the shorter the system reach the final steady state. From the figure, we can see that the decay rate has little effect on the range of the rumor spreading,.However, after we considered the noise disturbance, it became noticeably smaller.

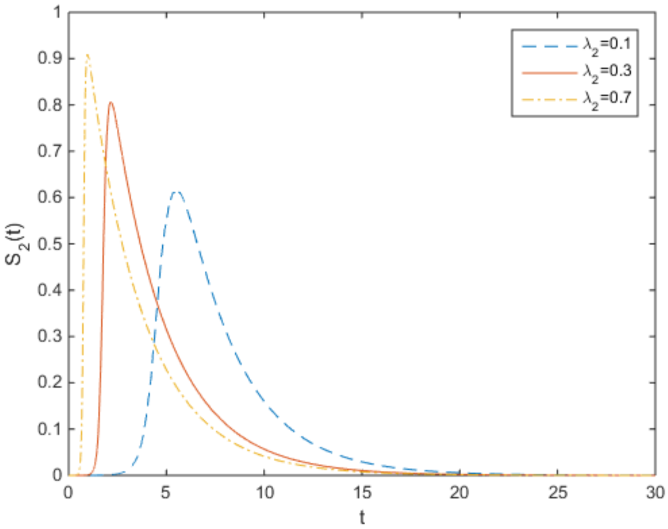


Fig.5. The density of $S_2(t)$ under different λ_2

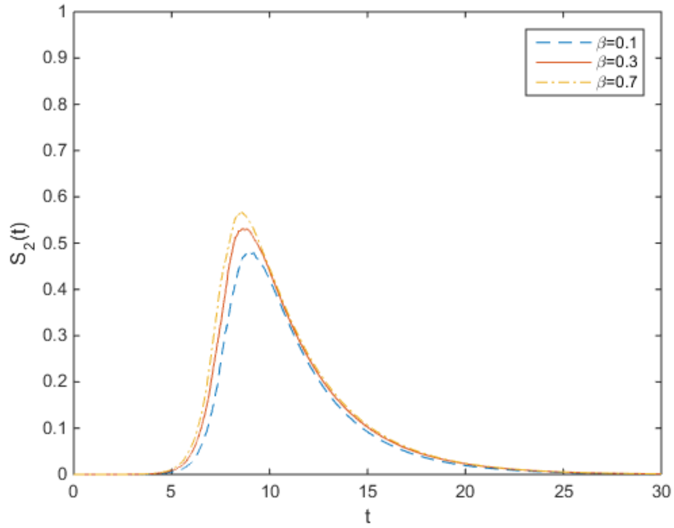


Fig.6. The density of $S_2(t)$ under different β

Fig.5 illustrates how the density of $S_2(t)$ varies with time at different rates of infection. It can be seen from the figure that $S_2(t)$ can reach the peak in a shorter time with a higher infection rate, and the lower the peak, the infection rate is lower than the peak of the higher propagation rate. The high transmission rate of the spreaders not only reflects that the spreaders itself is more concerned with rumors, but also reflects that the spreaders is more likely to spread rumors, relatively speaking, the value of λ_2 is greater. In view of the characteristics of rumor and their connection with the individual, the rumor itself is related to personal interests.

Fig.6 discusses the density of $S_2(t)$ change with time under different β. In this process, the density of $S_2(t)$ first rapidly reaches the peak and then decreases, and finally reaches to the balance. We can observe that the larger of β, the greater the number of $S_2(t)$.In the whole process of converting the low probability spreaders to the high probability spreaders, β more larger, the density of $S_2(t)$ to reach the peak faster. From the figure we can conclude that β has a certain effect on the density of $S_2(t)$.

4. Conclusions

In this paper, we propose a dynamic I2SR model based on rumors with noise interference at different infection rates, and complete the theoretical proof in the homogeneous network. In the numerical simulation, we draw the following conclusions:

(1) The addition of noise reduces the peak and the spreading range of rumors. we receive that the greater the noise, the infected individuals spends more time to reach the peak, and the smaller the diffusion range of rumors. It can be observed that the noise makes the system subject to a certain degree of interference, and reduces the spreading of rumors.

(2) The greater the probability of high probability spreaders to low probability spreaders, the bigger the density of low probability spreaders. Furthermore, We conclude that the different characteristics of rumors and the interests of the individual are very important in the process of communication.

(3) Through the analysis of different infection rates, we know that the probability from the ignorant to the spreaders has a great influence on the spreading of rumors. The higher the probability of spreaders from ignorant to high probability spreaders, the higher the probability that spreaders can reach a greater peak in a relatively short period of time. It shows that high probability spreaders not only pay more attention to rumors, but also more likely to spread rumors. Furthermore, rumors are related to the interests of the spreaders. In addition, we find that the decay rate has little effect on the final diffusion range of the system, while the addition of noise reduces the spreading range of rumors significantly.

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