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Haar wavelet collocation method to solve problems arising in induction motor

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Abstract. In this paper, Haar wavelet collocation method has been employed to solve problems occur in the mathematical modeling of induction motor. To approximate the solution algorithm based on Haar wavelet is considered. The order of convergence is estimated for discussed problems. The accuracy issues of the solutions are demonstrated by comparing with other numerical techniques existing in the literature.

Keywords: Haar wavelets; induction motor; collocation method; convergence analysis.

1. Introduction

Alfred Haar [1] introduced the notion of wavelets which are called Haar wavelets. These wavelets placed a crucial role for the numerical solution of differential or integral equations. At present there are two approaches to applying the Haar wavelet for integrating ordinary differential equations (ODE). In case of the first method for integrating ODE concept of operational matrix is introduced by Chen and Hsiao [2, 3]. Another approach is called direct method due to Lepik [4]. In this approach Haar functions are integrated directly. The direct method is easily applicable for calculating integrals of arbitrary order but the operational matrix method has been used mainly for first order integrals. Haar wavelets consists of piecewise constant functions and are therefore the simplest orthogonal wavelets with compact support. Here the fact that Haar wavelets are not continuous and hence derivatives do not exists at the braking points. So that it is not possible to apply the Haar wavelets for solving ODEs directly. The main advantage of the method is that it can be used directly without using restrictive assumptions.

The fifth-order differential equations arise in modeling of induction motor with two rotor circuits. This model contains two stator state variables, two rotor state variables and one shaft speed. Normally, two more variables must be added to account for the effects of a second rotor circuit representing deep bars, a starting cage or rotor distributed parameters. To avoid the computational burden of additional state variables when additional rotor circuits are required model is often limited to the fifth order and rotor impedance is algebraically altered as function of rotor speed. This is done under the assumption that the frequency of rotor currents depends on rotor speed. This approach is efficient for the steady state response with sinusoidal voltage [5].

The existence and uniqueness solutions of fifth order boundary value problems (BVPs) are discussed by Agarwal [6]. Over the years some researchers have worked on induction motor problems by using different methods for numerical solutions. Reddy et al. [7] have demonstrated the superiority of the HWCM for the solution of seventh order ODEs of induction motor with two rotor circuits. Siddiqi et al. [8] estimated the solution for linear special case fifth-order two-point boundary value problems by non-polynomial sextic spline method (NPSS). Farajeyan and Jalilian [9] have found the numerical solution by fifth order BVPs in off step points (OSPM). Mehdi Golomi et al. [10] solved fifth order differential equations by He's Variational iteration method (VIM). This paper deals with fifth order ODEs arising in modeling of induction motor; these problems have the following general representation:

$$y^{(5)}(x) = f(x, y, y^{(1)}, y^{(2)}, y^{(3)}, y^{(4)}), \qquad x \in (c, d), \tag{1}$$

subject to the following conditions:

Case I: Initial value problem:

$$y(c) = \alpha_1,\ y^{(1)}(c) = \beta_1,\ y^{(2)}(c) = \gamma_1,\ y^{(3)}(c) = \delta_1,\ y^{(4)}(c) = \eta_1. \tag{2}$$

Case II: Boundary value problems of **Type 1:**

$$y(c)=\alpha_2,\ y^{(1)}(c)=\beta_2,\ y^{(2)}(c)=\gamma_2,\ y(d)=\delta_2,\ y^{(1)}(d)=\eta_2. \tag{3}$$

Case III: Boundary value problem of **Type 2:**

$$y(c)=\alpha_3,\ y^{(1)}(c)=\beta_3,\ y^{(3)}(c)=\gamma_3,\ y(d)=\delta_3,\ y^{(1)}(d)=\eta_3. \tag{4}$$

Where α_i's, β_i's, γ_i's, δ_i's, η_i's, c and d are real constants for $i=1,2,3.$

This article is organized as, in section 2 notations of Haar wavelets and their integrals are introduced. In section 3, numerical algorithm based on wavelets is introduced. In section 4 convergence analysis is presented. In section 5 we reported our numerical results with comparison. In the final section conclusion of our work has been discussed.

2. Haar wavelets and their integrals

In this section, we obtain orthogonal basis for the subspaces of $L^2[c,d]$ called Haar wavelet family. For this notations introduced in Ref. [4] are used. The interval $[c,d]$ is divided into 2^{J+1} subintervals of equal length $\left(\Delta t=\frac{(d-c)}{2^{J+1}}\right)$, where J is called maximal level of resolution. We have coarser resolution values $j=0,1,2,\ldots,J$ and translation parameter $k=0,1,2,\ldots,2^j-1$. With these two parameters i^{th} Haar wavelet in Haar family is defined as

$$h_i(t)=\begin{cases}1, & for\ t\in[\zeta_1(i),\zeta_2(i)),\\ -1, & for\ t\in[\zeta_2(i),\zeta_3(i)),\\ 0, & otherwise,\end{cases} \tag{5}$$

here $i=m+k+1, \zeta_1(i)=c+2k\mu\Delta t, \zeta_2(i)=c+(2k+1)\mu\Delta t$ and $\zeta_3(i)=c+2(k+1)\mu\Delta t$, where $\mu=2^{J-j}$.

Above equations are valid for $i>2$. $h_1(t)$ and $h_2(t)$ are called father and mother wavelets in Haar wavelet family and are given by

$$h_1(t)=\begin{cases}1, & for\ t\in[c,d),\\ 0, & otherwise,\end{cases} \tag{6}$$

$$h_2(t)=\begin{cases}1, & for\ t\in\left[c,\ p\right),\\ -1, & for\ t\in\left[p,d\right),\\ 0, & otherwise,\end{cases} \tag{7}$$

where, $p=\frac{c+d}{2}$.

Any function which is having finite energy on $[c,d]$, i.e. $f\in L^2[c,d]$ can be decomposed as infinite sum of Haar wavelets:

$$f(x)=\sum_{i=1}^{\infty}a_ih_i(x), \tag{8}$$

where a_i's are called Haar coefficients. If f is either piecewise constant or wish to approximate by piecewise constant on each subinterval then the above infinite series will be terminated at a finite number of terms. Since, we have explicit expression for each member of Haar family (5- 7). We can integrate as many times depend upon the context. The following notations are used for γ times of integration of members in the family defined on $[c,d)$:

$$P_{\gamma,i}(t)=\int_c^t\int_c^t.....\int_c^t h_i(x)\,dx^{\gamma}, \tag{9}$$

$$E_{\gamma,i}=\int_c^d P_{\gamma,i}(t)dt. \tag{10}$$

For $i=1$, (9) becomes

$$P_{\gamma,1}(t)=\frac{1}{\gamma!}(t-c)^{\gamma}, \tag{11}$$

for $i\geq 2$, we have

$$P_{\gamma,i}(t)=\begin{cases}0, & if\ t\in[c,\zeta_1(i)),\\ \frac{1}{\gamma!}(t-\zeta_1(i))^{\gamma}, & if\ t\in[\zeta_1(i),\zeta_2(i)),\\ \frac{1}{\gamma!}\left\{(t-\zeta_1(i))^{\gamma}-2(t-\zeta_2(i))^{\gamma}\right\}, & if\ t\in[\zeta_2(i),\zeta_3(i)),\\ \frac{1}{\gamma!}\left\{(t-\zeta_1(i))^{\gamma}-2(t-\zeta_2(i))^{\gamma}+(t-\zeta_3(i))^{\gamma}\right\}, & if\ t\in[\zeta_3(i),d).\end{cases} \tag{12}$$

3. Method of solution

Haar wavelet collocation method: The proposed method is as follows [4, 7].

- Approximate highest derivative in terms of Haar wavelets

$$y^{(5)}(x)=\sum_{i=1}^{2^{J+1}}a_i h_i(x). \tag{13}$$

- Decompose $y^{(4)}(x), y^{(3)}(x), y^{(2)}(x), y^{(1)}(x)$ and $y(x)$ in terms of integrated Haar functions and replace these in to the given linear differential equation.

- Discritize equation obtained in above at collocation points: $x_l=\frac{(x_{l-1}+x_l)}{2},\ l=1,2,...2^{J+1}$,

where $x_n=c+n\Delta t,\quad n=0,1,2,...,2^{J+1}$. Resulting into $2^{J+1}\times 2^{J+1}$ linear algebraic system. Calculate the wavelet coefficients a_i's and obtain the approximate solution for problem of induction motor.

In this paper, problems originating in modeling of induction motor defined over $[0,1]$ are considered. The proposed method is further simplified with the help of particular initial or boundary conditions. For IVPs: $c=0$ and BVPs : $c=0, d=1$. Integrate equation (13) from 0 to x five times we obtained the approximate solution.

$$y(x)=y(0)+xy^{(1)}(0)+\frac{x^2}{2!}y^{(2)}(0)+\frac{x^3}{3!}y^{(3)}(0)+\frac{x^4}{4!}y^{(4)}(0)+\sum_{i=1}^{2^{J+1}}a_iP_{5,i}(x)\ . \tag{14}$$

Using equation (2) we can find the solution for initial value problem. Approximate solution for BVPs can be estimated by finding the $y^{(3)}(0), y^{(4)}(0)$ values using boundary conditions of type 1 and $y^{(2)}(0), y^{(4)}(0)$ values using boundary conditions of type 2, these unknowns are expressed as follows

Type 1:

$$y^{(3)}(0)=-24\alpha_2-18\beta_2-6\gamma_2+24\delta_2-6\eta_2+\sum_{i=1}^{2^{J+1}}a_i(-24E_{5,i}+6E_{4,i}), \tag{15}$$

$$y^{(4)}(0)=72\alpha_2+48\beta_2+12\gamma_2-72\delta_2+24\eta_2+\sum_{i=1}^{2^{J+1}}a_i(72E_{5,i}-246E_{4,i}), \tag{16}$$

Type 2:

$$y^{(2)}(0)=-4\alpha_3-3\beta_3-\frac{1}{6}\gamma_3+4\delta_3-\eta_3-\sum_{i=1}^{2M}a_i(4E_{5,i}-E_{4,i}), \tag{17}$$

$$y^{(4)}(0)=24\alpha_3+12\beta_3-2\gamma_3-24\delta_3+12\eta_3+\sum_{i=1}^{2M}a_i(24E_{5,i}-12E_{4,i}). \tag{18}$$

Where,

$$E_{4,i} = \int_0^1 P_{4,i}(x)\,dx \quad and \quad E_{5,i} = \int_0^1 P_{5,i}(x)\,dx. \tag{19}$$

4. Convergence analysis of Haar wavelet discretization method (HWDM)

The accuracy issue of the HWDM was open from year 1997. This issue is clarified by J. Majak et al. [11] in 2015. Following results are due to notations introduced in Ref. [11].The general form of fifth order ODE is

$$f\left(x, y, y^{(1)}, y^{(2)}, y^{(3)}, y^{(4)}, y^{(5)}\right) = 0. \tag{20}$$

Expand fifth order derivative into Haar wavelets as

$$\frac{d^5 y(x)}{dx^5} = \sum_{i=1}^{\infty} a_i h_i(x) \tag{21}$$

$$= a_1 h_1 + \sum_{j=0}^{\infty} \sum_{k=0}^{2^j-1} a_{2^j+k+1} h_{2^j+k+1}(x). \tag{22}$$

In equations (21) and (21) $2^j + k + 1 = i,\ k = 0,1,....,2^j - 1$. Integrating equation (22) five times, we obtain the solution of DE (20) as

$$y(x) = \frac{a_1}{5!} + \sum_{j=0}^{\infty} \sum_{k=0}^{2^j-1} a_{2^j+k+1} P_{5,\,2^j+k+1}(x) + B(x). \tag{23}$$

Here $P_{5,\,2^j+k+1}(x)$ can be calculated with aid of equation (12) and $B(x)$ is a boundary term.

Let us assume that $\frac{d^5 y(x)}{dx^5} \in L^2(R)$ is a continuous and its next derivative is bounded on $[0,1]$, i.e. $\exists\ \eta : \left|\frac{d^6 y(x)}{dx^6}\right| \le \eta$

Let $y_{2^{J+1}}(x) = \frac{a_1}{5!} + \sum_{j=0}^{J} \sum_{k=0}^{2^j-1} a_{2^j+k+1} P_{5,2^j+k+1}(x) + B(x)$ be the approximation to unknown y by Haar wavelets. The absolute error at the J^{th} resolution is denoted as $|E_{2^{J+1}}|$ and is given by

$$\left|E_{2^{J+1}}\right| = \left|y(x) - y_{2^{J+1}}(x)\right| = \left|\sum_{j=J+1}^{\infty} \sum_{k=0}^{2^j-1} a_{2^j+k+1} P_{5,2^j+k+1}(x)\right|. \tag{24}$$

Norm of the error in Hilbert space $L^2(R)$ [11] is defined as

$$\| E_{2^{J+1}} \|_2^2 = \int_0^1 \left(\sum_{j=J+1}^{\infty} \sum_{k=0}^{2^j-1} a_{2^j+k+1} P_{5,2^j+k+1}(x)\right)^2 dx$$

$$= \sum_{j=J+1}^{\infty} \sum_{k=0}^{2^j-1} \sum_{r=J+1}^{\infty} \sum_{s=0}^{2^r-1} a_{2^j+k+1} a_{2^r+s+1} \int_0^1 P_{5,2^j+k+1}(x) P_{5,2^r+s+1}(x)\,dx, \tag{25}$$

J. Majak et al. [11, 12] have shown that $a_i \le \frac{\eta}{2^{J+1}}$, for $i = 2^j + k + 1$ and $P_{5,i}(x)$ are monotonically increasing on $[0,1)$. Equation (25) can be estimated as

$$\left\|E_{2^{J+1}}\right\|_2^2 \le \frac{\eta^2}{4} \sum_{j=J+1}^{\infty} \sum_{k=0}^{2^j-1} \sum_{r=J+1}^{\infty} \sum_{s=0}^{2^r-1} \frac{1}{2^j} \frac{1}{2^r} \times \left[\frac{1}{6}\left(\frac{1}{2^{j+1}}\right)^2 + \frac{1}{12}\left(\frac{1}{2^{j+1}}\right)^4\right] \times \left[\frac{1}{6}\left(\frac{1}{2^{j+1}}\right)^2 + \frac{1}{12}\left(\frac{1}{2^{j+1}}\right)^4\right]. \tag{26}$$

Above equation can be simplified as

$$\left(\text{factrization and } \sum_{r=J+1}^{\infty}\left(\frac{1}{2^{r+1}}\right)=\left(\frac{1}{2^{2m}-1}\right)\times\left(\frac{1}{2^{J+1}}\right)^2, \ \mathrm{m}=1,2. \right)$$

$$\left\|E_{2^{J+1}}\right\|_2 \le \frac{\eta}{36}\left[\left(\frac{1}{2^{J+1}}\right)^2+\frac{1}{10}\left(\frac{1}{2^{J+1}}\right)^4\right]. \tag{27}$$

Therefore,

$$\left\|E_{2^{J+1}}\right\|_2 = O\left[\left(\frac{1}{2^{J+1}}\right)^2\right]. \tag{28}$$

From equation (28), we can conclude that the convergence is of order two.

5. Numerical studies

We considered some problems of induction motor whose exact solutions are known. The approximate solution for each problem is devised by the HWCM. All computations are carried out by MATLAB software.

Example 1: Consider the initial value problem [10],

$$y^{(5)}(x)=5(x-1)\sin(x)+5(x-x^2-5)\cos(x)-xy(x), \qquad x\in[0,1], \tag{29}$$

with initial conditions:

$$y(0)=5,\ y^{(1)}(0)=-5,\ y^{(2)}(0)=-5,\ y^{(3)}(0)=15,\ y^{(4)}(0)=5. \tag{30}$$

Using the method of solution solving equation (29), we get the Haar coefficients $[a_1,a_2,...,a_{2^{J+1}}]=[-21.8,-2.9,\ldots,-1.4]$ for J=4.. By the help of a_i's Haar wavelet solution y(x) is found. Obtained solution is represented with analytic solution $y(x)=5(1-x)\cos(x)$ in Table 1.

Example 2: Consider the induction motor problem [5],

$$y^{(5)}(x)+\sin(x)\,y(x)=\cos(x)(1+\sin(x))+\sin(x)(\sin(x)-1), \qquad x\in[0,1], \tag{31}$$

with boundary conditions:

$$y(0)=1,\ y(1)=\cos(1)+\sin(1),\ y^{(1)}(0)=1,\ y^{(1)}(1)=\cos(1)-\sin(1),\ y^{(2)}(0)=-1. \tag{32}$$

Worked out the equation (31) of Type1, we found the Haar coefficients $[a_1,a_2,...,a_{2^{J+1}}]=[0.38,0.33,...,0.08]$ for J=5. Deduced the solution using a_i's, equations (14-16) and is represented with the analytic solution $y(x)=\cos(x)+\sin(x)$ in Table 2.

Example 3: Consider the induction motor problem [5],

$$y^{(5)}(x)+y(x)=4e^x\cos(x)+2e^x(1-\sin(x))+5e^x\sin(x), \qquad x\in[0,1], \tag{33}$$

with boundary conditions:

$$y(0)=1,\ y(1)=e(1-\sin(1)),\ y^{(1)}(0)=0,\ y^{(1)}(1)=-e(\cos(1)+\sin(1)-1),\ y^{(2)}(0)=-1. \tag{34}$$

Haar coefficient vector of Type 1 BVP for J=4 is $[a_1,a_2,...,a_{2^{J+1}}]=[10.8,-3.24,...,-0.91]$. Approximate solution y(x) is found using equations (14-16) and compared with the exact solution $y(x)=e^x(1-\sin(x))$ in Table 3.

Example 4: Consider the boundary value problem [9],

$$y^{(5)}(x)=y(x)-(15+10x)e^x, \qquad x\in[0,1], \tag{35}$$

with boundary conditions:

$$y(0)=0,\ y(1)=0,\ y^{(1)}(0)=1,\ y^{(1)}(1)=-e,\ y^{(3)}(0)=0. \tag{36}$$

$[a_1,a_2,...,a_{2^{J+1}}]=[-35.4,12.73,...,5.16]$ are Haar coefficients of Type 2 BVP for J=6. These findings, equations (14), (17) & (18) are approximated the solution. The comparison of obtained solution and analytic solution $x(1-x)e^x$ are tabulated in Table 4.

6. Results and discussion

Absolute errors obtained by HWCM are compared to other existing techniques, exhibit the results that as the level of resolution increases absolute error curves of HWCM are goes closer to **x**-axis (Where the absolute errors are zero). The comparison of approximate solution, exact solution at collocations points with **J=3,4,5 & 6** for **Examples 1, 2, 3 & 4** have been demonstrated in **Figures 1, 3, 5, 7** respectively. Here in each figure approximate solution coincided with the exact solution, this assures the exactness of HWCM results. Absolute errors obtained by the **Examples 1, 2, 3 & 4** are drawn in **Figures 2, 4, 6& 8** for various resolutions. In all these figures as the level of resolution increased absolute errors tended towards **x** axis. These graphs concluded that decreasing of absolute errors show the accuracy of the HWCM solutions. To check the efficiency of the method quantitatively **6 Tables** are designed. In **Tables 1-4** approximate solution, exact solution, absolute errors are at the grid points **0.1, 0.2,…, 0.9** with **J=4, 5, 4 & 6** for **Example 1-4** are listed. Each tables explored the comparison of exact and Haar solution. By this observation concluded that approximate solutions are much closed to analytic solutions. In **Table 5** Maximum absolute errors obtained by **Examples 1, 2& 3** for different values of **J** are compared to non polynomial spline method. This table ensures that maximum absolute errors of HWCM are very less compared to NPSM. Accuracy of solutions obtained for **Example 4** is examined in **Table 6.** HWCM has given least maximum absolute errors for **J=7, 8, 9** compared to OSPM and NPSS methods. **Tables 1-6** depict as accuracy of the wavelet solutions are less satisfactory for small value of **J**; in case increases occur in the resolution better accuracy can be achieved.

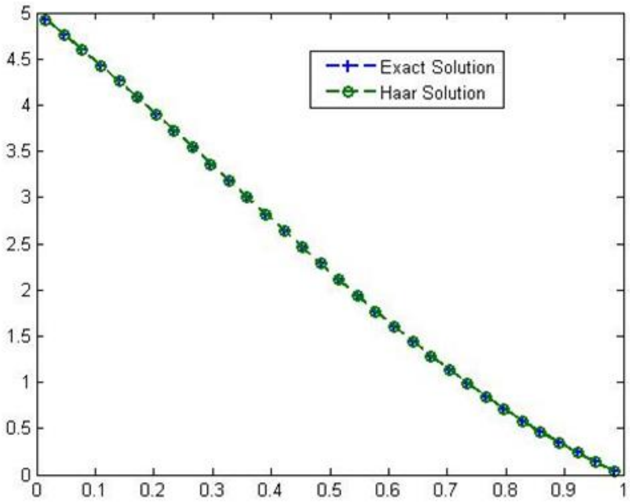

Fig.1. Comparison of exact and Haar solution of **Example 1** for J=4

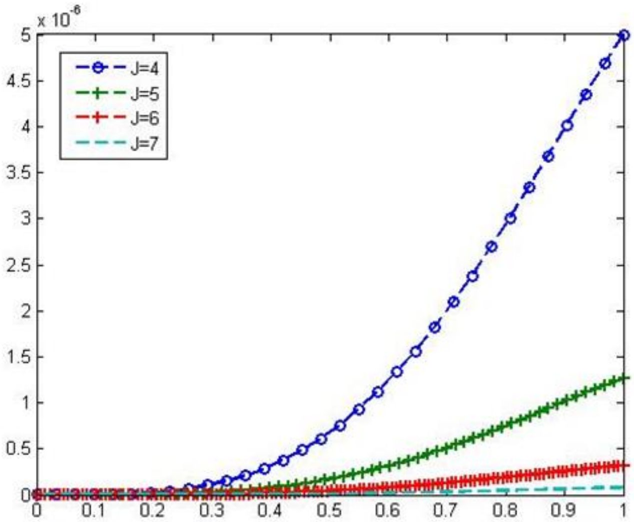

Fig.2. Absolute errors for various resolutions of **Example 1**

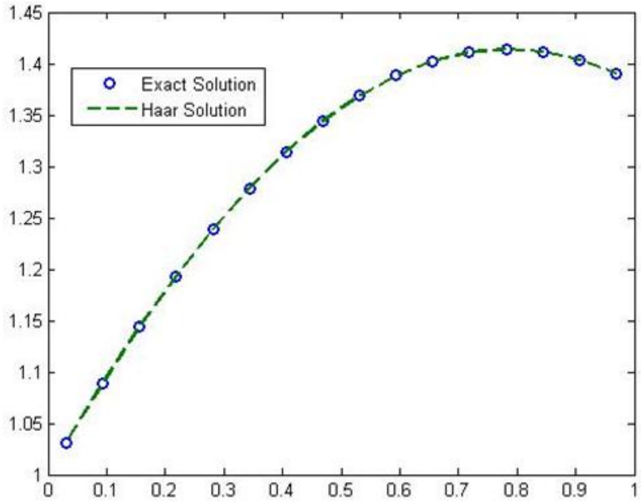

Fig.3. Comparison of Exact and Haar Solution for **Example 2** with J=3

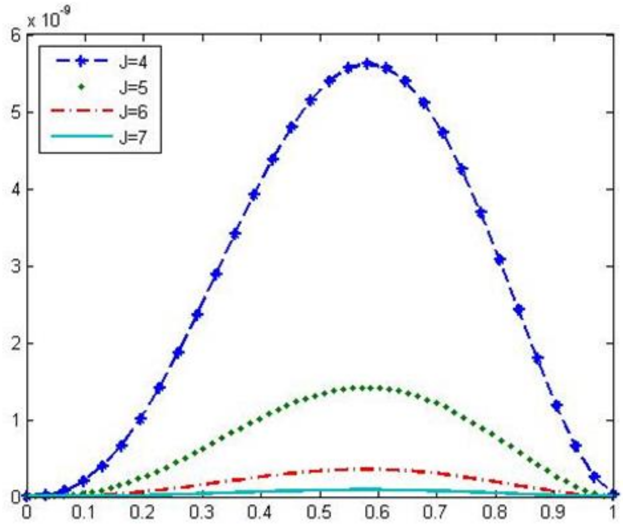

Fig.4. Absolute errors by HWCM with J=4, 5, 6&7 for **Example 2**

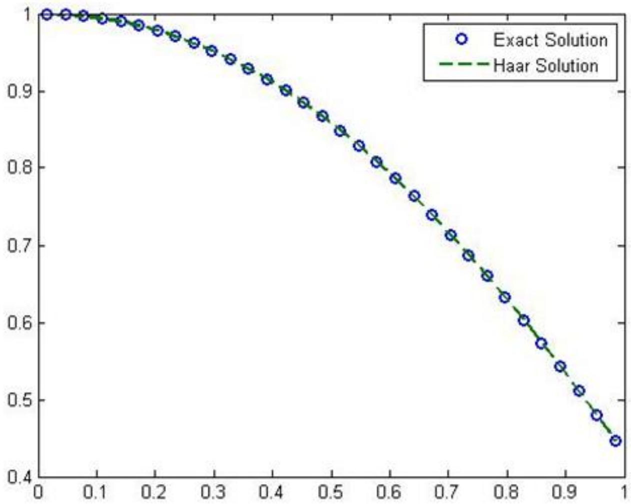

Fig.5. Comparison of Exact and Haar Solution for **Example 3** with J=4

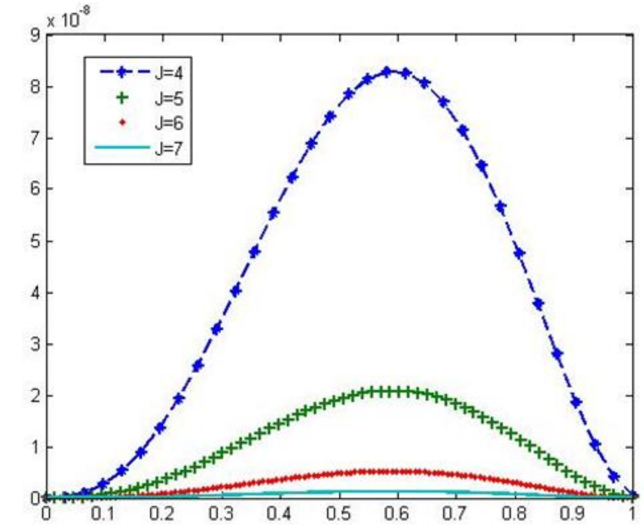

Fig.6. Absolute errors by HWCM with J=4, 5, 6&7 for **Example 3**

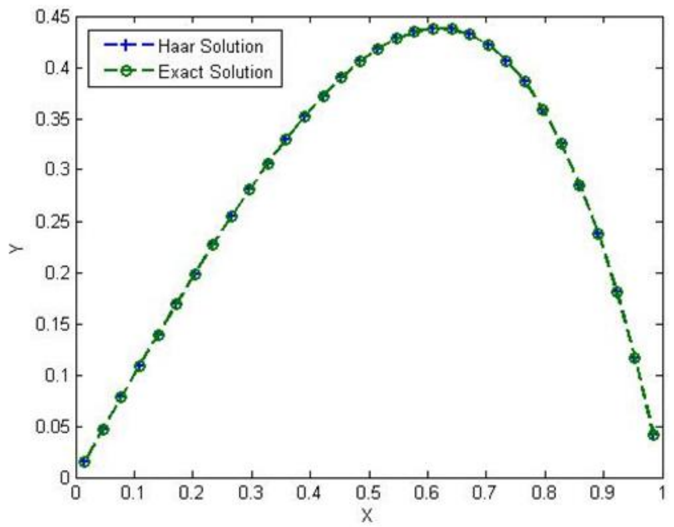

Fig.7. Comparison of Exact and Haar Solution for **Example 4** with J=5

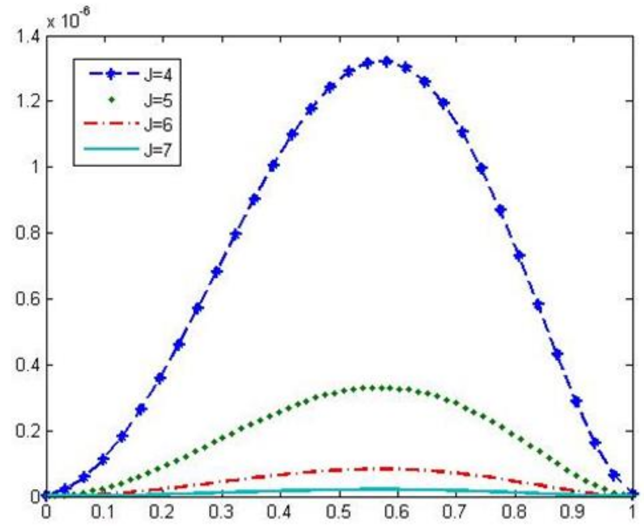

Fig.8. Absolute errors by HWCM with J=4, 5, 6&7 for **Example 4**

Table 1. Comparison of exact and Haar solution of **Example 1**

X	Exact Solution	Approximate Solution	Absolute error
0.1	4.4775	4.4775	1.5209E-09
0.2	3.9203	3.9203	2.2999E-08
0.3	3.3437	3.3437	1.0744E-07
0.4	2.7632	2.7632	3.0993E-07
0.5	2.1940	2.1940	6.8349E-07
0.6	1.6507	1.6507	1.2653E-06
0.7	1.1473	1.1473	2.0633E-06
0.8	0.6967	0.6967	3.0438E-06
0.9	0.3108	0.3108	4.1196E-06

Table 2. Comparison of exact and Haar Solution of **Example 2**

X	Exact Solution	Approximate Solution	Absolute error
0.1	1.0948	1.0948	4.0037E-11
0.2	1.1787	1.1787	2.4506E-10
0.3	1.2509	1.2509	6.1035E-10
0.4	1.3105	1.3105	1.0218E-09
0.5	1.3570	1.3570	1.3289E-09
0.6	1.3900	1.3900	1.4058E-09
0.7	1.4091	1.4091	1.1980E-09
0.8	1.4141	1.4141	7.5618E-10
0.9	1.4049	1.4049	2.5531E-10

Table 3. Comparison of exact and Haar Solution of **Example 3**

X	Exact Solution	Approximate Solution	Absolute error
0.1	0.9948	0.9948	2.0804E-09
0.2	0.9787	0.9787	1.3121E-08
0.3	0.9509	0.9509	3.3510E-08
0.4	0.9109	0.9109	5.7450E-08
0.5	0.8583	0.8583	7.6478E-08
0.6	0.7933	0.7933	8.2772E-08
0.7	0.7165	0.7165	7.2138E-08
0.8	0.6290	0.6290	4.6544E-08
0.9	0.5329	0.5329	1.6055E-08

Table 4. Comparison of exact and Haar Solution of **Example 4**

X	Exact Solution	Approximate Solution	Absolute error
0.1	0.0995	0.0995	5.8563E-09
0.2	0.1954	0.1954	2.1784E-08
0.3	0.2835	0.2835	4.3417E-08
0.4	0.3580	0.3580	6.4741E-08
0.5	0.4122	0.4122	7.9354E-08
0.6	0.4373	0.4373	8.1906E-08
0.7	0.4229	0.4229	6.9733E-08
0.8	0.3561	0.3561	4.4745E-08
0.9	0.2214	0.2214	1.5570E-08

Table 5. Comparison of Maximum absolute errors

Example No.	HWCM	Non-Polynomial Spline(NPS)[5]	
		h=1/10	h=1/20
1	9.46E—08	-	-
2	5.52E—12	1.63E--07	1.86E--08
3	8.13E—11	4.79E--06	1.05E--06

Table 6. Maximum absolute errors of HWCM compared with OSPM and [8]

Example No.	OSPM[9]		HWCM	NPSS [8]
	$\alpha_1 = 0$	$\alpha_2 = \frac{1}{240}$	J=7,8,9.	-
4	3.76E-09 1.80E-11 3.17E-11	2.85E-05 7.80E-06 1.99E-06	2.19E-10 5.46E-12 3.47E-12	1.46E-04 7.13E-06 4.75E-07

7. Conclusion

The present method has been tested on initial and boundary value problems (Type 1 and Type2) of induction motor. The order of convergence for fifth order ODEs is found. The approximate solutions obtained by HWCM are in good agreement with exact solutions. We concluded from graphs and tables that the numerical results obtained by HWCM are better than other existing methods. Analyzing the numerical studies we observed that the proposed method gives more precise results by increasing level of resolution. So that the method is efficient and reasonable for induction motor problems.

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