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Pinning hybrid synchronization of time-delay hyperchaotic Lü systems via single linear control

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Abstract. The pinning hybrid synchronization of time-delay hyperchaotic Lü systems is investigated via linear control. Based on Lyapunov stability theory, the coexistence of pinning anti-synchronization and complete synchronization of time-delay hyperchaotic Lü systems is obtained via one single controller. Sufficient conditions are obtained to achieve the hybrid Pinning synchronization. Numerical simulations are presented to demonstrate the effectiveness of the proposed schemes.

Keywords: pinning hybrid synchronization; time-delay; hyperchaotic system; linear control.

1. Introduction

During the last decades, synchronization of chaotic systems, an important topic in nonlinear science, has attracted more and more attention and has been explored intensively. Many kinds of synchronizations, such as complete synchronization [1], phase synchronization [2], generalized synchronization [3], lag synchronization [4], Q-S synchronization [5], anti-synchronization [6], time-scale synchronization [7], functional synchronization [8], projective synchronization [9], cluster synchronization [10], etc., have been proposed and successfully applied to the chaos synchronization of chaotic or hyperchaotic systems.

Many theoretical studies of chaos synchronization have been carried out for coupled systems[11,12]. In most cases of drive-response synchronization, all the states of the response system synchronize to the corresponding states of drive system in terms of the same synchronization regime. As far as we know, complete synchronization is characterized by the equality of state variables while evolving in time. Anti-synchronization is characterized by the disappearance of the sum of relevant variables [5]. Does the phenomenon that some states of the interactive systems are synchronized in terms of one type of synchronization regime, and other states synchronized in terms of another type exist in unidirectionally and linearly coupled chaotic systems? There is no doubt that it is an interesting problem. Therefore, inspired by [13,14], it is invited to investigate the coexistence synchronization problems of time delay chaotic systems by using single control input both for theoretical research and practical applications. However, up to our knowledge, there have been few (if any) results of an investigation for the delay chaotic systems via a single controller with one variable in the literature, especially the time-delay hyperchaotic systems.

In this paper, we will show that the pinning hybrid synchronization of time-delay hyperchaotic Lü systems can be achieved only by a single variable controller. Other parts of this paper are arranged as follows. In section 2, dynamical behavior of time delay Hyperchaotic system is explored and 2D overview hyperchaotic attractors are given. In section 3, schemes to achieve the pinning hybrid synchronization are proposed. Section 4 demonstrates the numerical simulations to verify the theoretical results. Some conclusions are drawn in section 4.

2. Dynamical behavior of time delay Hyperchaotic system

In this paper, the considered hyperchaotic system with time delay is described as

$$\begin{aligned} \dot{x} &= a(y-x), \\ \dot{y} &= cy - xz + w(t-\tau), \\ \dot{z} &= xy - bz, \\ \dot{w} &= -\alpha_1 x - \alpha_2 y, \end{aligned} \tag{1}$$

where $\tau > 0$ is the time delay. When $\tau = 0$, system (1) is the hyperchaotic system constructed from the Lü system by Pang S, Liu Y in [15]. For convenience, we call it delay Hyperchaotic Lü system. When a = 35,

b = 3, c = 20, α_1 = 2, α_2 = 2 and the time delay τ is chosen as 1, by the Galerkin approximation technique, an algorithms considered by Ghosh D, Chowdhury R, Saha P for calculating Lyapunov exponents for system with time delay[15], system (1) has two positive Lyapunov exponents, i.e., λ_1= 1.4523, λ_2 = 0.3562, and the hyperchaotic attractors of system (1) are shown in Fig. 1 (2D overview).

Furthermore, in real application, smaller number of controller and simpler form of controller are practical greatly. In following sections, we will investigate the coexistence of pinning anti- synchronization and complete synchronization of time-delay hyperchaotic 4D systems via simple feedback controller.

3. Pinning Hybrid synchronization of time-delay hyperchaotic systems

In this section, we describe the synchronization effects in large spatially ordered ensembles of oscillators, i.e., the systems are arranged in a regular spatial structure. The simplest example is a chain, where each element interacts with its nearest neighbors, if the first and last elements of the chain are also coupled, then it becomes a ring structure.

In the multiple structure coupled in a ring with hyperchaotic 4D systems, only the second variable of each chaotic system interacts with its nearest neighbors, the first and last systems of the chain are also coupled, which can be given as the following forms:

$$\begin{aligned} \dot{x}_i &= a(y_i - x_i), \\ \dot{y}_i &= cy_i - x_i z_i + w_i(t-\tau) + \rho\, v_i, \\ \dot{z}_i &= x_i y_i - bz_i, \\ \dot{w}_i &= -\alpha_1 x_i - \alpha_2 y_i, \end{aligned} \tag{2}$$

where $i = 1,2,...,n, v_i = (y_{i+1} - y_i)\ (i \neq n), v_n = (y_1 - y_n)$ and ρ is the coupling coefficient.

We choose the states x, y, z and w of an isolated node dynamical system (1) as a synchronous solution of the controlled complex dynamical network (2) because it is a diffusive coupling network. The target of this paper is to find a single controller, which makes the states variable x_i, y_i and w_i in response system pinning anti-synchronize x, y and w in drive system, respectively, while the third state variable z_i in response system is pinning complete -synchronized to z in drive system. For this purpose, let

$$e_{ix} = x_i + x,\ e_{iy} = y_i + y,\ e_{iz} = z_i - z,\ e_{iw} = w_i + w. \tag{3}$$

The aim is to propose simple input controller such that the state errors in (3) satisfy

$$\lim_{t\to\infty} e_{ix}(t) = 0,\ \lim_{t\to\infty} e_{iy}(t) = 0,\ \lim_{t\to\infty} e_{iz}(t) = 0,\ \lim_{t\to\infty} e_{iw}(t) = 0. \tag{4}$$

Firstly, we will show the coexistence of anti-synchronization and complete-synchronization via single linear controller. The controller are imposed on the second formula to change the dynamics of the nonlinear response system as shown in Eq. (2). The result is given in **Theorem 1**.

Theorem 1. For suitable value of ε, the state variables x_i, y_i and w_i in the following controlled complex dynamical network (5) are anti-synchronized to the synchronous solutions x, y and w, while the third state variable z_i in (5) is complete-synchronized to z.

$$\begin{aligned} \dot{x}_i &= a(y_i - x_i), \\ \dot{y}_i &= cy_i - x_i z_i + w_i(t-\tau) + \rho\, v_i + u_i, \\ \dot{z}_i &= x_i y_i - bz_i, \\ \dot{w}_i &= -\alpha_1 x_i - \alpha_2 y_i, \end{aligned} \tag{5}$$

where $u_1 = -\varepsilon e_{1y}$, u_i =0 ($i = 2,3,...,n$).

Proof. The error system of (5) and (1) can be governed by the following dynamical system

$$\begin{aligned} \dot{e}_{ix} &= a(e_{iy} - e_{ix}), \\ \dot{e}_{iy} &= ce_{iy} - z_i e_{ix} + xe_{iz} + e_{iw}(t-\tau) + \rho\, \vartheta_i + u_i, \\ \dot{e}_{iz} &= y_i e_{ix} - xe_{iy} - be_{iz}, \end{aligned}$$

$$\dot{e}_{iw} = -\alpha_1 e_{ix} - \alpha_2 e_{iy}\,, \tag{6}$$

where $\vartheta_i = e_{(i+1)y} - e_{iy}, (i \neq n), \vartheta_n = e_{1y} - e_{ny}$.

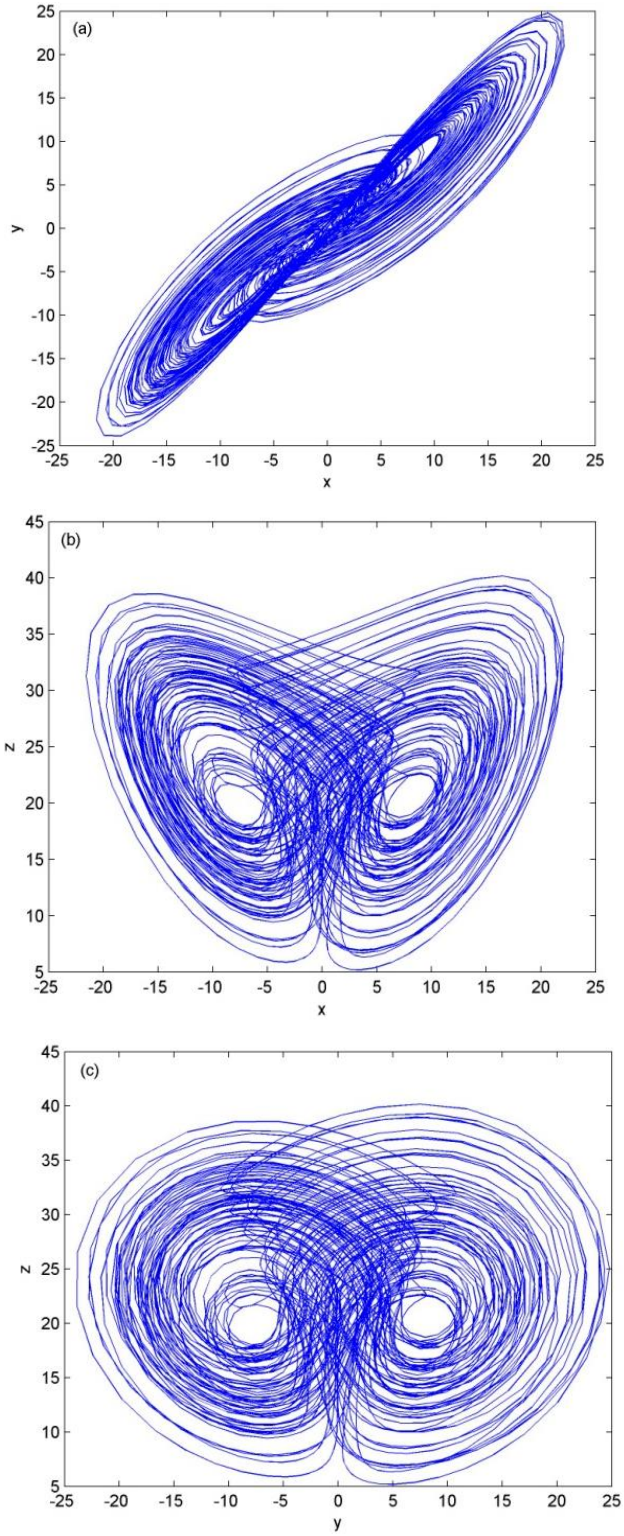

Fig.1. 2D overview hyperchaotic attractor of Eq. (1) with a = 20, b = 10.6, c = 2.8, $\alpha_1 = 2$, $\alpha_2 = 2$, $\tau = 1$, (a) (x, y), (b) (x, z), (c) (y, z).

The positive definite Lyapunov function is constructed as

$$V = \exp[\sum_{i=1}^{n}(e_{ix}^2 + e_{iy}^2 + e_{iz}^2 + e_{iw}^2)/2 + \beta\sum_{i=1}^{n}\int_{t-\tau}^{t} e_{iw}^2 dt - \beta\sum_{i=1}^{n}\int e_{iw}^2 dt$$
$$-\sum_{i=1}^{n}\int (a - z_i)e_{ix}e_{iy}dt - \sum_{i=1}^{n}\int y_i e_{ix}e_{iz}dt + \alpha_1\sum_{i=1}^{n}\int e_{ix}e_{iw}dt$$
$$+\alpha_2\sum_{i=1}^{n}\int e_{iy}e_{iw}dt - \rho\sum_{i=1}^{n-1}\int e_{iy}e_{(i+1)y}dt - \rho\int e_{1y}e_{ny}dt]. \tag{7}$$

Calculate the time derivative of the Lyapunov function (7), and we can obtain

$$\dot{V} = V\{\sum_{i=1}^{n}(e_{ix}\dot{e}_{ix} + e_{iy}\dot{e}_{iy} + e_{iz}\dot{e}_{iz} + e_{iw}\dot{e}_{iw}) - \beta\sum_{i=1}^{n}e_{iw}^2(t-\tau) - \sum_{i=1}^{n}(a - z_i)e_{ix}e_{iy}$$
$$-\sum_{i=1}^{n}y_i e_{ix}e_{iz} + \alpha_1\sum_{i=1}^{n}e_{ix}e_{iw} + \alpha_2\sum_{i=1}^{n}e_{iy}e_{iw} - \rho\sum_{i=1}^{n-1}e_{iy}e_{(i+1)y} - \rho e_{1y}e_{ny}\}$$
$$= V\{\sum_{i=1}^{n}[-ae_{ix}^2 - (\rho - c)e_{iy}^2 - be_{iz}^2] - \varepsilon e_{1y}^2 + \sum_{i=1}^{n} e_{iy}e_{iw}(t-\tau) - \beta\sum_{i=1}^{n}e_{iw}^2(t-\tau)\}$$
$$\le V\{\sum_{i=1}^{n}[-ae_{ix}^2 - (\rho + 0.5/\lambda - c)e_{iy}^2 - be_{iz}^2] - \varepsilon e_{1y}^2 - (\beta - 2\lambda)\sum_{i=1}^{n} e_{iw}^2(t-\tau)\}. \tag{8}$$

With suitable values of positive β, λ, ε and the coupling intensity ρ, we can get

$$\dot{V} \le V\{\sum_{i=1}^{n}[-ae_{ix}^2 - (\rho + 0.5/\lambda - c)e_{iy}^2 - be_{iz}^2] - \varepsilon e_{1y}^2 - (\beta - 2\lambda)\sum_{i=1}^{n} e_{iw}^2(t-\tau)\} < 0. \tag{9}$$

According to the Lyapunov stability theory, the zero solution of the error system (6) is globally asymptotically stable. It means that the hybrid synchronization of systems (1) and (5) can be obtained.

The single linear feedback controller proposed in **Theorem 1** must have a sufficiently large feedback gain ε for any initial values, which often induces a kind of waste in practice. To overcome these difficulties, an effectively adaptive synchronization approach is proposed based on adaptive control technology.

Theorem 2. If the control input is generated by a simple adaptive feedback law

$$u_1 = -\varepsilon e_{1y},\ u_i = 0\ (i = 2,3,...,n), \tag{10}$$

where the adaptive gains ε satisfies

$$\dot{\varepsilon} = e_{1y}^2, \tag{11}$$

then the state variables x_i, y_i and w_i in the controlled complex dynamical network (5) can anti-synchronize to the synchronous solutions x, y and w, while the third state variable z_i in (5) is complete-synchronized to z.

Proof. The positive definite Lyapunov function is constructed as following:

$$V = \exp[\sum_{i=1}^{n}(e_{ix}^2 + e_{iy}^2 + e_{iz}^2 + e_{iw}^2)/2 + (\varepsilon - \hat{\varepsilon})^2/2 + \beta\sum_{i=1}^{n}\int_{t-\tau}^{t} e_{iw}^2 dt - \beta\sum_{i=1}^{n}\int e_{iw}^2 dt$$
$$-\sum_{i=1}^{n}\int (a - z_i)e_{ix}e_{iy}dt - \sum_{i=1}^{n}\int y_i e_{ix}e_{iz}dt + \alpha_1\sum_{i=1}^{n}\int e_{ix}e_{iw}dt$$
$$+\alpha_2\sum_{i=1}^{n}\int e_{iy}e_{iw}dt - \rho\sum_{i=1}^{n-1}\int e_{iy}e_{(i+1)y}dt - \rho\int e_{1y}e_{ny}dt], \tag{12}$$

where $\hat{\varepsilon}$ is a positive constant to be determined. If the values of positive β, λ, $\hat{\varepsilon}$ and the coupling intensity ρ are chosen correctly, then we have

$$\dot{V} \le V\{\sum_{i=1}^{n}[-ae_{ix}^2 - (\rho + 0.5/\lambda - c)e_{iy}^2 - be_{iz}^2] - \hat{\varepsilon} e_{1y}^2 - (\beta - 2\lambda)\sum_{i=1}^{n} e_{iw}^2(t-\tau)\} < 0. \tag{13}$$

According to the Lyapunov stability theory, the pinning hybrid synchronization in **Theorem 2** can be obtained.

4. Numerical simulations

In this section, some numerical simulations are presented to illustrate the theoretical results. In the following numerical simulations, the system parameters are selected as above mentioned.

When the controller is taken as in **Theorem 1**, time evolution curves of system (1) and system (5) (where $\varepsilon = 300$) are depicted in Fig.2 and the synchronization errors are shown in Fig.3. From Figs.2-3, it is obvious to see that the synchronization errors converge asymptotically to zero, which suggests that the coexistence of pinning anti-synchronization and complete synchronization of system (1) and system (5) can be obtained under the condition of Theorem 1.

When the controller is chosen as in Theorem 2, the time evolution curves of system (1) and system (5) are shown in Fig. 4 and the synchronization errors dynamics are shown in Fig.5. The estimation of feedback gain ε is shown in Fig.6. From the numerical simulation results, it is obvious to see that the pinning hybrid synchronization of the hyperchaotic systems can be achieved and the unknown feedback gain can be estimated via numerical simulation.

5. Conclusion

In this paper, we investigate the pinning hybrid synchronization of the network with time-delay hyperchaotic systems via linear feedback control methods. Furthermore, adaptive linear feedback control scheme is given to estimate the feedback gain. The proposed methods are simple, efficient, and easy to implement in practical applications. The correctness of the proposed methods is verified not only by theoretical analysis, but also by numerical simulations.

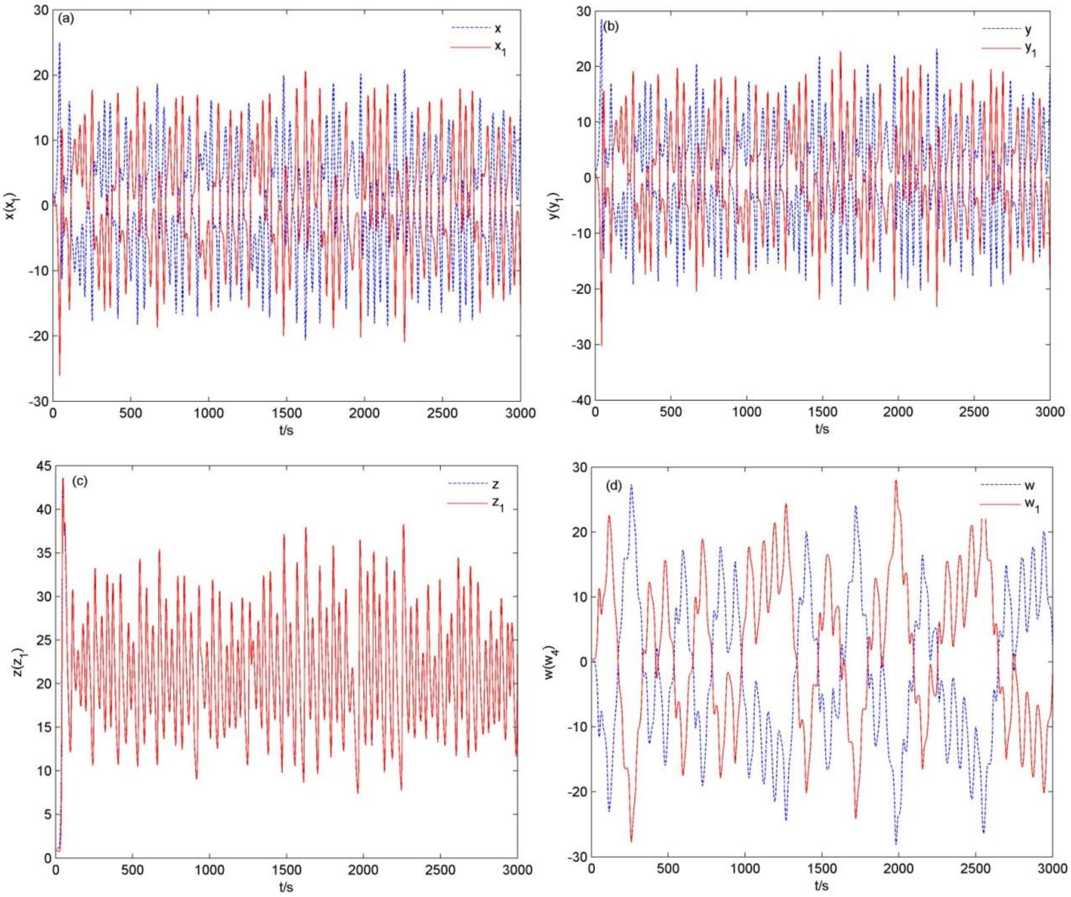

Fig.2. Time evolution curves of system (1) and system (5) with the controller in Theorem 1, (a) (x, x_1), (b) (y, y_1), (c) (z, z_1), (d) (w, w_1).

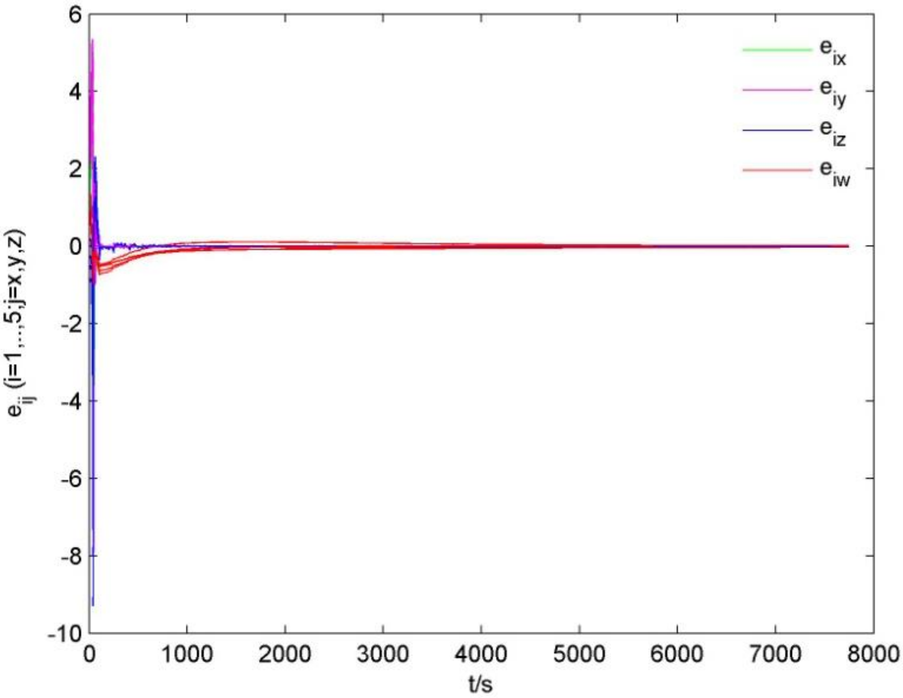

Fig.3. Dynamics of synchronization error states e_{ix}, e_{iy}, e_{iz} and e_{iw} $(i=1,2,...,5)$ with the controller in Theorem 1.

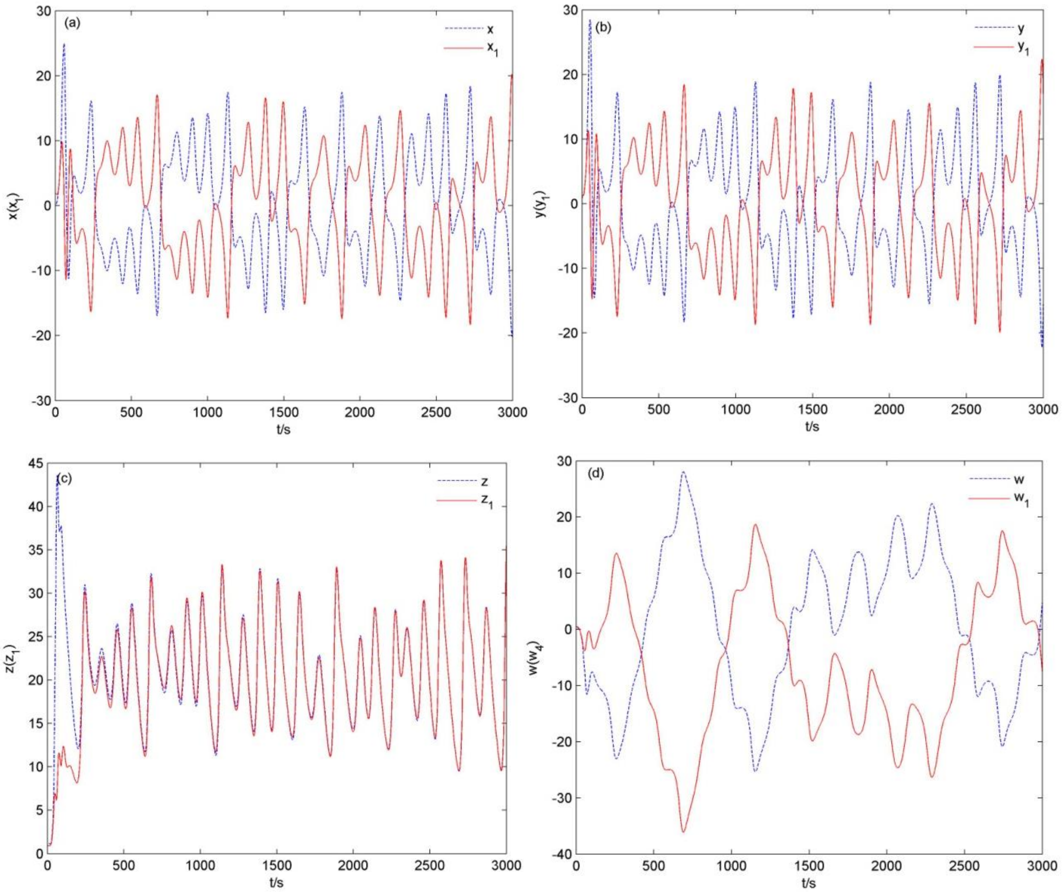

Fig.4. Time evolution curves of system (1) and system (5) with controller in Theorem 2, (a) (x , x_1), (b) (y , y_1), (c) (z , z_1), (d) (w , w_1).

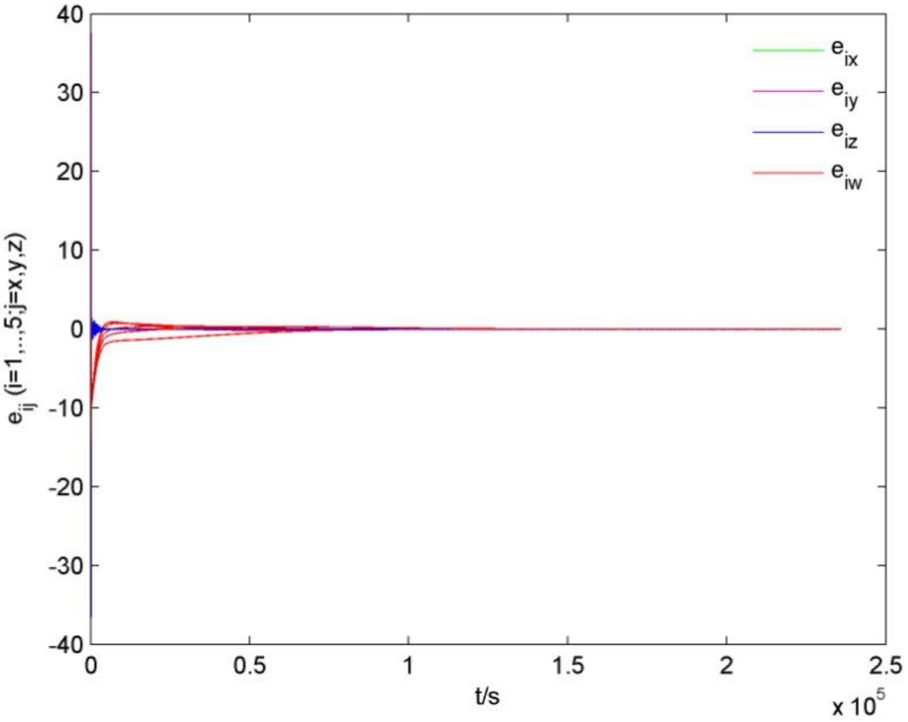

Fig.5. Dynamics of synchronization error states e_{ix}, e_{iy}, e_{iz} and e_{iw} $(i = 1, 2, ..., 5)$ with the controller in Theorem 2.

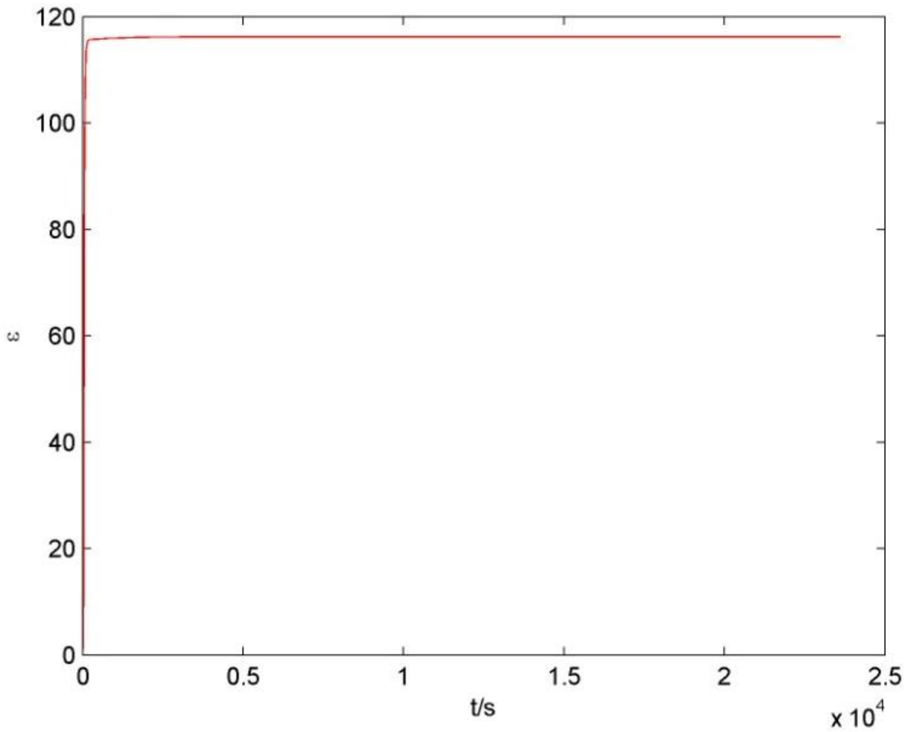

Fig.6. The estimation of feedback gain ε.

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7. References

[1] Ma J, Li F, Huang L, et al, Complete synchronization, phase synchronization and parameters estimation in a realistic chaotic system, Communications in Nonlinear Science & Numerical Simulation. 16(9):3770-3785 (2011).

[2] Fujisaka H, Yamada T, Kinoshita G, et al, Chaotic phase synchronization and phase diffusion, Physica D Nonlinear Phenomena. 205(1-4):41-47(2005).

[3] Yang J, Hu G, Three types of generalized synchronization, Physics Letters A. 361(4-5):332-335(2007).

[4] Hu T C, Wu Y Q, Li S X, Lag Synchronization between two coupled networks via open-plus- closed-loop and adaptive controls. Communications in Theoretical Physics. 65(1): 33-38(2016).

[5] Wang Z L, Shi X R, Adaptive Q-S synchronization of non-identical chaotic systems with unknown parameters, Nonlinear Dynamics. 59(4):559-567(2010).

[6] Mossa Al-Sawalha M, Noorani M S M, Al-Dlalah M M, Adaptive anti-synchronization of chaotic systems with fully unknown parameters, Computers & Mathematics with Applications. 59(10): 3234-3244(2010).

[7] Hramov A E, Koronovskii A A, Time scale synchronization of chaotic oscillators. 206(3-4):252-264(2006).

[8] Banerjee S, Chowdhury A R, Functional synchronization and its application to secure communications[J], Journal of Modern Physics B. 23(09):2285-2295(2012).

[9] Qin W X, Chen G R, Coupling schemes for cluster synchronization in coupled Josephson equations, Physica D. 197(3-4):375-391(2004).

[10] Horikawa Y, Stabilization of metastable dynamical rotating waves in a ring of unidirectionally coupled sigmoidal neurons due to shortcuts, Neural Networks. 85:140-156(2017).

[11] Tchakui M V, Woafo P, Dynamics of three unidirectionally coupled autonomous Duffing oscillators and application to inchworm piezoelectric motors: Effects of the coupling coefficient and delay, Chaos. 26(11): 113108(2016).

[12] Chen J H, Chen H K, Lin Y K, Synchronization and anti-synchronization coexist in Chen-Lee chaotic systems, Chaos Solitons Fractals. 39(2): 707-716(2009).

[13] Wu ZY, Leng H, Complex hybrid projective synchronization of complex-variable dynamical networks via open-plus-closed-loop control, Journal of the Franklin Institute-Engineering And Applied Mathematics. 354(2): 689-701(2017).

[14] Pang S Q, Liu Y J, A new hyperchaotic system from the Lü system and its control, Journal of Computational and Applied Mathematics. 235(8): 2775-2789(2011).

[15] Ghosh D, Chowdhury A R, Saha P, Multiple delay Rossler system-bifurcation and chaos control, Chaos Solitons Fractals. 35(3): 472-485(2008).