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Finite-time chaos synchronization of the delay Lorenz system with disturbance via a single controller

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Abstract. This paper deals with the finite-time chaos synchronization of the delay Lorenz system with disturbance via a single controller. Based on the finite-time stability theory, a control law is proposed to realize finite-time chaos synchronization of delay Lorenz system with disturbance. Finally, numerical simulation results are given to demonstrate the effectiveness and robustness of the proposed scheme.

Keywords: Finite-time chaos synchronization; delay Lorenz system; disturbance

1. Introduction

Chaos synchronization has attracted much attention of many researches since the seminal work of Pecora and Carroll [1]. From then on, chaos synchronization has been developed extensively and intensively due to its potential application in many fields, such as secure communication [2, 3], complex networks [4-7], biotic science [8-13] and so on [14-26].

Nowadays, most of the results about chaos control and synchronization are derived based on the asymptotic stability of the chaotic systems. In fact, it is more valuable to control or synchronize chaotic systems as fast as possible. To obtain faster convergence, the finite-time control approach is an effective technique. Moreover, the finite-time techniques have demonstrated better robustness and disturbance rejection properties than that of asymptotic methods [27-37]. Therefore, the finite-time chaos control and synchronization have attracted a great deal of attention over the last few decades.

On the other hand, it is difficult to know the external disturbance always occurs in the system. Thus, the chaos control and synchronization of chaotic system in the presence of external disturbance are effectively crucial in practical applications.

In this paper, we present a controller to realize finite-time synchronization of delay Lorenz system with disturbance. The controller is robust and simple to be constructed. Numerical simulations are presented to demonstrate the effectiveness and robustness of the proposed scheme.

2. Preliminary definitions and lemmas

Finite-time synchronization means that the state of the slave system can track the state of the master system after in finite-time. The precise definition of finite-time synchronization is given as below.

Definition 1. Consider the following two chaotic systems:

$$\begin{aligned} \dot{x}_m &= f(x_m), \\ \dot{x}_s &= h(x_m, x_s), \end{aligned} \tag{1}$$

where x_m, x_s are two n-dimensional state vectors. The subscripts '*m*' and '*s*' stand for the master and slave systems, respectively. $f: R^n \to R^n$ and $h: R^n \to R^n$ are vector-valued functions. If there exists a constant $T > 0$, such that

$$\lim_{t \to T} \|x_m - x_s\| = 0,$$

and

$$\|x_m - x_s\| \equiv 0, \text{ if } t \geq T,$$

then synchronization of the system (1) is achieved in a finite-time.

Lemma 1 [32]**.** Assume that a continuous, positive-definite function $V(t)$ satisfies differential inequality

$$\dot{V}(t) \le -cV^{\eta}(t),\ \forall t \ge t_0, V(t_0) \ge 0, \tag{2}$$

where $c > 0$, $0 < \eta < 1$ are constants, then, for any given t_0, $V(t)$ satisfies inequality

$$V^{1-\eta}(t) \le V^{1-\eta}(t_0) - c(1-\eta)(t-t_0),\ t_0 \le t \le t_1, \tag{3}$$

and

$$V(t) \equiv 0,\ \forall t \ge t_1, \tag{4}$$

with t_1 given by

$$t_1 = t_0 + \frac{V^{1-\eta}(t_0)}{c(1-\eta)}. \tag{5}$$

Proof. Consider differential equation

$$\dot{X}(t) = -cX^{\eta}(t),\ X(t_0) = V(t_0), \tag{6}$$

although differential equation (6) does not satisfy the global Lipschitz condition, the unique solution of Eq.(6) can be found as

$$X^{1-\eta}(t) = X^{1-\eta}(t_0) - c(1-\eta)(t-t_0). \tag{7}$$

Therefore, from the comparison Lemma, one obtains

$$V^{1-\eta}(t) \le V^{1-\eta}(t_0) - c(1-\eta)(t-t_0),\ t_0 \le t \le t_1, \tag{8}$$

and

$$V(t) \equiv 0,\ \forall t \ge t_1.$$

with t_1 given in (5).

Lemma 2 [34]. If $\alpha > (\frac{2}{3})^{\frac{2}{3}}$, it can be gotten that

$$(\alpha|x_1| + \frac{1}{2}x_2^{\,2})^{\frac{3}{2}} + x_1x_2 \ge 0, \tag{9}$$

where x_1 and x_2 are any real numbers .

Corollary 1 [34]. If $\alpha > (\frac{2}{3})^{\frac{2}{3}}$, it can be obtained that

$$|x_1x_2| \le (\alpha|x_1| + \frac{1}{2}x_2^{\,2})^{\frac{3}{2}}, \tag{10}$$

where x_1 and x_2 are any real numbers .

3. Main results

A chaotic system is extremely sensitive to disturbance. In actual situation, the system is disturbed and cannot be exactly predicted. These uncertainties will destroy the synchronization and even break it. Therefore, it is important and necessary to study the synchronization of systems with disturbance. In this section, the dynamic behaviors of the delay Lorenz system is to be explored and the finite-time synchronization of the delay Lorenz systems will be discussed.

3.1 Dynamics of delay Lorenz system with disturbance

Delay Lorenz system with disturbance is considered as

$$\begin{aligned} \dot{x}_1 &= a(x_2 - x_1), \\ \dot{x}_2 &= cx_1 - x_2 - x_1x_3 + Ax_2\sin(\omega t), \\ \dot{x}_3 &= x_1x_2 - bx_3(t-\tau), \end{aligned} \tag{11}$$

where a, b, c, τ, A, ω are real positive constants. In this section, initial conditions of system (11) is chosen as $(-0.12, -0.4, -0.7)$ and the parameters of the system are selected as $a = 10$, $b = 3$, $c = 28$, $A = 0.001$, $\omega = 0.01$. Figs.1-4 depict the dynamics of system (11) for different values of τ. Fig.1 and Fig.3 indicate that the delay Lorenz system with disturbance is chaotic for certain values of τ. Fig.2 shows that the system has periodic-1 solution for $\tau = 0.24$. Fig.4 reflects that the system has periodic-2 solution for $\tau = 0.37$.

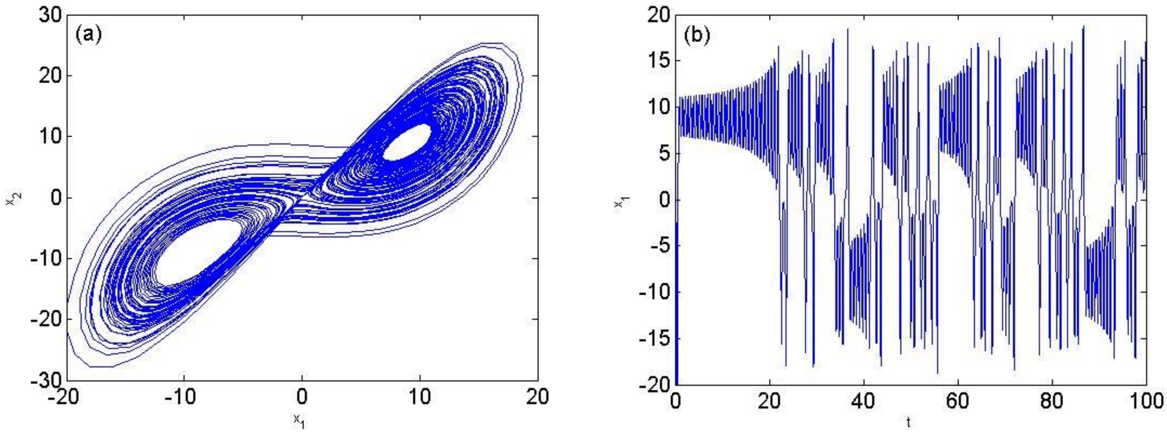

Fig.1. The phase portrait and time series of variables in system (11) for $\tau = 0.0019$, (a) phase portrait of x_1 and x_2, (b) time series of x_1.

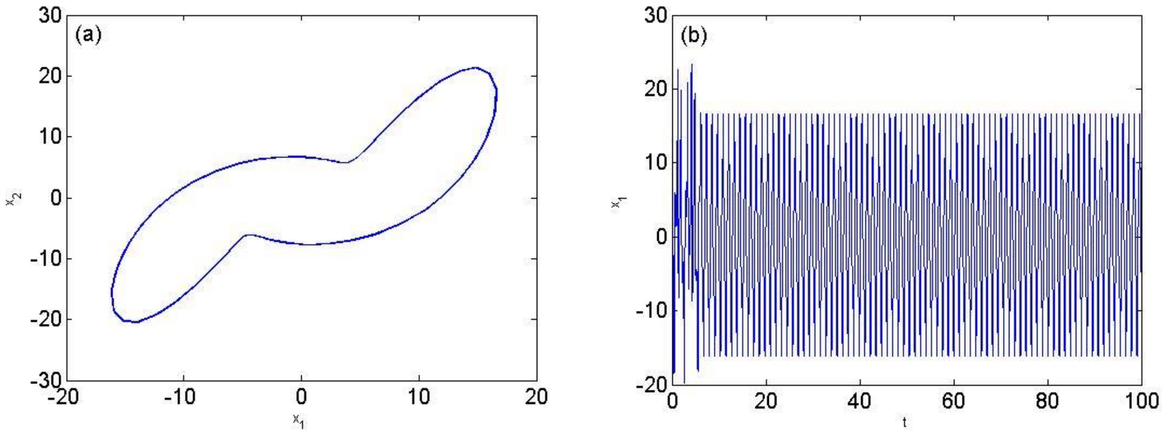

Fig.2. The phase portrait and time series of variables in system (11) for $\tau = 0.24$, (a) phase portrait of x_1 and x_2, (b) time series of x_1.

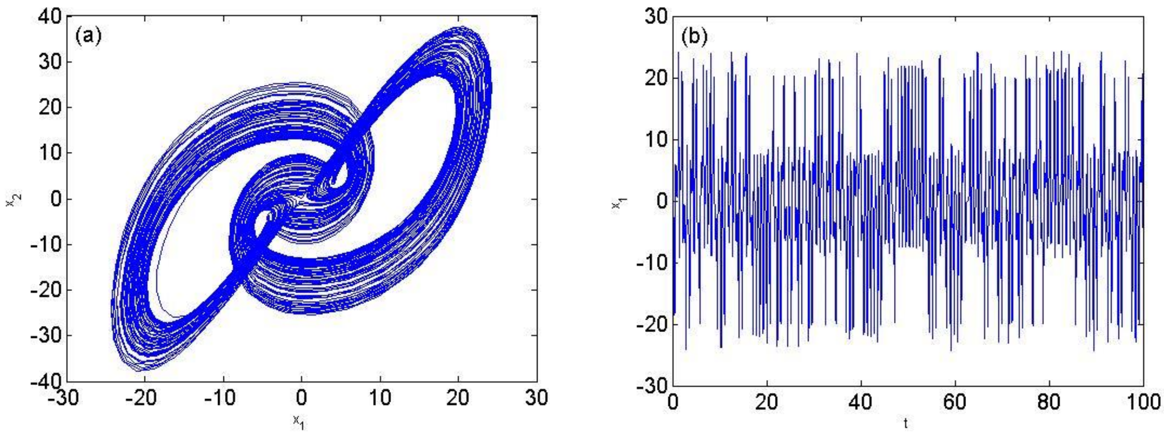

Fig.3. The phase portrait and time series of variables in system (11) for $\tau = 0.25$, (a) phase portrait of x_1 and x_2, (b) time series of x_1.

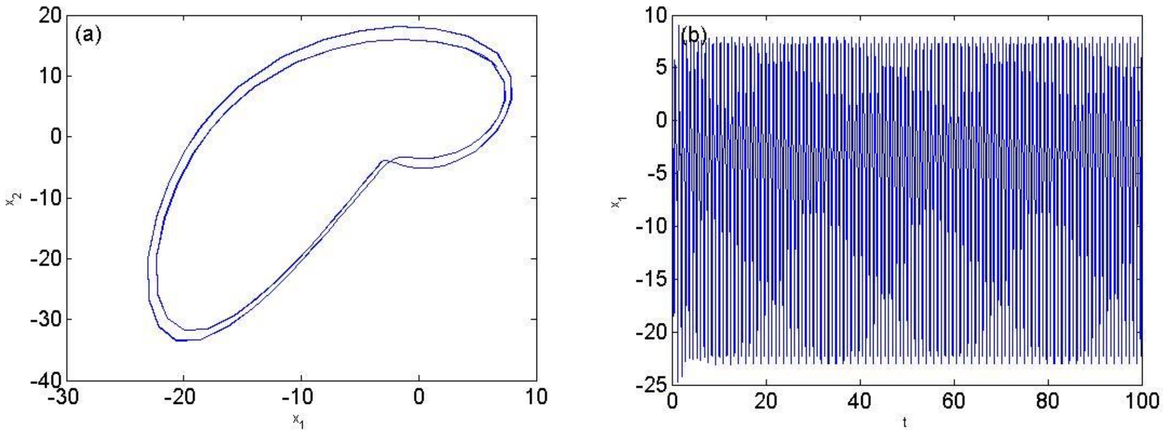

Fig.4. The phase portrait and time series of variables in system (11) for $\tau = 0.37$, (a) phase portrait of x_1 and x_2, (b) time series of x_1.

3.2 Chaos synchronization of delay Lorenz system with disturbance

System (11) is considered as the master system and the slave system is a controlled system as

$$\dot{y}_1 = a(y_2 - y_1),$$
$$\dot{y}_2 = cy_1 - y_2 - y_1 y_3 + Ay_2 \sin(\omega t) + u_2,$$
$$\dot{y}_3 = y_1 y_2 - by_3(t-\tau). \quad (12)$$

Let $e_1 = y_1 - x_1$, $e_2 = y_2 - x_2$, $e_3 = y_3 - x_3$, and subtract Eq.(11) from Eq.(12), the error system between systems (11) and (12) can be gotten as

$$\dot{e}_1 = a(e_2 - e_1),$$
$$\dot{e}_2 = ce_1 + e_1 e_3 - e_1 y_3 - y_1 e_3 - e_2 + Ae_2 \sin(\omega t) + u,$$
$$\dot{e}_3 = -e_1 e_2 + e_1 y_2 + y_1 e_2 - be_3(t-\tau). \quad (13)$$

Our aim is to design a controller that can achieve the finite-time synchronization of the delay Lorenz system (11) and the controlled system (12). The problem can be converted to design a controller to attain finite-time stable of the error system (13).

To achieve the finite-time stabilization, the controller u is taken as

$$u = -ce_1 - e_1 e_3 + e_1 y_3 + y_1 e_3 + e_2 - k_1 sign(e_1) - k_2 sign(e_2) - k_3 sign(e_3), \quad (14)$$

where k_1, k_2, k_3 are positive parameters to be designed.

Substitute (14) into (13), we can get the closed-loop plant dynamics

$$\dot{e}_1 = a(e_2 - e_1),$$
$$\dot{e}_2 = Ae_2 \sin(\omega t) - k_1 sign(e_1) - k_2 sign(e_2) - k_3 sign(e_3),$$
$$\dot{e}_3 = -e_1 e_2 + e_1 y_2 + y_1 e_2 - be_3(t-\tau). \quad (15)$$

Choose a candidate Lyaupunov function for the system (15) as

$$V = (\alpha |e_1| + \frac{1}{2} e_2^{\,2})^{\frac{3}{2}} + e_1 e_2,$$

then the derivative of V along the trajectory of (15) can be derived as

$$\dot{V} = \frac{3}{2}(\alpha |e_1| + \frac{1}{2} e_2^{\,2})^{\frac{1}{2}} (\alpha sign(e_1)\dot{e}_1 + e_2 \dot{e}_2) + \dot{e}_1 e_2 + e_1 \dot{e}_2$$

$$= \frac{3}{2}(\alpha |e_1| + \frac{1}{2} e_2^{\,2})^{\frac{1}{2}} [\alpha sign(e_1) a(e_2 - e_1) + e_2 (Ae_2 \sin(\omega t) - k_1 sign(e_1)$$
$$-k_2 sign(e_2) - k_3 sign(e_3))] + a(e_2 - e_1)e_2 + e_1[Ae_2 \sin(\omega t) - k_1 sign(e_1) \quad -k_2 sign(e_2) - k_3 sign(e_3)]$$

$$\leq -\frac{3}{2}(\alpha|e_1|+\frac{1}{2}e_2^{2})^{\frac{1}{2}}|e_2|[(k_1-a\alpha)sign(e_1e_2)-Ae_2sign(e_2)\sin(\omega t)+k_2$$
$$+k_3sign(e_2e_3))]+ae_2^{2}+|e_1|[Ae_2\sin(\omega t)sign(e_1)-k_1-k_2sign(e_1e_2)-k_3sign(e_1e_3)-ae_2sign(e_1)].$$

Let $-AM+k_1-k_2-k_3-aM\geq 0$, $a\alpha-k_1\geq 0$, $k_1-a\alpha-AM+k_2-k_3\geq 0$, $|e_2|\leq M$, then we have

$$\dot{V}\leq -\frac{3}{2}(\alpha|e_1|+\frac{1}{2}e_2^{2})^{\frac{1}{2}}|e_2|[k_1-a\alpha-AM+k_2-k_3]+ae_2^{2}-|e_1|[-AM+k_1-k_2-k_3-aM].$$

Let

$$u=k_1-a\alpha-AM+k_2-k_3-\frac{2}{3}\sqrt{2}a,$$
$$v=-AM+k_1-k_2-k_3-aM,$$

then we can arrive

$$\dot{V}\leq -\frac{3}{2\sqrt{2}}ue_2^{2}-v|e_1|$$
$$\leq -m[(\alpha|e_1|+\frac{1}{2}e_2^{2})^{\frac{3}{2}}]^{\frac{2}{3}}, \tag{16}$$

where $m=\min\left\{\frac{v}{\alpha},\frac{3u}{\sqrt{2}}\right\}$.

Based on Corollary 1, we have

$$e_1e_2+(\alpha|e_1|+\frac{1}{2}e_2^{2})^{\frac{3}{2}}\leq 2((\alpha|e_1|+\frac{1}{2}e_2^{2})^{\frac{3}{2}}). \tag{17}$$

Substituting (16) into (15) leads to the inequation

$$\dot{V}\leq -m\frac{1}{2^{\frac{2}{3}}}[(\alpha|e_1|+\frac{1}{2}e_2^{2})^{2}+e_1e_2]^{\frac{2}{3}}=-\beta V^{\frac{2}{3}}, \tag{18}$$

where $\beta=m\frac{1}{2^{\frac{2}{3}}}$.

By solving the above inequality, one gets

$$V(t)\leq (V_0^{\frac{1}{3}}-\frac{\beta t}{3})^{3}. \tag{19}$$

Due to $V(t)\geq 0$, it follows that $\frac{\beta t}{3}\leq V_0^{\frac{1}{3}}$, which means that $t\leq\frac{3}{\beta}V_0^{\frac{1}{3}}$. Therefore, there exists constant $T_1=\frac{3}{\beta}V_0^{\frac{1}{3}}$ such that $\lim_{t\to T_2}e_1=\lim_{t\to T_2}e_2=0$. Based on the second of system (15), we can conclude that $\lim_{t\to T_2}e_1=\lim_{t\to T_2}e_2=0$ implies $\lim_{t\to T_2}e_3=0$. From Lemma 1, the error system (15) is finite-time stable. That is to say $e_1\equiv 0, e_2\equiv 0, e_3\equiv 0$ after a finite-time T_1. Therefore, when $t>T_1$, $y_1\equiv x_1, y_2\equiv x_2, y_3\equiv x_3$.

4. Simulation results

In this section, initial conditions of the master system and slave system are chosen as $(-0.12,-0.4,-0.7)$ and $(-0.13,-0.38,-0.71)$, respectively. The system parameters of are taken as $a=10$, $b=3$, $c=28$, $A=0.001$, $\omega=0.01$, $k_1=0.0021$, $k_2=0.001$, $k_3=0.001$. Fig.5 and Fig.6 show the dynamical behaviors of error systems of the delay Lorenz system with different τ , which proves that time-delay has influence on synchronous time.

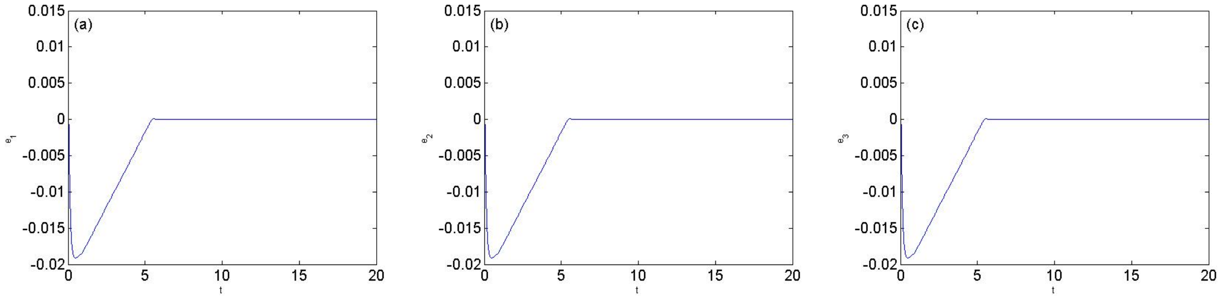

Fig.5. Synchronization errors of the delay Lorenz system when $\tau = 0.0019$.

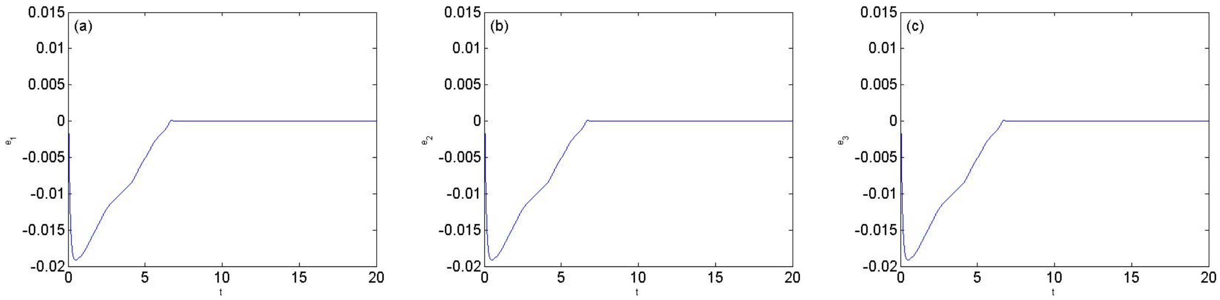

Fig.6. Synchronization errors of the delay Lorenz system when $\tau = 0.25$.

5. Conclusion

This paper is concerned with finite-time synchronization of the delay Lorenz systems with disturbance. Based on the finite-time stability theory, a control law is proposed to realize finite-time chaos synchronization of the delay Lorenz systems with disturbance via a single controller. Finally, numerical simulations are given to demonstrate the effectiveness and robustness of the proposed scheme. From the proof process we can see that the proposed method can be extended to other systems.

6. Acknowledgement

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