

# A coupling segmentation method based on CV model for high-noise image

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**Abstract.** For image segmentation methods, a clear image is often the object. High-quality segmentation is possible in many experiments. However, in the actual image, noise is inevitable. Many segmentation methods for high-noise images are not satisfactory. This paper puts forward a method of image coupling denoising and segmentation for high-noise image. A new variational model is adopted, then the denoised image is segmented using the improved CV model. The numerical calculation uses multiple directions difference to approximate the partial derivative, obtaining a rapid and stable effect. The experimental results show that the proposed coupling denoising and segmentation method could demonstrate validity. Where the image's, high noise is concerned, segmentation is obviously superior to the Li's[15] model.

**Keywords:** image segmentation; high noise; denoising; coupling model; CV model.

## 1. Introduction

In brief, image segmentation serves to separate the background and objective of an image. It is a key technology in image processing. Since the 1970s, it has been a subject of great interest. Thousands of algorithms have been proposed in relation. Image segmentation methods can be divided into four categories: threshold segmentation methods, edge detection methods, region extraction methods and methods based on specific theories. Threshold segmentation involves determining a threshold value in the range of the image gray value, then comparing the threshold with each of the pixel gray values in the image, moreover, the pixels are divided into two categories[2-5]. The most basic feature of the image is its edge, which is the result of a discontinuity in the local characteristics. Edge detection uses the first-order derivative of the image of the extreme value or the two-order derivative of the zero-point information to provide the basic basis for judging the edge point[6]. We use the first-order differential operators such as Roberts Operator, Sobel Operator, Prewitt Operator in practical work, the second-order differential operators Laplacian Operator, and Krish Operator. The advantages of the differential operator method are that it is fast and easy to calculate, and the disadvantage is sensitive to noise. The essence of region segmentation is to link up the pixels with some similar properties to form the final segmentation region. It uses the local spatial information of the image, which can effectively overcome the shortcomings of image segmentation in other ways, but it usually leads to over-segmentation of the image. In recent years, people have put forward a lot of segmentation technology combined with some specific theories, methods, and tools. Image segmentation based on mathematical morphology, image segmentation based on fuzzy theory, and image segmentation based on neural network[7-12] are proposed and so on. The active contour model has become a research hotspot in the field of image segmentation. It can effectively deal with variable topology. The variable level set method not only has the above advantages, but also can effectively integrate multiple model components. It is widely used in image segmentation, motion tracking, 3D reconstruction and so on. Experts have proposed many different manifestations of the active contour model: Mumford and Shah[13] proposed the MS model; Chan and Vese put forward the CV model[14].

This article will focus on high-noise images using the improved PM model to deal with the noise of the image; the variational model is obtained by Euler-Lagrang equation and gradient descent flow, differential equation with finite difference approximation of the pixels each direction derivative, makes full use of its local information. Using improved CV model segmentation, increase the punishment term, make maintain throughout the image area keep the property of signal distance, to avoid the cyclical to initialize, get a optimal segmentation image.

## 2. CV model and improved model

In 2001, Chan and Vese put forward the CV model based on M-S model. The CV model considers the simplest of segmentation, namely the image is divided into two parts, the background and target, the target and the background of gray level distribution constant values. The CV model of energy functional is obtained:

$$E(C, c_1, c_2) = \mu \text{Length}(C) + \lambda_1 \int_{inC} |I(x, y) - c_1|^2 dx dy + \lambda_2 \int_{outC} |I(x, y) - c_2|^2 dx dy \quad (1)$$

Among these,  $I(x, y)$  is for image segmentation,  $C$  is evolution curve, and  $\mu \geq 0, \lambda_1, \lambda_2$  is weight coefficient. In the energy functional, the first term corresponds to the length of the curve evolution, a regular role, after the two as binary fitting. Get the energy function level set representation:

$$E(\phi, c_1, c_2) = \mu \int_{\Omega} \delta_{\varepsilon}(\phi) |\nabla(\phi)| dx dy + \lambda_1 \int_{\Omega} |I - c_1|^2 H_{\varepsilon}(\phi) dx dy + \lambda_2 \int_{\Omega} |I - c_2|^2 (1 - H_{\varepsilon}(\phi)) dx dy \quad (2)$$

In the functional,

$$\delta_{\varepsilon}(\phi) = H_{\varepsilon}'(\phi) = \frac{1}{\pi} \frac{\varepsilon}{\varepsilon^2 + \phi^2} \quad (3)$$

is regularization form of the  $\delta(\phi)$  function. So  $\delta_{\varepsilon}(\phi)$  is not zero, makes the energy functional drive equation play a role in all levels, it has nothing to do with the curve of the initial position of the global minimum. In addition, this will help automatically detect the target of internal and external contours. Given the evolution curve corresponding to the level set function of  $\phi$ , minimise the energy functional  $E(\phi, c_1, c_2)$  available  $c_1, c_2$  expression.

$$c_1 = \frac{\int_{\Omega} I(x, y) H(\phi(x, y)) dx dy}{\int_{\Omega} H(\phi(x, y)) dx dy}, c_2 = \frac{\int_{\Omega} I(x, y) (1 - H(\phi(x, y))) dx dy}{\int_{\Omega} (1 - H(\phi(x, y))) dx dy} \quad (4)$$

Make the  $c_1, c_2$  remains invariant, according to the variational method and the steepest descent method, the partial differential equation of the controlled level set evolution is as follow:

$$\frac{\partial \phi}{\partial t} = \delta(\phi) [\mu \text{div}(\frac{\nabla \phi}{|\nabla \phi|}) - \lambda_1 (I - c_1)^2 + \lambda_2 (I - c_2)^2] \quad (5)$$

The CV model involves comprehensive utilisation of the global information of the image and gets the global optimal result of image segmentation. The model also has obvious defects because of inherent characteristics: (1) can't deal with gray inhomogenous images; (2) have to periodically to initialize level set function; (3) although the segmentation result is insensitive to the initial position of natural evolution, the evolution speed is obviously dependent on the initial position of the evolution curve; (4) despite the reduced sensitivity to noise, can't split images of the noise pollution seriously; antinoise ability is still not strong. To overcome the defect of the re-initialisation, Li et al[15] proposed the LBF(local binary fitting) method, namely the level set function  $\phi$ , adding the term

$$P(\phi) = \iint_{\Omega} \frac{1}{2} (|\nabla \phi| - 1)^2 dx dy \quad (6)$$

Obviously, minimising  $P(\phi)$  means that requires  $|\nabla \phi| = 1$ , which is then required in the evolution of the level set function, always staying as the signal distance function in the process. By the variational method and gradient descent method, we can obtain its gradient descent of flow.

$$\frac{\partial \phi}{\partial t} = \Delta \phi - \text{div}(\frac{\nabla \phi}{|\nabla \phi|}) = \text{div}[(1 - \frac{1}{|\nabla \phi|}) \nabla \phi] \quad (7)$$

This is a nonlinear heat equation, its conductivity rate

$$\alpha = 1 - \frac{1}{|\nabla \phi|} \quad (8)$$

When  $|\nabla\phi| > 1$ ,  $\alpha > 0$ , "heat" is transferred positively, and decreases  $|\nabla\phi|$ , and vice versa, when  $|\nabla\phi| < 1$ ,  $\alpha < 0$ , increased  $|\nabla\phi|$ . Thus, for any deviation form,  $|\nabla\phi| = 1$  local will be the "correction" in the subsequent evolution. So the re-initialisation is completely unnecessary.

### 3. PM model

Because of the hardware, the environment, human factors, and the influence of such elements as the image is always inevitably with noise, the noise decreases the image quality to some extent. The high-noise image segmentation result is not accurate, thus usually requiring the denoising work. Common denoising methods are based on the spatial domain, the methods based on frequency domain, and the one based on variational and partial differential equations[16-19]. In 1990, Perona and Malik[20] first proposed the anisotropic image smoothing method, to achieve denoising and protect the image edge details at the same time. A very natural idea is that the "transmission coefficient", which depends on the local characteristics of the image, is introduced in the process of diffusion. Specifically, in the plain area of the image, increase the coefficient automatically. This can be achieved in the flat area, smoothed out small irregular ups and downs; at the edge of the image area, conduction coefficient can be automatically reduced, so that it can hardly affect the image edge. The mathematical expression of PM diffusion equation is as follows:

$$\begin{cases} \frac{\partial u(x, y, t)}{\partial t} = \text{div}[g(|\nabla u|)\nabla u] \\ u(x, y, 0) = u_0(x, y) \end{cases} \quad (9)$$

where  $u(x, y, t)$  stands for evolving images,  $u_0(x, y, t)$  for the initial image, and  $g(\cdot)$  for conduction function. The conduction coefficient value is determined by the regional value of gradient modulus; this suggests that the different regions have different values of gradient modulus, thus the conduction coefficient is different. This shows that the PDE algorithm based on the PM equation can realise the nonlinear diffusion of anisotropy. It makes use of the gradient modulus value of the image. It combined the image edge detection with the image filtering process, the flat area to achieve smoothness, and the border area to achieve the sharpening.

The function  $g$  is a decreasing function of the value of gradient modulus.  $g(0) = 1$ ; when  $r$  tends to be infinite,  $g(r) = 0$ . An image in the border area is  $|\nabla u|$  very big, so  $g(|\nabla u|)$  is very small, in the image smooth area  $|\nabla u|$  is very small, so  $g(|\nabla u|)$  is very big to selective smoothing, image smoothing effect in the border area is small, on the contrary, the one on the flat area is large. So Perona and Malik proposed two forms of  $g(r)$ :

$$g(r) = \exp[-(r/k)^2] \quad (10)$$

$$g(r) = \frac{1}{1 + (r/k)^2} \quad (11)$$

However, the PM equation has two shortcomings: the gray value difference between the noise pixels and the surrounding pixels is great,  $|\nabla I|$  is big at this moment, function  $g$  is close to 0, diffusion velocity is reduced, does not favour the denoising; the PM equation of steady state solutions have no continuous dependence on initial conditions.

### 4. Denoising segmentation coupling model

To achieve a good segmentation for a high-noise image, we put forward a kind of denoising segmentation coupling model. We measure the denoising before image segmentation for a high-noise image. We use the improved model of the PM to denoise. It inherits the advantages of the PM model; it can also improve the denoising speed and keep the details of the boundary. In terms of segmentation, because the object of this paper is to contain a high-noise image, we not only need to deal with the noise of image before segmentation, and as far as possible in the segmentation of image smoothly, reduce the influence of noise to the segmentation result. This is the model based on the above ideas. The model proposed by Li et al in  $I(x, y)$  improved  $K = |\nabla G_\delta(x, y) * u_0(x, y)|$ , the influence of noise to a minimum.

Consider the following differential equation:

$$\frac{\partial u}{\partial t} = \text{div}\left[\frac{\nabla u}{1 + \left(\frac{\nabla u}{k}\right)^\alpha}\right] \tag{12}$$

$$\alpha = \alpha(|\nabla(G_\delta * f)|) = 2 - \frac{2}{1 + m(|\nabla(G_\delta * f)|)^2} \tag{13}$$

$G_\delta$  stands for  $\delta$  parameters of the Gaussian function. Catta[21] proposed to control the diffusion process by replacing the original image gradient with the gradient of the gradient after Gauss smoothing. The Gauss smoothing function can effectively suppress Gauss noise. After smoothing, the gradient model can more accurately reflect the changes of the image edge and not be affected by the Gauss noise.

When  $\alpha = 0$ , the original type into  $\frac{\partial u}{\partial t} = \Delta u$ , as the heat conduction equation, isotropic diffusion.

When  $\alpha = 2$ , the original type into  $\frac{\partial u}{\partial t} = \text{div}\left(\frac{\nabla u}{1 + (|\nabla u|/k)^2}\right)$ , as the heat conduction equation, anisotropic diffusion.

For the segmentation method, consider the following energy functional:

$$E(c_1, c_2, \phi) = \mu \text{Length}(C) + \nu \text{Area}(\text{inside}(C)) + \lambda_1 \int_{inC} |K - c_1|^2 dx dy + \lambda_2 \int_{outC} |K - c_2|^2 dx dy + \tau \int_{\Omega} \frac{1}{2} (|\nabla \phi(x)| - 1)^2 dx dy \tag{14}$$

Here  $\mu, \nu, \lambda_1, \lambda_2, \tau$  are positive constants. In the energy functional, the first three items are the special case of the CV of energy functional; the last item represents the level set function, the degree of deviating from the signal distance function, ensuring the level set function in the process of evolution to always be a similar signal distance function, which can overcome the CV model must periodically initialise level set function defects.

Using the variational principle and the steepest descent method[20,21], the corresponding partial differential equation is obtained.

$$\frac{\partial u}{\partial t} = \delta_\epsilon \left[ \mu \text{div}\left(\frac{\nabla \phi}{|\nabla \phi|}\right) - \lambda_1 (K - c_1)^2 - \lambda_2 (K - c_2)^2 \right] + \tau (\Delta \phi - \text{div}\left(\frac{\nabla \phi}{|\nabla \phi|}\right)) \tag{15}$$

The  $K(x, y) = |\nabla G_\delta(x, y) * u_0(x, y)|$ .

The method on the basis of the CV model joined the penalty term  $\frac{1}{2} (|\nabla \phi(x)| - 1)^2$ . To avoid the traditional level set segmentation periodically to initialisation, replace Li's model's  $I$  with  $K$ ; in the process of segmentation minimise the effects of noise on the experimental results.

### 5. Numerical implementation and experiment result

In the denoising process in this paper, to make full use of local information, all directions of derivative are approximated by finite difference equation. The PM model adopted the four directions to approximate the partial derivatives; this paper adopted the partial derivatives of the following eight directions, using N,S,E,W,NE,SE,SW,and NW to represent north, south, east, west, northeast, southeast, southwest, northwest.

$$\frac{\partial u}{\partial t} = \frac{u_{i,j}^{n+1} - u_{i,j}^n}{\Delta t} \tag{16}$$

$$(C_d^n)_{i,j} = \frac{1}{1 + (|\nabla_N u_{i,j}| / K)^{\alpha_{i,j}}} \tag{17}$$

$d \in \{N, S, E, W, NE, SE, SW, NW\}$ .

$$\text{div}_{i,j}^n = (C_N^n)_{i,j} \nabla_N u_{i,j} + (C_S^n)_{i,j} \nabla_S u_{i,j} + (C_E^n)_{i,j} \nabla_E u_{i,j} + (C_W^n)_{i,j} \nabla_W u_{i,j} + (C_{NE}^n)_{i,j} \nabla_{NE} u_{i,j} + (C_{SE}^n)_{i,j} \nabla_{SE} u_{i,j} + (C_{SW}^n)_{i,j} \nabla_{SW} u_{i,j} + (C_{NW}^n)_{i,j} \nabla_{NW} u_{i,j} \tag{18}$$

We take the following approach for boundary conditions:

$$\begin{aligned} u_{i,j}^0 &= f_{i,j} = f(ih, jh), u_{i,1}^n = u_{i,2}^n \\ u_{1,j}^n &= u_{2,j}^n, u_{I,j}^n = u_{I-1,j}^n, u_{i,J}^n = u_{i,J-1}^n \end{aligned}$$

Among them:

$$\begin{aligned} \nabla_N u_{i,j} &= u_{i-1,j}^n - u_{i,j}^n, \nabla_S u_{i,j} = u_{i+1,j}^n - u_{i,j}^n \\ \nabla_E u_{i,j} &= u_{i,j+1}^n - u_{i,j}^n, \nabla_W u_{i,j} = u_{i,j-1}^n - u_{i,j}^n \\ \nabla_{NE} u_{i,j} &= u_{i+1,j+1}^n - u_{i,j}^n, \nabla_{SE} u_{i,j} = u_{i+1,j-1}^n - u_{i,j}^n \\ \nabla_{NW} u_{i,j} &= u_{i-1,j+1}^n - u_{i,j}^n, \nabla_{SW} u_{i,j} = u_{i-1,j-1}^n - u_{i,j}^n \\ 1 \leq i \leq I, 1 \leq j \leq J \end{aligned}$$

Through this new method, we can control the spread of the form. Close to the edge area, the new model is similar with the PM model, with a backward diffusion to preserve and even enhance image boundary; in the inner region of the image, the new model has diffusion way forward. The heat diffusion equation removes the noise points and smooths the image. Based on the extremum principle and the Lyapunov equation[22], solutions to the equation are stable and converge to a constant.

This model uses the simple finite difference method approximating the partial derivative. The first set  $h$  is the spatial step length,  $\Delta t$  is time step, grid point  $(x_i, y_i) = (ih, jh)$ , the  $1 \leq i, j \leq M$ ,  $n$  time  $\phi(t, x, y)$  in the grid point approximation for  $\phi_{i,j}^n = \phi(n\Delta t, x_i, y_i)$ , here  $n \geq 0$ ,  $\phi^0 = \phi_0$ , use forward difference approximation instead of  $\phi_x, \phi_y$

$$\phi_x \approx \frac{\phi_{i+1,j}^n - \phi_{i,j}^n}{h}, \phi_y \approx \frac{\phi_{i,j+1}^n - \phi_{i,j}^n}{h} \text{ for}$$

$$c_1 = \frac{\int_{\Omega} I(x, y) H(\phi(x, y)) dx dy}{\int_{\Omega} H(\phi(x, y)) dx dy}, c_2 = \frac{\int_{\Omega} I(x, y) (1 - H(\phi(x, y))) dx dy}{\int_{\Omega} (1 - H(\phi(x, y))) dx dy} \quad (19)$$

$$|\nabla \phi| = \sqrt{\frac{\partial \phi^2}{\partial x} + \frac{\partial \phi^2}{\partial y} + s} \quad (20)$$

$s$  is a very small constant, taking  $s = 10^{-6}$  in experiment. This paper chose the regularisation of the Dirac function  $\delta_{\varepsilon}$ , same as the CV model,  $\varepsilon = 1.5$ .

In this paper, we added an image to the noise, the image does not denoise and denoise, and then compared with the segmentation result. The experiment selected the image which Li et al. selected in the experiment. The initial contour captured the edges by two ways: "expansion" and "contraction". So we implement the result by two ways. In this experiment, the threshold  $T$  off for 70. Time step  $\Delta t = 3/44$ , stability of the eight iterations are implemented. We make the time step in the process of segmentation as 1, parameter  $\mu = 0.2$ ,  $\lambda_1 = \lambda_2 = 5$ ,  $\alpha = -3$ . Dirac function of  $\varepsilon = 1.5$ ; in the image, Gaussian convolution via  $\sigma = 0.8$ . In segmentation process evolution, in the first example a total of 210 times were carried out, and 200 times is the capture of image contour, to fine-tune outline 10 times. In the second example a total of 230 times were carried out, and 220 times is the capture of image contour, to fine-tune outline 10 times.

Comparing figure 4 and figure 6, we can get the complete contour after denoising; figure 14 is also better than figure 11. And we can get a steady result by a few iterations.



Figure 1. Original Image.

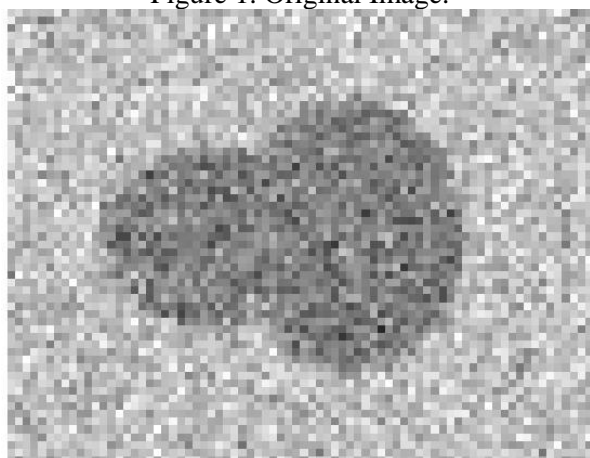


Figure 2. Noise Image.

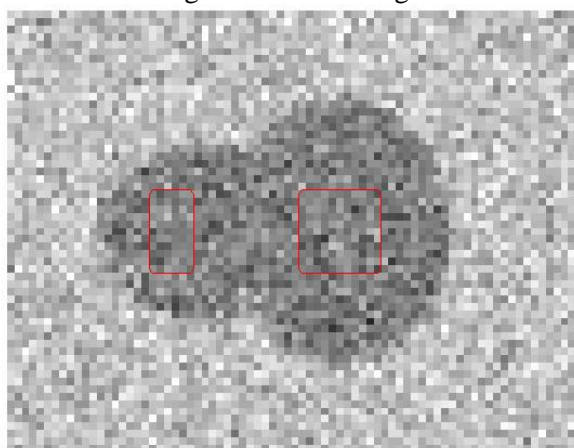


Figure 3. Initial Level Contour of Noise Image.

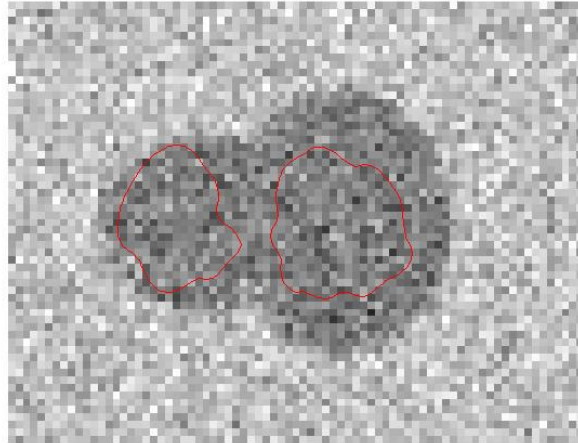


Figure 4. Noise Image Segmentation.

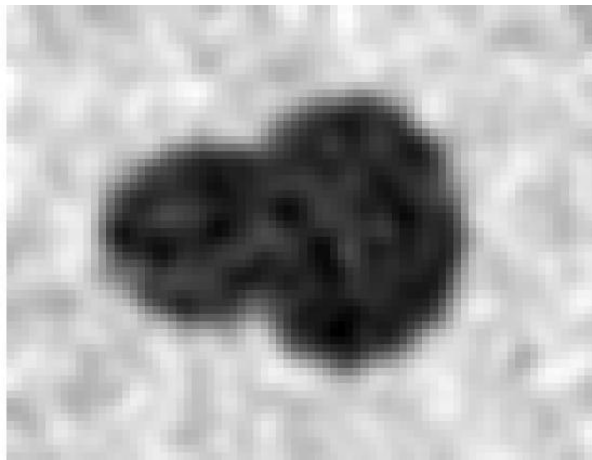


Figure 5. Noise Image After Denoised.

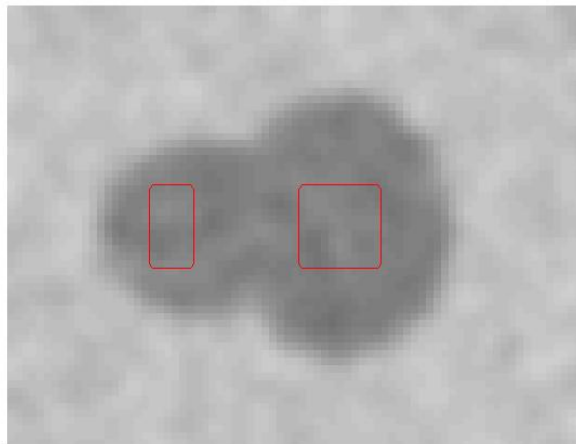


Figure 6. Initial Level Contour of Denoise Image.

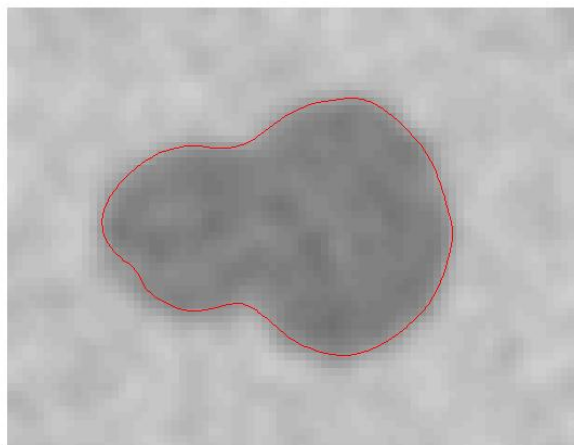


Figure 7. Noise Image Segmentation After Denoising.

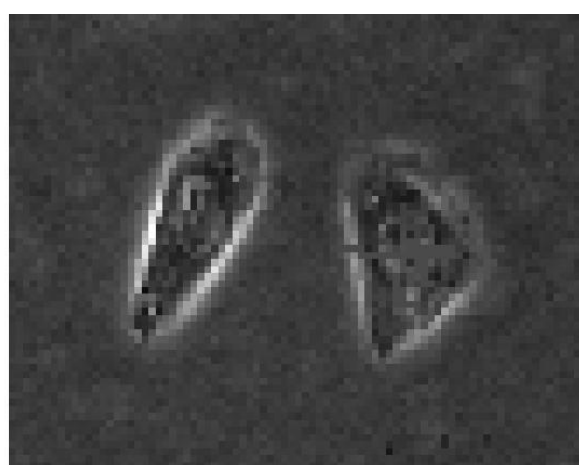


Figure 8. Original Image.

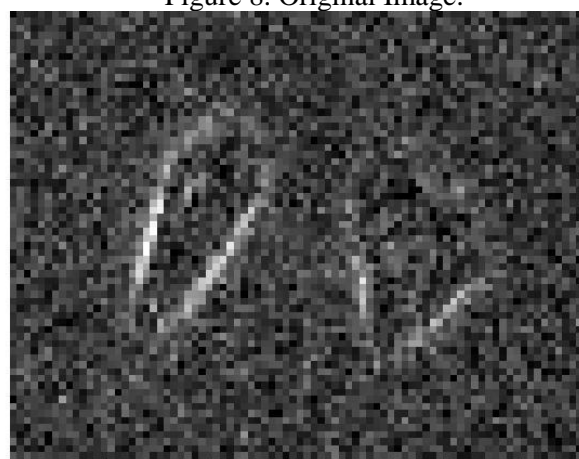


Figure 9. Noise Image.



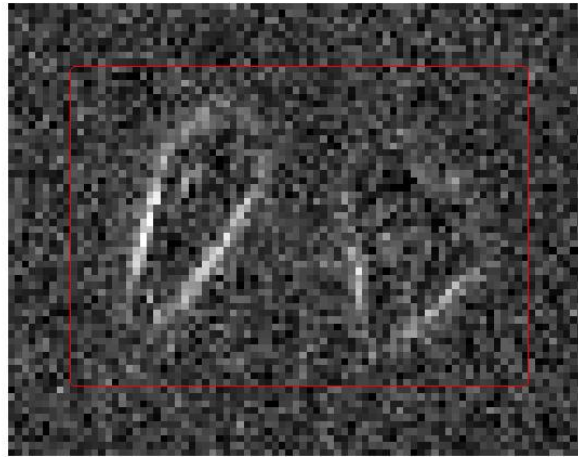


Figure 10. Initial Level Contour of Noise Image.

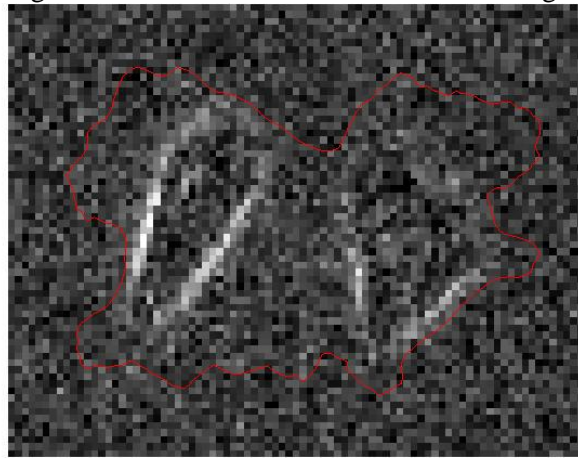


Figure 11. Noise Image Segmentation.

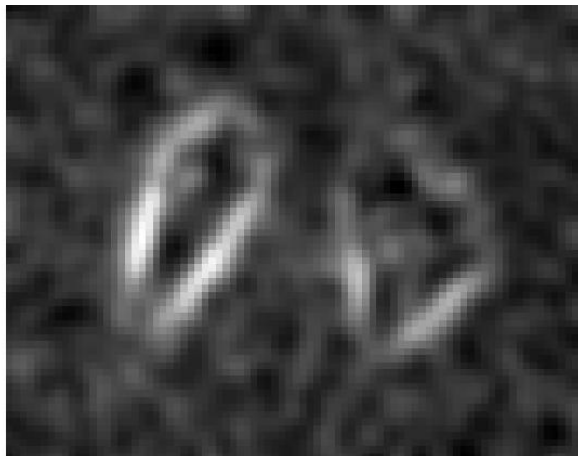


Figure 12. Noise Image After Denoising.

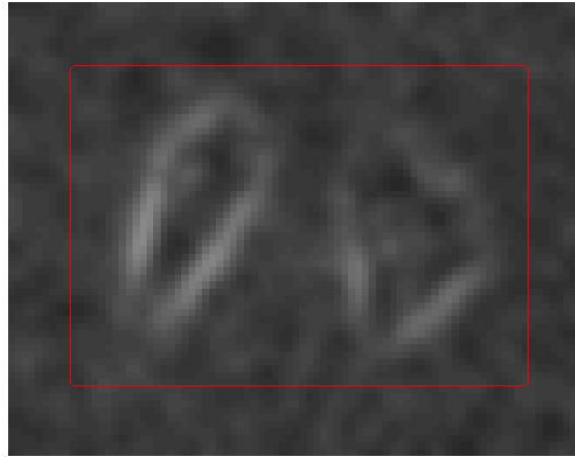


Figure 13. Initial Level Contour of Denoised Image.



Figure 14. Noise Image Segmentation After Denoising.

## 6. Conclusion

For high-quality pictures, we have been able to get good result, but for noise image, especially the high-noise image, our segmentation method result is not satisfactory. In real life, however, the presence of noise is inevitable. So improving image segmentation resulting in high-noise conditions is very necessary. This article starts working in this context. In this paper, for high-noise images, use a method of denoising segmentation coupled to denoise images first, then segment. During the process of denoising, we used an improved PM model, accelerated the diffusion rate, and the equation of the steady state solution of continuous dependence on the initial conditions, and made up for the deficiency of the original PM model. In the process of segmentation, we increased the penalty term of the CV model, which need not be a cyclical level set evolution process of initialisation, and substitute  $I$  in  $K$  in the model, the effects of the image noise to a minimum. Without denoising directly for high-noise contrast of image segmentation, we found that our proposed model can get a better result. But it is a long way to get our imagine "perfect"; we believe we have a lot of work to do in the future.

## 7. References

- [1] Yujin Zhang, Image Segmentation, Beijing: Beijing Science Press. 29-51(2001).
- [2] Otsu N, A threshold selection method from gray-level histogram, IEEE transation on systems. 33(6): 130-136(2009).
- [3] Xueqiang Yan, Xiuqing Ye, Jilin Liu, Maximum entropy thresholding algorithm based on quantization image histogram, Pattern recognition and artificial intelligence. 10(6): 111-114(1999).
- [4] Papamarkes N, Gatos B, A new approach for multilevel threshold selection, Graphical models and Image Process. 56(5): 357-370(1994).

- [5] Olivo J C, Automatic threshold selection using the wavelet transform, *Graphical models and Image Process.* 56(5): 357-370(1994).
- [6] Galambosc, Kittlerj, Matasj, Gradient Based Progressive Probabilistic Hough Transform, *Image Signal Process.* 148(3): 158-165(2001).
- [7] Victor O.R, Juan I G L, Nicolas S L, Pedro G V, An improved watershed algorithm based on efficient computation of shortest paths, *Pattern Recognition.* 40(3): 1078-1090(2001).
- [8] Masooleh M G, Moosavi SAS, An improved fuzzy algorithm for image segmentation, *Proceeding of World Academy of Science Engineering and Technology.* 28(4): 400-404(2008).
- [9] H.Berg, R.Olsson, T.Lindblad et al, Automatic design of pulse coupled neurons for image segmentation, *Neuro-computing.* 71(6): 1980-1993(2008).
- [10] Chengxin Yan, Nong Sang, Tianxu Zhang, Research progress of image segmentation based on graph theory, *Computers and Applications.* 42(5): 11-14(2006).
- [11] Pavan M, Pelillo M, A new graph-theoretic approach to clustering and segmentation, *Pro IEEE conf Computer Vision and Pattern Recognition[C]: USA: IEEE.* 145-152(2003).
- [12] Bingtao Liu, Zhen Tian, Xiaobing Li, Qiangfeng Zhou, Multi scale segmentation of SAR images based on graph theory Gomory-Hu algorithm, *Journal of Astronautics.* 29(3): 1002-1007(2008).
- [13] Mumford D, Shah J, Optimal approximations by piecewise smooth functions and associated variational problems, *Communications on Pure and Applied Mathematics.* 42: 577-685(1989).
- [14] Chan T.F., Vese L.A, Active contours without edges, *IEEE Trans. Image Processing.* 10(2): 266-277(2001).
- [15] Chunming Li, Chenyang Xu, Changfeng Gui, Martin D.Fox, Distance Regularized Level Set Evolution and Its Application to Image Segmentation, *Transactions on Image Processing.* 19(12): (2010).
- [16] Zhihong Luo, Guocan Feng, Guan Yang, A Novel Variation Segmentation Method for High-noise Image, *National Conference on image and graphics.* (2009).
- [17] Rafael C.Gonzalez, Richard E.Woods, *Digital Image Processing,* *Pattern Recognition.* 19(1): 41-47(1986).
- [18] J.Weickert.B.M.ter Haar Romeny, M.A.Viergever, Efficient and reliable schemes for nonlinear diffusion filtering, *IEEE Trans.Image Process.* 7(3): 398-410(1998).
- [19] Haiping Xu, Meiqing Wang, Choi-Hong Lai, Generalized Newton Method for Minimization of A Region-based Active Contour Model, *12th International Symposium on Distributed Computing and Applications to Business Engineering Science.* (2013).
- [20] P.Perona and J.Malik, Scale-space and edge detection using anisotropic diffusion, *IEEE Trans, Pattern Analysis and Machine Intelligence.* 12(7): 629-639(1990).
- [21] F.Catte, T.Coll, P.L.Lions, Image selective smoothing and edge detection by nonlinear diffusion II, *SIAM J. Numer. Anal.* 29: 845-866(1992).
- [22] JB Pomet, L Praly, Adaptive nonlinear regulation: estimation from the Lyapunov equation, *IEEE Transaction.* 37(6): 729-740(1992).