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Study on the price Stackelberg game model under different competitive environment

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Abstract. The product sold in different stores may be not same price, mainly caused by diverse retailers' situations. Meanwhile, maximizing relative interests is the businesses' common goal. Based on the 2-2 suppliers-retailers situation, this paper researches on the market by using dynamic price Stackelberg game model. We consider two cases, one is retailers under the win-win environment (case 1) and the other is they under the malicious competition environment (case 2). Through analyzing and comparing the two cases, this paper finds that the influence of different parameters on the Nash Equilibrium stable range in size is same, while the amplitude of stable range, the prices of goods, sales as well as the businessman profit are all different. These results will be revealed in a series of charts, including the stability diagram, bifurcation diagram, phase plot and histogram. And the conclusions may have a certain reference value for businesses on how to stabilize the market and which competitive measures should be taken.

Keywords: 2-2 suppliers-retailers situation, dynamic price Stackelberg game, the stable range of Nash Equilibrium, different competitive environment ,bifurcation diagram.

1. Introduction

Commodity pricing has been the first thing that businessmen need to consider. Due to different conditions and competitive environment, similar goods may be set into various price by merchants. This paper mainly researches the difference between the effects of goods under different environment, including the prices of goods, sales, the influence of different parameters on the Nash Equilibrium and the businessman profit.

The factors that affect pricing have been studied by many scholars and they found that sales volume, the freight, sales cost are all important for price[1-2]. The price Stackelberg game model is often used to study such economic problems, but the specific models structure is not the same[3-6]. Wang et al.[7] pointed out that the competitive environment had a certain impact on the price strategy. At present, the competitive environment is divided into two categories, namely, win-win and malicious competition situation. Interest allocation in cooperative situation was researched in [8]. The view that businesses may pay more attention to maximize the relative interests in malicious competition was put forward in [9]. Sales volume and prices of products are the most direct factors that affect business benefits. Pi[10] gave the market demand function researching on pricing and cooperating of the dual-channel supply chain under the competitive environment of more brand. This function can be used to reflect the relationship between sales volume and prices. The complexity pricing game and coordination of the duopoly air conditioner market with disturbance demand were researched in [11]. Even if the products are alternative, the basic needs of the market should have a certain degree of difference. When sales volume are expressed in terms of prices, the profit function can be constructed. As Zeng[12] mentioned that retailers can obtain the optimal retail pricing by using the differential extremum principle in the strategy of enterprise sales pricing. Then according to the price Stackelberg game theory, businesses can put the optimal retail price into the profit function and the optimal wholesale price can be got by using differential extremum principle again. Repeating this process, the price adjustment system of wholesale can be obtained. Many scholars[13-15]use this method to deal with the problems of supply chains. The stability of the system is also analyzed in this paper. Ma[16] considered the effect of delay variation on system stability. Yang[17]studied rich dynamics of a nonlinear economic model, then found Chaotic and bubbling phenomena which clearly agreed with phenomena from technology bubbling. The existence of Nash equilibrium point and its local stability of the game as well as the route to chaos was investigated in [18]. Many papers have proved that the increase of the adjustment speed can make the system lose the stability. The Bertrand duopoly game with differentiated goods was researched, and the conclusion that the fast loss of the stability with fast adjustment speed was drawn[19-20]. Chen[21] analyzed the existence of bifurcation.

Based on the previous studies, the difference between similar businesses (mainly the freight cost) are fully considered, and the special 2-2 suppliers - retailers model is used in this paper. We assume two cases, including retailers in the win-win environment (case 1) and retailers in the malicious competition environment(case 2). Through analyzing and comparing these two environment, we can find a series of conclusions, including the influences of different parameters on the stable range, the prices of goods, sales and so on. These conclusions will be expressed by many graphical representation such as bifurcation diagram ,two dimensional graph,phase plot and histogram.

This paper is organized as follows. In section 2, we introduce the basic game model. In section 3, the dynamic game model will be given and the system stability will be analyzed in section 4.Then,in section 5,we will compare two environment and find the difference between them. At the last section, the conclusion will be given as well as the prospect.

2. The basic game model

Assuming that there exists two manufacturers (A and B) and two retailers (R_1 and R_2) in the market. Two manufacturers provide two similar products (a and b), which are alternative products. Two retailers sell the products by using traditional selling channel.

Firstly, the manufacturers set the wholesale price. Then, the retailers decide the retailer price. According to the market situation, the manufacturers need to adjust the wholesale price again until they achieve the maximization of interests and at the same time ,the market presents the steady state. The manufacturers are the leaders and the retailers are the followers. Manufacturers are competing and so are retailers.

2.1. Assumptions

Face In order to clearly clarify, the following assumptions are proposed :

(1) There are two manufacturers (A and B), two alternative products (a and b) and two retailers(1 and 2) . Retailers bear the freight.

(2) The cost of manufacturing products is normalized to zero. The operating cost and the cost of selling for each retailer are normalized to zero. In fact, this assumption is used for the convenience of calculation. The optimal individual prices of the goods in practical application is equal to the the optimal price in this assumption conditions adding the cost of individual goods.

(3) Sales without stockout or oversupply.

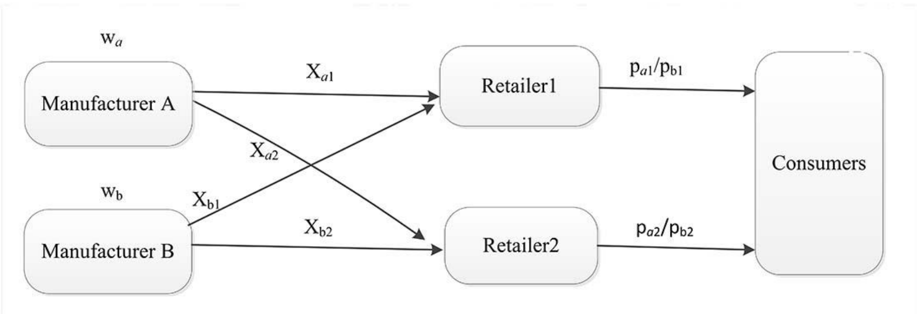


Fig.1. The supply chain system with two manufacturers and two retailers.

2.2. Assumptions

In the market, the demand is affected by the price of itself and competitors. Even if the products are alternative, the market potential of them should have a certain degree of difference.Thus, the demand function[10] is needed to modify. In this paper, sales without stockout or oversupply and then, the sales volume model can be written :

$$\begin{cases} q_{a1} = a_a - bp_{a1} + k_1 p_{a2} + k_2 p_{b1} + k_3 p_{b2}, \\ q_{a2} = a_a - bp_{a2} + k_1 p_{a1} + k_2 p_{b2} + k_3 p_{b1}, \\ q_{b1} = a_b - bp_{b1} + k_1 p_{b2} + k_2 p_{a1} + k_3 p_{a2}, \\ q_{b2} = a_b - bp_{b2} + k_1 p_{b1} + k_2 p_{a2} + k_3 p_{a1}. \end{cases} \tag{1}$$

where q represents the sales volume of product (q_{a1} and q_{a2} represent the a product's sales volume in retailer1 and retailer2 respectively, q_{b1} and q_{b2} represent b commodity's sales volume in retailer1 and retailer2 respectively). And p represents the price of product (the similar representations are defined for p_{a1}, p_{a2}, p_{b1} and p_{b2}). In addition, a_a and a_b represent the market potential of a product and b product respectively. k_1, k_2, k_3 and b are all price sensitivity coefficient. b＞(k_1+k_2+k_3),this because the one's price has more influences on its sales than others' prices. k_i＞0（i=1,2,3), as long as others' prices are in existence, the sales of this product is favorable.

The relation of the supply chain system can be seen in Fig.1. As A and B simultaneously determine the wholesale price w_a and w_b, then the retailer1 determines the sale price p_{a1} and p_{b1} for the product a and b, the retailer2 determines the retail price p_{a2} and P_{b2} for the product a and b. Therefore, the profits of the two manufacturers and the two retailers can be obtained as :

$$\begin{cases} \pi_A = (q_{a1} + q_{a2}) w_a, \\ \pi_B = (q_{b1} + q_{b2}) w_b, \\ \pi_{R1} = (p_{a1} - w_a) q_{a1} + (p_{b1} - w_b) q_{b1} - cX_{a1} q_{a1} - cX_{b1} q_{b1}, \\ \pi_{R2} = (p_{a2} - w_a) q_{a2} + (p_{b2} - w_b) q_{b2} - cX_{a2} q_{a2} - cX_{b2} q_{b2}. \end{cases} \tag{2}$$

where π represents the profit of merchant. X is used to indicate the distance between the manufactures and retailers and q represents the sales volume. c is the unit price of freight.

3. The dynamic game model

In this paper, the manufactures A and B are the leaders, and the retailer1 and the retailer2 are the followers. This relationship implies the dominance of the manufactures over the retailers. The manufacturers are duopoly, and so are retailers. There are competition between themselves.

As the initial retail price is given, the manufacturer will adjust the wholesale price according to the market situation. Meanwhile, in order to guarantee the interests, retailers will also make the corresponding price regulation. If the adjusted speed is too slow, it can make their own interests to get loss, but it can make the market unstable if adjusting too quickly. Thus, in order to ensure the market stability, the price adjustment speed should be in a reasonable range.

3.1. Case1: under the win-win environment

Without getting the idea from the opponents , the retailers have no doubt to keep the price at the state which is best for their own interests when they are in competitive condition.

To maximize the profits, the retailers need to give a reasonable retail price according to the wholesale price.Through the demand interests on the price partial derivative equation, we can obtain the optimal solution for each price which is related with the wholesale price. In the case of no malicious degrade price, retailers will be in a win-win state at this time.

According to (2), the following partial derivative equation is obtained as :

$$\begin{cases} \dfrac{\partial \pi_{R1}}{\partial p_{a1}} = a_a - bp_{a1} + k_1 p_{a2} + k_2 p_{b1} + k_3 p_{b2} - b(p_{a1} - w_a) + k_2 (p_{b1} - w_b) + bcX_{a1} - ck_2 X_{b1}, \\ \dfrac{\partial \pi_{R1}}{\partial p_{b1}} = a_b - bp_{b1} + k_1 p_{b2} + k_2 p_{a1} + k_3 p_{a2} - b(p_{b1} - w_b) + k_2 (p_{a1} - w_a) + bcX_{b1} - ck_2 X_{a1}, \\ \dfrac{\partial \pi_{R2}}{\partial p_{a2}} = a_a - bp_{a2} + k_1 p_{a1} + k_2 p_{b2} + k_3 p_{b1} - b(p_{a2} - w_a) + k_2 (p_{b2} - w_b) + bcX_{a2} - ck_2 X_{b2}, \\ \dfrac{\partial \pi_{R1}}{\partial p_{b2}} = a_b - bp_{b2} + k_1 p_{b1} + k_2 p_{a2} + k_3 p_{a1} - b(p_{b2} - w_b) + k_2 (p_{a2} - wa) + bcX_{b2} - ck_2 X_{a2}. \end{cases} \tag{3}$$

Let formula(3) be equivalent to zero, we can obtain the optimal solution for each price which is related with the wholesale price as follows:

$$
\begin{cases}
p_{a1} = [2(a_a + bw_a - k_2w_b + bcX_{a1} - ck_2X_{b1})(-4b^3 + bk_1^2 + 4bk_2^2 + bk_3^2 + 2k_1k_2k_3) - (a_a + bw_a - k_2w_b + bcX_{a2} \\
\quad - ck_2X_{b2})(4b^2k_1 + 8bk_2k_3 - k_1^3 + 4k_1k_2^2 + k_1k_3^2) - 2(a_b + bw_b - k_2w_a + bcX_{b1} - ck_2X_{a1})(4k_2b^2 + 2bk_1k_3 \\
\quad + k_1^2k_2 - 4k_2^3 + k_2k_3^2) - (a_b + bw_b - k_2w_a + bcX_{b2} - ck_2X_{a2})(4k_3b^2 + 8bk_1k_2 + k_1^2k_3 - k_3^2 + 4k_3k_2^2)]/ \\
\quad [-16(b^4 - k_2^4) + 8b^2(k_1^2 + 4k_2^2 + k_3^2) + 32bk_1k_2k_3 - k_1^4 + 8k_1^2k_2^2 + 2k_1^2k_3^2 + 8k_2^2k_3^2 - k_3^4], \\
p_{b1} = [2(a_b + bw_b - k_2w_a + bcX_{b1} - ck_2X_{a1})(-4b^3 + bk_1^2 + 4bk_2^2 + bk_3^2 + 2k_1k_2k_3) - (a_b + bw_b - k_2w_a + bcX_{b2} \\
\quad - ck_2X_{a2})(4b^2k_1 + 8bk_2k_3 - k_1^3 + 4k_1k_2^2 + k_1k_3^2) - 2(a_a + bw_a - k_2w_b + bcX_{a1} - ck_2X_{b1})(4k_2b^2 + 2bk_1k_3 \\
\quad + k_1^2k_2 - 4k_2^3 + k_2k_3^2) - (a_a + bw_a - k_2w_b + bcX_{a2} - ck_2X_{b2})(4k_3b^2 + 8bk_1k_2 + k_1^2k_3 - k_3^2 + 4k_3k_2^2)]/ \\
\quad [-16(b^4 - k_2^4) + 8b^2(k_1^2 + 4k_2^2 + k_3^2) + 32bk_1k_2k_3 - k_1^4 + 8k_1^2k_2^2 + 2k_1^2k_3^2 + 8k_2^2k_3^2 - k_3^4], \\
p_{a2} = [2(a_a + bw_a - k_2w_b + bcX_{a2} - ck_2X_{b2})(-4b^3 + bk_1^2 + 4bk_2^2 + bk_3^2 + 2k_1k_2k_3) - (a_a + bw_a - k_2w_b + bcX_{a1} \\
\quad - ck_2X_{b1})(4b^2k_1 + 8bk_2k_3 - k_1^3 + 4k_1k_2^2 + k_1k_3^2) - 2(a_b + bw_b - k_2w_a + bcX_{b2} - ck_2X_{a2})(4k_2b^2 + 2bk_1k_3 \\
\quad + k_1^2k_2 - 4k_2^3 + k_2k_3^2) - (a_b + bw_b - k_2w_a + bcX_{b1} - ck_2X_{a1})(4k_3b^2 + 8bk_1k_2 + k_1^2k_3 - k_3^2 + 4k_3k_2^2)]/ \\
\quad [-16(b^4 - k_2^4) + 8b^2(k_1^2 + 4k_2^2 + k_3^2) + 32bk_1k_2k_3 - k_1^4 + 8k_1^2k_2^2 + 2k_1^2k_3^2 + 8k_2^2k_3^2 - k_3^4], \\
p_{b2} = [2(a_b + bw_b - k_2w_a + bcX_{b2} - ck_2X_{a2})(-4b^3 + bk_1^2 + 4bk_2^2 + bk_3^2 + 2k_1k_2k_3) - (a_b + bw_b - k_2w_a + bcX_{b1} \\
\quad - ck_2X_{a1})(4b^2k_1 + 8bk_2k_3 - k_1^3 + 4k_1k_2^2 + k_1k_3^2) - 2(a_a + bw_a - k_2w_b + bcX_{a2} - ck_2X_{b2})(4k_2b^2 + 2bk_1k_3 \\
\quad + k_1^2k_2 - 4k_2^3 + k_2k_3^2) - (a_a + bw_a - k_2w_b + bcX_{a1} - ck_2X_{b1})(4k_3b^2 + 8bk_1k_2 + k_1^2k_3 - k_3^2 + 4k_3k_2^2)]/ \\
\quad [-16(b^4 - k_2^4) + 8b^2(k_1^2 + 4k_2^2 + k_3^2) + 32bk_1k_2k_3 - k_1^4 + 8k_1^2k_2^2 + 2k_1^2k_3^2 + 8k_2^2k_3^2 - k_3^4].
\end{cases}
\tag{4}
$$

As (4) is the optimal decision prices of the common retailer on the premise of w_a and w_b, the retailers can obtain the decision after they observe the behaviors of the manufacturers. Similarly, substitute (4) into (2), the marginal profit of two manufacturers is obtained as follows:

$$
\begin{cases}
\frac{\partial \pi_A}{\partial w_a} = 2k_2w_b - 4bw_a - bc(X_{a1} + X_{a2}) + ck2(X_{b1} + X_{b2}) + \\
\frac{(b + k_2)(2(a_a - a_b) + (4w_a - 2w_b)(b + k_2) + (bc + ck_2)(X_{a1} + X_{a2} - X_{b1} - X_{b2}))}{2(2b - k_1 + 2k_2 + k_3)} - \\
\frac{(b - k_2)(2(a_a + a_b) + (4w_a - 2w_b)(b - k_2) + (bc - ck_2)(X_{a1} + X_{a2} + X_{b1} + X_{b2}))}{2(k_1 - 2b + 2k_2 + k_3)}, \\
\frac{\partial \pi_B}{\partial w_b} = 2k_2w_a - 4bw_b - bc(X_{b1} + X_{b2}) + ck2(X_{a1} + X_{a2}) - \\
\frac{(b + k_2)(2(a_a - a_b) + (2w_a - 4w_b)(b + k_2) + (bc + ck_2)(X_{a1} + X_{a2} - X_{b1} - X_{b2}))}{2(2b - k_1 + 2k_2 + k_3)} - \\
\frac{(b - k_2)(2(a_a + a_b) + (2w_a + 4w_b)(b - k_2) + (bc - ck_2)(X_{a1} + X_{a2} + X_{b1} + X_{b2}))}{2(k_1 - 2b + 2k_2 + k_3)}.
\end{cases}
\tag{5}
$$

3.2. Case2: under the malicious competition environment

In the commercial competition, merchants pursue their own interests and also want to less the relative interests of the other side.In this paper, we make the retailer R1 have the malicious competition by setting the price P_{a1}. Meanwhile, retailer R2 still maximize its own interest without taking any measures.Similarly, in order to achieve their purposes, they will adjust their prices. For different purposes, they will adopt different ways to fix the price.

According to (2), the marginal profits of retailers on p_{a1},p_{b1},p_{a2} and p_{b2} are obtained as follows:

$$
\begin{cases}
\frac{\partial(\pi_{R2} - \pi_{R1})}{\partial p_{a1}} = bp_{a1} - a_a - k_1p_{a2} - k_2p_{b1} - k_3p_{b2} + b(p_{a1} - w_a) + k_1(p_{a2} - w_a) - k_2(p_{b1} \\
\qquad - w_b) + k_3(p_{b2} - w_b) - bcX_{a1} - ck_1X_{a2} + ck_2X_{b1} - ck_3X_{b2}, \\
\frac{\partial \pi_{R1}}{\partial p_{b1}} = a_b - bp_{b1} + k_2p_{a1} + k_3p_{a2} + k_1p_{b2} - b(p_{b1} - w_b) + k_2(p_{a1} - w_a) + bcX_{b1} - ck_2X_{a1}, \\
\frac{\partial \pi_{R2}}{\partial p_{a2}} = a_a - bp_{a2} + k_1p_{a1} + k_2p_{b2} + k_3p_{b1} - b(p_{a2} - w_a) + k_2(p_{b2} - w_b) + bcX_{a2} - ck_2X_{b2}, \\
\frac{\partial \pi_{R2}}{\partial p_{b2}} = a_b - bp_{b2} + k_2p_{a2} + k_3p_{a1} + k_1p_{b1} - b(p_{b2} - w_b) + k_2(p_{a2} - w_a) + bcX_{b2} - ck_2X_{a2}.
\end{cases}
\tag{6}
$$

Let formula(6) be zero. Then, the optimal price of the retailers is obtained as:

$$\left\{\begin{aligned}
p_{a1} =\ & [(-4b^3 + bk_1^{\ 2} + 4bk_2^{\ 2} + bk_3^{\ 2} + 2k_1k_2k_3)(a_a + bw_a + k_1w_a - k_2w_b + k_3w_b + bcX_{a1} + ck_1X_{a2} - \\
& ck_2X_{b1} + ck_3X_{b2}) - 2(k_3k_2^{\ 2} + bk_1k_2)(a_b + bw_b - k_2w_a + bcX_{b2} - ck_2X_{a2}) - 2(k_1k_2^{\ 2} + bk_3k_2) \\
& (a_a + bw_a - k_2w_b + b\,cX_{a2} - ck_2X_{b2}) - 4\,(b^2k_2 - k_2^{\ 3})(a_b + bw_b - k_2w_a + bcX_{b1} - ck_2X_{a1})]/ \\
& [2(-4b^4 + b^2k_1^{\ 2} + 8b^2k_2^{\ 2} + b^2k_3^{\ 2} + 4bk_1k_2k_3 + k_1^{\ 2}k_2^{\ 2} - 4k_2^{\ 4} + k_2^{\ 2}k_3^{\ 2})], \\
p_{b1} =\ & [4(-b^3 + bk_2^{\ 2})(a_b + bw_b - k_2w_a + bcX_{b1} - ck_2X_{a1}) - 2(k_1b^2 + k_2\,k_3b)(a_b + bw_b - k_2w_a + bc \\
& X_{b2} - ck_2X_{a2}) - (4b^2k_2 + 2bk_1k_3 + k_1^{\ 2}\,k_2 - 4k_2^{\ 3} + k_2k_3^2)\,(a_a + bw_a + k_1w_a - k_2w_b + k_3w_b + \\
& bcX_{a1} + ck_1X_{a2} - ck_2X_{b1} + ck_3X_{b2}) - 2(k_3b^2 + k_1k_2b)(a_a + bw_a - k_2w_b + bcX_{a2} - ck_2X_{b2})]/ \\
& [2(-4b^4 + b^2k_1^{\ 2} + 8b^2k_2^{\ 2} + b^2k_3^{\ 2} + 4bk_1k_2k_3 + k_1^{\ 2}k_2^{\ 2} - 4k_2^{\ 4} + k_2^{\ 2}k_3^{\ 2})], \\
p_{a2} =\ & [2(-4b^3 + bk_1^{\ 2} + 4bk_2^{\ 2} + k_1k_2k_3)(a_a + bw_a - k_2w_b + bcX_{a2} - ck_2X_{b2}) - 4(k_3b^2 + 2bk_1k_2 + k_3 \\
& k_2^2)(a_b + bw_b - k_2w_a + bcX_{b1} - ck_2X_{a1}) - (4b^2k_1 + 8bk_2k_3 - k_1^{\ 3} + 4k_1k_2^2 + k_1k_3^2)(a_a + bw_a + \\
& k_1w_a - k_2w_b + k_3w_b + bcX_{a1} + ck_1X_{a2} - ck_2X_{b1} + ck_3X_{b2}) - 2\,(4b^2k_2 + bk_1k_3 + k_1^{\ 2}k_2 - 4k_2^{\ 3}) \\
& (a_b + bw_b - k_2w_a + bcX_{b2} - ck_2X_{a2})]\,/\,[4(-4b^4 + b^2k_1^{\ 2} + 8b^2k_2^{\ 2} + b^2k_3^{\ 2} + 4bk_1k_2k_3 + k_1^{\ 2}k_2^{\ 2} \\
& - 4k_2^{\ 4}\ + k_2^{\ 2}k_3^{\ 2})], \\
p_{b2} =\ & [2(-4b^3 + bk_3^{\ 2} + 4bk_2^{\ 2} + k_1k_2k_3)(a_b + bw_b - k_2w_a + bcX_{b2} - ck_2X_{a2}) - 4(k_1b^2 + 2bk_3k_2 + k_1 \\
& k_2^2)\,(a_b + bw_b - k_2w_a + bcX_{b1} - ck_2X_{a1}) - (4b^2k_3 + 8bk_2k_1 - k_3^{\ 3} + 4k_3k_2^2 + k_3k_1^2)(a_a + bw_a + \\
& k_1w_a - k_2w_b + k_3w_b + bcX_{a1} + ck_1X_{a2} - ck_2X_{b1} + ck_3X_{b2}) - 2(4b^2k_2 + bk_1k_3 + k_3^{\ 2}k_2 - 4k_2^{\ 3}) \\
& (a_a + bw_a - k_2w_b + bcX_{a2} - ck_2X_{b2})]\,/\,[4(-4b^4 + b^2k_1^{\ 2} + 8b^2k_2^{\ 2} + b^2k_3^{\ 2} + 4bk_1k_2k_3 + k_1^{\ 2}k_2^{\ 2} \\
& - 4k_2^{\ 4} + k_2^{\ 2}k_3^{\ 2})].
\end{aligned}\right. \tag{7}$$

Substituting (7) into (2), the marginal profits of manufactures on w_a and w_b are obtained as:

$$\left\{\begin{aligned}
\frac{\partial\pi_A}{\partial w_a} =\ & [a_a(-3bk_1^3 - k_1^4 + k_1^2(2k_2^2 - 3k_2k_3 + 2k_3^2) + k_1(12b^3 - 12bk_2^2 + 4bk_2k_3 + 3bk_3^2) + 16b^4 - 32b^2k_2^2 + 12 \\
& b^2k_2k_3 + 2b^2k_3^2 + 16k_2^4 - 12k_2^3k_3 + 3k_3^3k_2 - k_3^4) + 2a_b(bk_3 + k_1k_2)(6b^2 + 5bk_1 + k_1^2 - 6k_2^2 + 5k_2k_3 - \\
& k_3^2) + w_b(k_1^4(k_2 - k_3) + k_1^3b(5k_2 - k_3) + k_1^2(2b^2k_2 + 8b^2k_3 - 10k_2^3 + 9k_2^2k_3 - 3k_2k_3^2 + 2k_3^3) + k_1(14b^3 \\
& k_3 - 30bk_2^2k_3 + 19bk_2k_3^2 + bk_3^3) + 16k_2^5 + 4k_2^4k_3 - (32b^2 + 12k_3^2)k_2^3 + (3k_3^3 - 8b^2k_3)k_2^2 + (16b^4 + 4b^2 \\
& k_3^2 + 2k_3^4)k_2 + 4b^4k_3 + 4b^2k_3^3 - k_3^5) + w_a(\text{-4b}k_1^4 - 2k_1^5 + k_1^3(6b^2 + 8k_2^2 - 2k_2k_3 + 4k_3^2) + k_1^2(24b^3 - 8 \\
& bk_2^2 + 38bk_2k_3 + 6bk_3^2) + k_1(8b^4 - 16b^2k_2^2 + 60b^2k_2k_3 + 18b^2k_3^2 + 8k_2^4 - 28k_2^3k_3 + 16\,k_2^2k_3^{\ 2} + 2k_3^3k_2 \\
& - 2k_3^4) + c(X_{a1}(-8b^5 + 4b^4k_1 + b^3(6k_1^2 + 16k_2^2 + 6k_3^2) + b^2(16k_1k_2k_3 + k_1k_3^2 - k_1^3 - 8k_1k_2^2) + b(2k_1^2k_3^2 \\
& - k_1^4 - 2k_1^2k_2^2 - k_1^{\ 2}k_2k_3 - 8k_2^4 - 2k_2^2k_3^2 + k_2k_3^3 - k_3^4) - k_1^5 + 4k_1k_2^4 - 8k_1k_2^3k_3) + X_{a2}(-bk_1^4 + k_1^3(4b^2 + \\
& 4k_2^2 - k_2k_3 + 2k_3^2) + k_1^2(6b^3 - 2bk_2^2 + 20bk_2k_3 + bk_3^2) + k_1(14b^2\,k_2k_3 + 8b^2k_3^2 - 6k_2^3k_3 + 8k_2^2k_3^{\ 2} + k_2 \\
& k_3^3 - k_3^4) - 8b^5 + 16b^3k_2^2 + 4b^3k_3^2 - 8bk_2^4 + 4bk_2k_3^3) + X_{b1}(k_2k_1^4 + bk_2k_1^3 + k_1^2(2b^2k_2 - 6k_2^3 + k_2^2k_3 - 2 \\
& k_2k_3^2) + k_1(8b^3k_3 - 16bk_2^2k_3 - bk_2k_3^2) + 8b^4k_2 + 4b^4k_3 - 16b^2k_2^3 - 8b^2k_2k_3^2 + 2b^2k_3k_2^2 + 8k_2^5 + 4k_2^4k_3 \\
& - 6k_2^2k_3^{\ 2} - k_2^2k_3^3 + k_2k_3^4) + X_{b2}(-k_3k_1^4 + k_1^3(4bk_2 - bk_3) + k_1^2(8b^2k_3 - 4k_2^3 + 8k_3k_2^2 - k_2k_3^2 + 2\,k_3^3) + \\
& k_1(6b^3k_3 - 14bk_2^2k_3 + 20bk_3k_2^2 + bk_3^3) + 8b^4k_2 - 16b^2k_2^3 + 2b^2k_2k_3^2 + 4b^2k_3^3 + 8k_2^5 - 6k_2^3k_3^{\ 2} + 4k_2^2k_3^3 \\
& + k_2k_3^4 - k_3^5)]/[-4(b^2k_1^2 - 4b^4 + 8b^2k_2^2 + b^2k_3^2 + 4bk_1k_2k_3 + k_1^2k_2^2 - 4k_2^4 + k_2^2k_3^2)], \\
\frac{\partial\pi_B}{\partial w_b} =\ & [-cX_{a1}(bk_2k_1^3 + k_1^2(k_3b^2 - 4k_2^3) + k_1(4b^3k_3 - 12bk_2^2k_3 - bk_2k_3^2) + 8b^4k_2 + 4b^4k_3 - 16b^2k_2^3 - 8b^2k_2^2k_3 \\
& - b^2k_3^3 + 8k_2^5 + 4k_2^4k_3 - 4k_2^3k_3^2) - cX_{a2}(k_2k_1^4 + k_1^3(4bk_2 + bk_3) + k_1^2(2b^2k_2 + 4b^2k_3 - 6k_2^3 + 4k_2^2k_3 - \\
& k_2k_3^2) + k_1(6b^3k_3 - 14bk_2^2k_3 + 4bk_2k_3^2 - b\,k_3^3) + 8b^4k_2 + 4b^4k_3 - 16b^2k_2^3 - 8b^2k_2^2k_3 + 8k_2^5 + 4k_2^4k_3 - \\
& 4k_2^{\ 3}k_3^2) - cX_{b1}(-k_2^2k_1^3 + k_1^2(4b^3 - bk_2k_3) + k_1(4b^4 - 8b^2k_2^2 + 12b^2k_2k_3 + 4k_2^4 - 4k_2^3k_3 + k_2^2k_3^2) - 8b^5 \\
& + 16b^3k_2^2 + 4b^3k_3^2 - 8bk_2^4 + bk_2k_3^3) - cX_{b2}(k_2k_3k_1^3 + k_1^2(4b^3 + bk_3^2 + 4bk_2k_3) + k_1(4b^4 - 8b^2k_2^2 + 14 \\
& b^2k_2k_3 + 4b^2k_3^2 + 4k_2^4 - 6k_2^3k_3 + 4k_2^2k_3^2 - k_2k_3^3) - 8b^5 + 16b^3k_2^2 + 6b^3k_3^2 - 8bk_2^4 - 2bk_2^2k_3^2 + 4bk_2k_3^3 \\
& - bk_3^4) + w_a(-k_2k_1^4 - k_1^3(5bk_2 + bk_3) + k_1^2(-2b^2k_2 - 5b^2k_3 + 10k_2^3 - 4k_2^2k_3 + k_2k_3^2) + k_1(26bk_2^2k_3 - \\
& 10b^3k_3 - 3bk_2k_3^2 + bk_3^3) - 16b^4k_2 - 8b^4k_3 + 32b^2k_2^3 + 16b^2\,k_2^2k_3 + b^2k_3^3 - 16k_2^5 - 8k_2^4k_3 + 8k_2^3\,k_3^2) + \\
& w_b(k_1^3(2k_2^2 - 2k_3k_2) + k_1^2(-16b^3 - 2bk_3^2 - 6k_2bk_3) + k_1(2k_2k_3^3 - 16b^4 + 32b^2k_2^2 - 52b^2k_2k_3 - 8b^2k_3^2 \\
& - 16k_2^4 + 20k_2^3k_3^2 - 10k_2^2k_3^2) + 32b^5 - 64b^3k_2^2 - 20b^3k_3^2 + 32bk_2^4 + 4bk_2^2k_3^2 - 10bk_2k_3^3 + 2bk_3^4) - a_ak_2 \\
& k_1^3 + k_1^2(2a_bk_2^2 - 6a_abk_2 - a_abk_3) + k_1(8a_ak_2^3 + k_2^2(8a_bb - 6a_ak_3) + k_2(4a_bbk_3 - 8a_ab^2 + a_ak_3^2) - 8a_b \\
& b^3 - 6a_ak_3b^2) - 16a_bb^4 - 8a_ab^3k_3 + 32a_bb^2k_2^2 - 8a_bb^2k_2k_3 + 2a_bb^2k_3^2 + 8a_abk_2^2k_3 - 6a_abk_2k_3^2 + a_ab \\
& k_3^3 - 16a_bk_2^4 + 8a_bk_2^3k_3]/\,4(b^2k_1^2 - 4b^4 + 8b^2k_2^2 + b^2k_3^2 + 4bk_1k_2k_3 + k_1^2k_2^2 - 4k_2^4 + k_2^2k_3^2)].
\end{aligned}\right. \tag{8}$$

3.3. The dynamic model

In this paper, the manufactures are the leaders and the retailers are the followers. The manufactures A and B set the w_a and w_b, respectively. Then the retailers set the retail price of the goods, the manufacturers will adjust the wholesale price according to the market situation and the retailers will adjust their price at the same time.The wholesale prices we set in period *t* are $w_a(t)$ and $w_b(t)$. Obviously, the wholesale prices $w_a(t+1)$

and $w_b(t+1)$ in period $t+1$ are adjusted based on the previous price $w_a(t)$ and $w_b(t)$. Therefore, the price regulation model of manufactures is given by:

$$\begin{cases} w_a(t+1) = w_a(t) + s_1 w_a(t)\dfrac{\partial \pi_A}{\partial w_a}, \\ w_b(t+1) = w_b(t) + s_2 w_b(t)\dfrac{\partial \pi_B}{\partial w_b}. \end{cases} \tag{9}$$

where s_1 and s_2 are coefficients that capture the speed at which the two manufacturers adjust its price according to the consequent marginal change in its profit respectively.

4. The system stability

An In system (9),let $w_a(t+1)=w_a(t)$, $w_b(t+1)=w_b(t)$.Then we have:

$$\begin{cases} \dfrac{\partial \pi_A}{\partial w_a} = 0, \\ \dfrac{\partial \pi_B}{\partial w_b} = 0. \end{cases} \tag{10}$$

Four fixed points can be available, but only the Nash equilibrium point $E^*(w_a, w_b)$ has the economic meaning. To study the stability of the Nash equilibrium point, the Jacobian matrix of E* is needed:

$$J = \begin{vmatrix} 1 + s_1\dfrac{\partial \pi_A}{\partial w_a} + s_1 w_a \dfrac{\partial^2 \pi_A}{\partial^2 w_a} & s_1 w_a \dfrac{\partial^2 \pi_A}{\partial w_a \partial w_b} \\ s_2 w_b \dfrac{\partial^2 \pi_B}{\partial w_b \partial w_a} & 1 + s_2\dfrac{\partial \pi_A}{\partial w_b} + s_2 w_b \dfrac{\partial^2 \pi_A}{\partial^2 w_b} \end{vmatrix} \tag{11}$$

4.1.Case1: under the win-win environment

When retailers under the win-win environment, the Nash equilibrium point meet Eq.(10) and formula.(5).We can obtain the $E^*(w_a, w_b)$:

$$\begin{cases} w_a = -c(X_{a1}+X_{a2})/4 - [4(a_a-a_b)(2(b^2-k_2{}^2)-(b+k_2)(k_1+k_3)) + c(X_{a1}+X_{a2}-X_{b1}-X_{b2}) \\ \quad (-k_1{}^2 k_2 + 2bk_1k_2 + 2k_2{}^3 - 2b^2k_2 + 3k_2{}^2k_3 + k_2k_3{}^2 - b^2k_3)]/[8(-4b^3 + b^2(6k_1 - 2k_2 - k_3) + \\ \quad b(-2k_1{}^2 + 2k_1k_2 + 4k_2{}^2 + 4k_2k_3 + 2k_3{}^2) - k_1{}^2k_2 - 2k_1k_2{}^2 + 2k_2{}^3 + 3k_2{}^2k_3 + k_2k_3{}^2)] - [(a_a + \\ \quad a_b)(4(-k_1k_2 + k_2k_3 + 2k_2{}^2 - 2b^2)) + 4b(a_a - a_b)(k_1 - k_3) + c(X_{a1}+X_{a2}-X_{b1}-X_{b2})((-2 \\ \quad k_2 - k_3)b^2 + 2bk_1k_2 - k_1{}^2k_2 + 2k_2{}^3 - 2bk_2{}^2 + 3k_2{}^2k_3 + k_2k_3{}^2)]/[8(4b^3 - b^2(6k_1 + 2k_2 + k_3) \\ \quad + b(2k_1{}^2 + 2k_1k_2 - 4k_2{}^2 - 4k_2k_3 - 2k_3{}^2) - k_1{}^2k_2 + 2k_1k_2{}^2 + 2k_2{}^3 + 3k_2{}^2k_3 + k_2k_3{}^2)], \\ w_b = -c(X_{b1}+X_{b2})/4 + [4(a_a-a_b)(2(b^2-k_2{}^2)-(b+k_2)(k_1+k_3)) + c(X_{a1}+X_{a2}-X_{b1}-X_{b2}) \\ \quad (-k_1{}^2k_2 + 2bk_1k_2 + 2k_2{}^3 - 2b^2k_2 + 3k_2{}^2k_3 + k_2k_3{}^2 - b^2k_3)]/[8(-4b^3 + b^2(6k_1 - 2k_2 - k_3) + \\ \quad b(-2k_1{}^2 + 2k_1k_2 + 4k_2{}^2 + 4k_2k_3 + 2k_3{}^2) - k_1{}^2k_2 - 2k_1k_2{}^2 + 2k_2{}^3 + 3k_2{}^2k_3 + k_2k_3{}^2)] + [4(a_a + \\ \quad a_b)(2(k_2{}^2 - b^2) - (b-k_2)(k_1-k_3)) + c(X_{a1}+X_{a2}-X_{b1}-X_{b2})(b^2(-2k_2 - k_3) + 2bk_1k_2 - \\ \quad k_1{}^2k_2 + 2k_2{}^3 - 2bk_2{}^2 + 3k_2{}^2k_3 + k_2k_3{}^2)]/[8(4b^3 - b^2(6k_1 + 2k_2 + k_3) + b(2k_1{}^2 + 2k_1k_2 - 4k_2{}^2 \\ \quad - 4k_2k_3 - 2k_3{}^2) - k_1{}^2k_2 + 2k_1k_2{}^2 + 2k_2{}^3 + 3k_2{}^2k_3 + k_2k_3{}^2)]. \end{cases} \tag{12}$$

Substituting formula(5) and (12) into (11), the Jacobian matrix of the E* in this condition can be determined as J_1 and denoting λ_1 and λ_2 as its characteristic values. Only when the absolute value of both λ_1 and λ_2 are less than 1, the system(9) of Nash equilibrium is stable.

Then when $a_a=4$, $a_b=3$, $b=2$, $k_1=0.8$, $k_2=0.6$, $k_3=0.4$, $c=0.02$, $X_{a1}=2$, $X_{b1}=4$, $X_{a2}=3$, $X_{b2}=5$, we get $E^*(w_a, w_b) = (308/83, 3332/981)$.

$$J_1 = \begin{vmatrix} 1 - \dfrac{1586151557786831320 8313}{1565563820464668672000}s_1 & \dfrac{5091443419633056 83}{1351079888211148 80}s_1 \\ \dfrac{77807271990547021}{22517998136852480}s_2 & 1 - \dfrac{1454370976989205507 6967}{1565563820464668672000}s_2 \end{vmatrix} \tag{13}$$

$$\lambda_1 = 1 - 4.64s_2 - 5.07s_1 - 0.5 * \sqrt{102.88s_1^2 - 136.22s_1s_2 + 86.19s_2^2},$$

$$\lambda_2 = 1 - 4.64s_2 - 5.07s_1 + 0.5 * \sqrt{102.88s_1^2 - 136.22s_1s_2 + 86.19s_2^2}.$$

The necessary and sufficient condition for the local stability of E* should satisfy the following conditions:

$$\begin{cases} |\lambda_1| < 1, \\ |\lambda_2| < 1. \end{cases}$$

From Fig.2 to Fig.4,we can find that with the change of the wholesale price adjustment speed s_1, s_2,the system(9) presents the complex dynamic characteristics.When $s_2=0.1, s_1 \in (0,0.174)$,the system(9) keeps stable;when $s_2=0.1, s_1>0.174$,the system(9) appears bifurcation even chaos.

When the other parameters are kept constant, the stable region of the Nash equilibrium in(s_1,s_2) is changed with the single parameter. As are shown in Fig.5-Fig.7 ,when the parameter *b* becomes larger, the stable region also becomes larger. For convenience,we only give the two-dimensional map about other parameter in the follows. As are shown in Fig.8-Fig.11, when the k_1, k_2 or k_3 becomes larger, the stable region becomes smaller. This shows that the sensitive coefficient of their own prices becomes larger, the system will be more stable. On the contrary, the greater the impact of other prices on sales, the smaller the system stable region is.So, it is important for retailers to improve the influence of their products. The distance has little effects on the stability ,which is caused by that the cost is equal to the value of the goods.

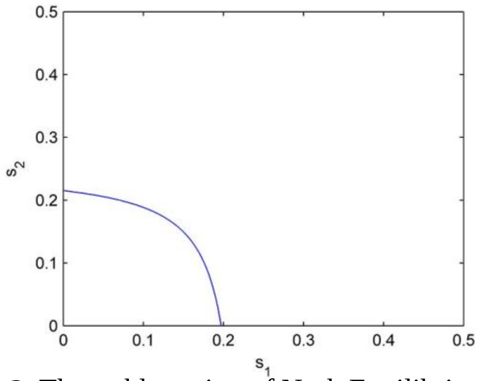


Fig.2. The stable region of Nash Equilibrium in (s_1,s_2) under the win-win environment .

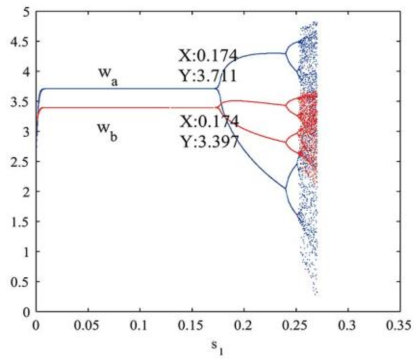
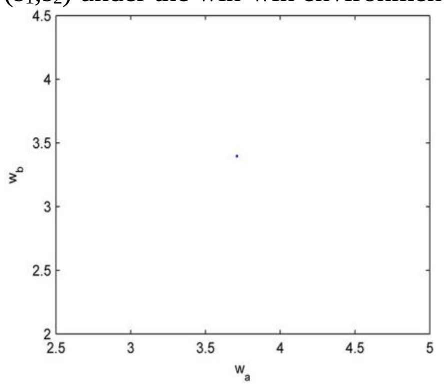
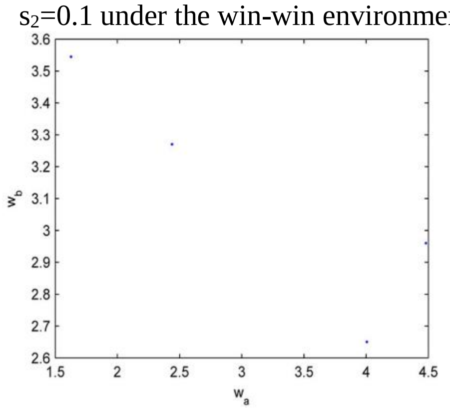


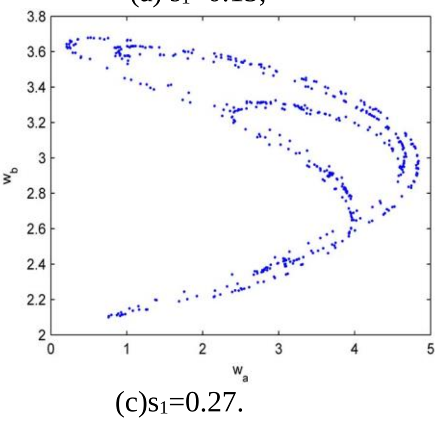
Fig.3. Bifurcation diagram with $s_1 \in (0,0.27)$, $s_2=0.1$ under the win-win environment.



(a) $s_1=0.15$;



(b) $s_1=0.25$;



(c)$s_1=0.27$.

Fig.4. Phase plot of system(9) with $s_2=0.1$ under win-win environment.

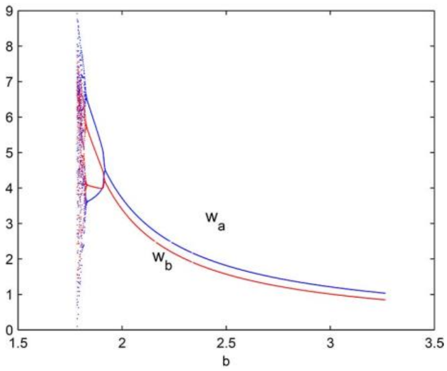


Fig.5. Bifurcation diagram of system(9) with $b \in (1.5, 3.5)$ when $s_1=0.15$, $s_2=0.1$.

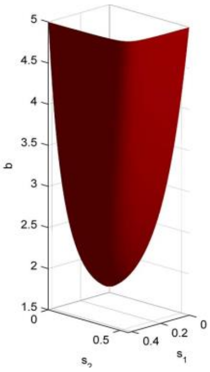


Fig.6. The 3D diagram of system(9) on the variable s_1, s_2,

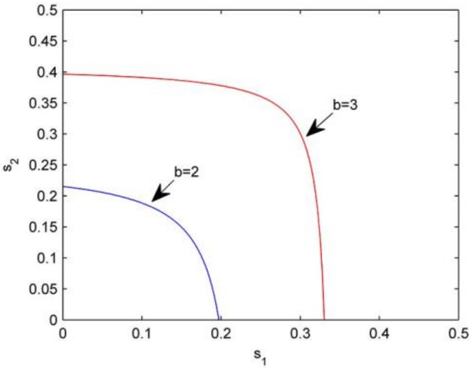


Fig.7. The stable region of Nash Equilibrium in(s_1,s_2) plan with different b .

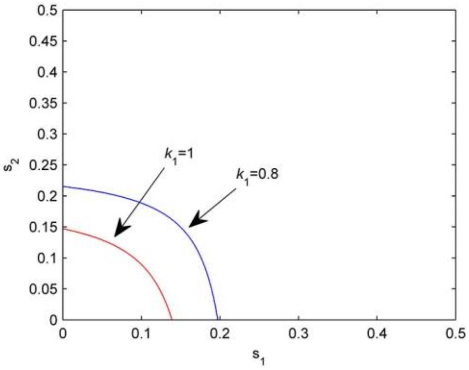


Fig.8. The stable region of Nash Equilibrium in(s_1,s_2) plan with different k_1.

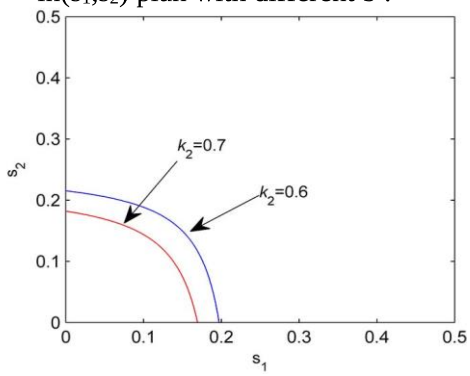


Fig.9. The stable region of Nash Equilibrium in(s_1,s_2) plan with different k_2 .

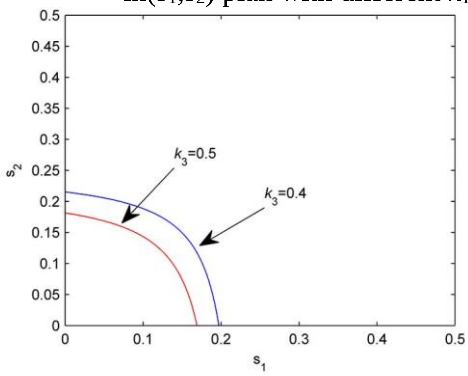


Fig.10. The stable region of Nash Equilibrium in(s_1,s_2) plan with different k_3.

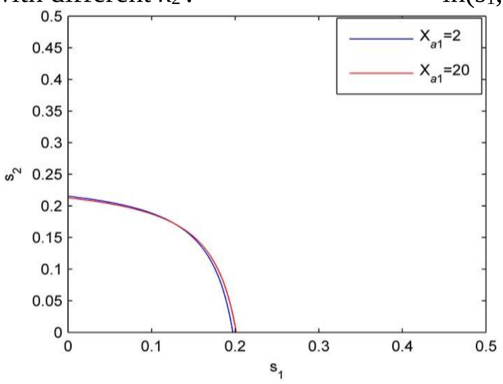


Fig.11. The stable region of Nash Equilibrium in(s_1,s_2) plan with different X_{a1}.

4.2. Case2: under the malicious competition environment

When retailers under the malicious competition environment, the Nash equilibrium point meet Eq.(10) and formula.(8).We can obtain the optimal solution of w_a and w_b（The formula is too complex, and the following is abbreviated):

$$\begin{cases} w_a = [-cX_{a1}(k_1^7(bk_2^2 - bk_3k_2) + k_1^6(4k_2^4 - 4k_2^3k_3) + 256b^{10} - 1152b^8k_2^2 - 4k_2^3k_3^7) \\ \quad - cX_{a2}(k_1^8(k_2^2 - k_3k_2) - k_1^7(16b^3 + 7bk_2^2 - 4bk_2k_3 - bk_3^2) + 256b^{10} - 4k_2^3 k_3^7) \\ \quad + cX_{b1}(k_1^7 (k_2^3 - k_3k_2^2) - k_1^6 (12b^3k_2 + 4b^3k_3 + bk_2k_3^2,) + 128k_2b^9 + bk_2k_3^8) \\ \quad - cX_{b2}(k_1^7(k_2^2k_3 - k_2k_3^2) + k_1^6(bk_2^2k_3 - 12b^3k_3 - bk_3^3) - 128b^9k_2 + bk_3^9) + k_1^7 \\ \quad (a_ak_3k_2 - a_a k_2^2) + \ldots + k_1^6(16 a_ab^3 - 2a_bk_2^2k_3) + 8a_bk_2^5k_3^4 + 8a_bk_2^3k_3^6)]/[(3 \\ \quad k_2{}^2 - 3k_2k_3)k_1^8 - (32b^3 + 2bk_2^2 - 15bk_2k_3 - 3bk_3^2)k_1^7 + 1024b^{10} - 4352b^8k_2^2 - \\ \quad 192b^8k_2k_3 - 1312b^8k_3^2 + 7168b^6k_2^4 + 288k_2^8k_3^2 + 80k_2^7k_3^3 - 152k_2^6k_3^4 + 24k_2^5k_3^5 \\ \quad + 24k_2^4k_3^6 - 8k_2^3k_3^7], \\ w_b = [cX_{a1}(bk_2k_1^8 + (-2b^2k_2 + k_3b^2 - 8k_2^3)k_1^7 - b^3k_3^7 + 14b^5k_3^5 - 56b^7k_3^3 + 64bk_2^8 \\ \quad k_3) - cX_{a2}(-k_2k_1^9 - (6bk_2 + bk_3)k_1^8 - 128b^9k_2 - 64b^9k_3 + 512b^7k_2^3 - 16bk_2^4 \\ \quad k_3^5) - cX_{b1}(k_2^2k_1^8 + (bk_2k_3 - 2bk_2^2 - 8b^3)k_1^7 + 256b^{10} - 1152b^8k_2^2 - 128k_2^{10} + \\ \quad + 8k_2^4k_3^6) - cX_{b2}(-k_2k_3k_1^8 - (8b^3 - 4bk_2^2 - 6bk_2k_3 - bk_3^2)k_1^7 + 256b^{10} - 1152b^8 \\ \quad k_2^2 - 8k_2^3k_3^7) + \ldots + k_1^7(8a_ab(k_2 + k_3) - 2a_bk_2^2) - k_1(64a_bb^6k_2^2 + \ldots 16a_bk_2^3k_3^5) + \\ \quad 8a_ak_2^3k_3^6 - 1344a_bb^5k_2^3k_3)]/[(3k_2{}^2 - 3k_2k_3)k_1^8 - (32b^3 + 2bk_2^2 - 15bk_2k_3 - 3b \\ \quad k_3^2)k_1^7 + 1024 b^{10} - 4352 b^8k_2^2 - 192 b^8k_2k_3 - 1312b^8k_3^2 + 7168b^6k_2^4 + 288 k_2^8 \\ \quad k_3^2 + 80k_2^7k_3^3 - 152k_2^6k_3^4 + 24k_2^5k_3^5 + 24k_2^4k_3^6 - 8k_2^3k_3^7]. \end{cases} \tag{14}$$

Substituting formula(8) and (14) into (11), the Jacobian matrix of the E* in this condition can be determined as J_2 and denoting λ_3 and λ_4 as its characteristic values. Only when the absolute value of both λ_3 and λ_4 are less than 1, the Nash equilibrium is stable.

Then when a_a=4, a_b=3, b=2, k_1=0.8, k_2=0.6, k_3=0.4, c=0.02, X_{a1}=2, X_{b1}=4, X_{a2}=3, X_{b2}=5, we can get E*(w_a ,w_b)=(2036/533,1066/321).

$$J_2 = \begin{vmatrix} 1 - \frac{387723159160821112479473 3}{359870756720160077250560} s_1 & \frac{9739206}{2497105} s_1 \\ \frac{10403627}{3007770} s_2 & 1 - \frac{394175454533973230311545 1}{433465339238917016125440} s_2 \end{vmatrix} \tag{15}$$

$$\lambda_3 = 1 - 4.55s_2 - 5.39s_1 - 0.5 * \sqrt{116.08s_1^2 - 141.99s_1s_2 + 82.69s_2^2},$$

$$\lambda_4 = 1 - 4.55s_2 - 5.39s_1 + 0.5 * \sqrt{116.08s_1^2 - 141.99s_1s_2 + 82.69s_2^2}$$

The necessary and sufficient condition for the local stability of E* should satisfy the following conditions:

$$\begin{cases} |\lambda_3| < 1, \\ |\lambda_4| < 1. \end{cases}$$

Similar to the win-win environment,from Fig.12 to Fig.14,we can find that under the malicious competition environment,when s_2=0.1,$s_1 \in (0,0.167)$,the system(9) keeps stable;when s_2=0.1,s_1>0.167,the system(9) appears bifurcation even chaos.

Under the malicious competitive environment, we also make the same comparison of the parameters. As is shown in Fig.15-Fig.19, when the parameter b becomes larger, the stable region also becomes larger.But when k_1, k_2 or k_3 becomes larger, the stable region becomes smaller. This also shows that the sensitive coefficient of their own prices becomes larger, the system will be more stable. The greater the impact of other prices on sales, the smaller the system stability region is. The distance has little effects on the system stability.This shows that whether it is a condition of win-win or malicious competition, the impact of price sensitivity coefficients on the the stable region of the system (9) are same.

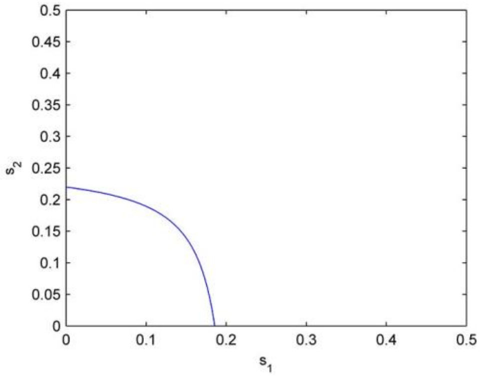


Fig.12. The stable region of Nash Equilibrium in (s_1,s_2) under the malicious competition environment.

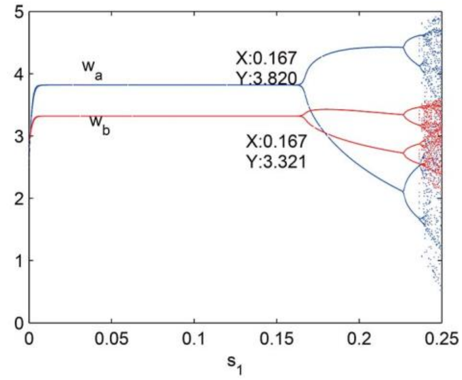
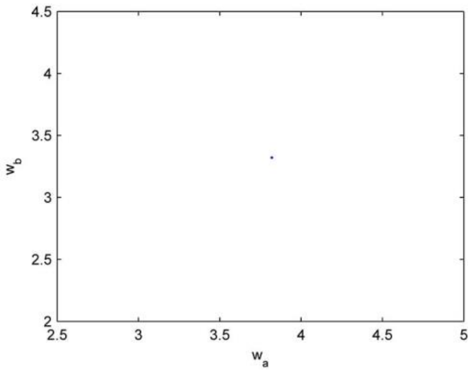
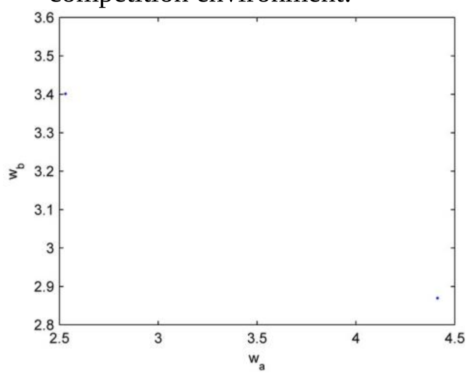


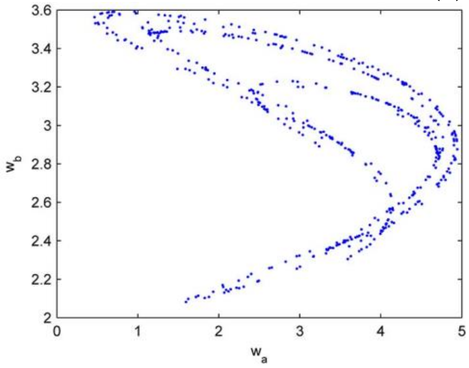
Fig.13. Bifurcation diagram of system(9)with $s_1 \in (0,0.25)$, s_2=0.1 under the malicious competition environment.



(a)s_1=0.15;



(b)s_1=0.20;



(c)s_1=0.25.

Fig.14. Phase plot of system(9) with s_2=0.1 under the malicious competition environment.

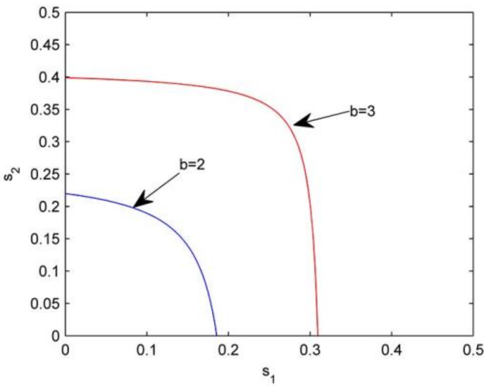


Fig.15. The stable region of Nash Equilibrium in (s_1,s_2) plan with different b.

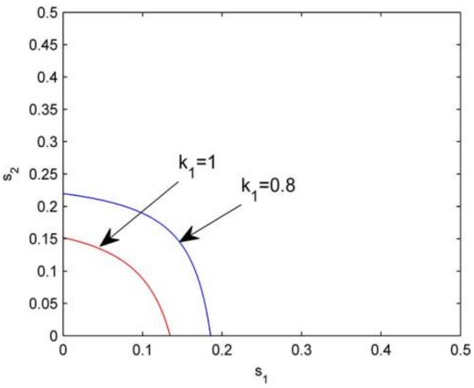


Fig.16.The stable region of Nash Equilibrium in(s_1,s_2) plan with different k_1.

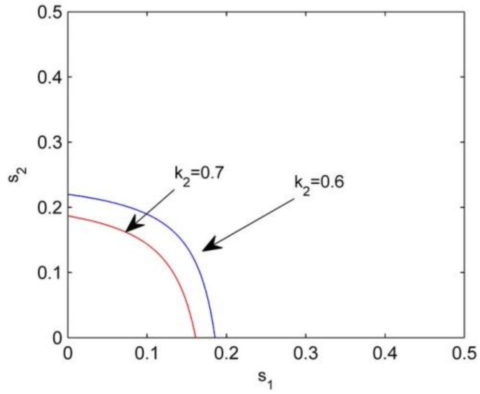


Fig.17. The stable region of Nash Equilibrium in(s_1,s_2) plan with different k_2..

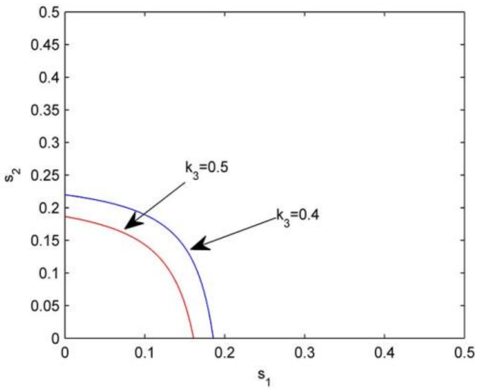


Fig.18. The stable region of Nash Equilibrium in(s_1,s_2) plan with different k_3.

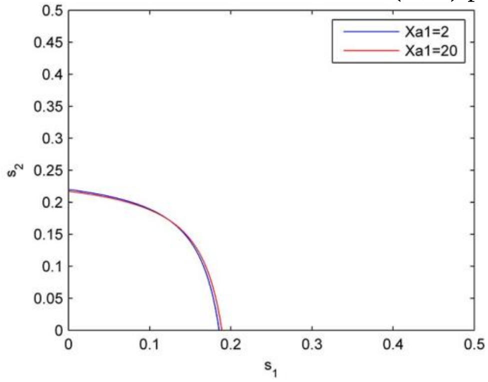


Fig.19. The stable region of Nash Equilibrium in(s_1,s_2) plan with different X_{a1} .

5. Comparison

5.1. The stable range of Nash Equilibrium in price adjustment speed

Comparing Fig.7-11 with Fig.15-19, we find that the parameters influence on the stable range of Nash Equilibrium in (s1,s2) are the same. When the sensitive coefficient of their own prices b becomes larger, the range of price adjustment speed becomes bigger. But the greater the impact of other prices on sales, the smaller the system stability region is.In a reasonable range, distance has little effect on stability.This is because the freight has been calculated into the price of goods.So,it is necessary for retailers to improve their prices' influence for both stable market and better competition.

However, when using the same parameters, the change range of s2 under the malicious competition is larger than the range under the win-win environment , but s1 is the opposite(Fig.20).Visible, malicious competition is more beneficial to the price adjustment of commodity a.This may be due to the the malicious competition environment,which is formed by the retailer1's price adjustment of product a.

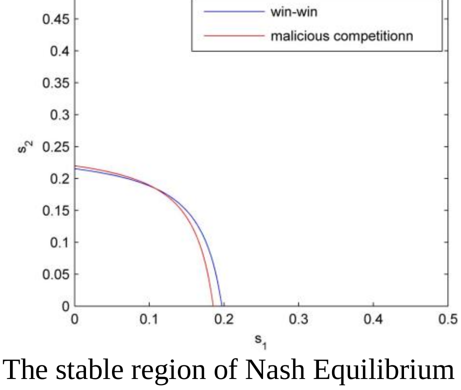


Fig.20. The stable region of Nash Equilibrium in (s_1,s_2) with different competitive environment.

5.2. Price of goods

In two environment, wa＞wb, this is because the total cost of transporting goods b is relatively higher.When comparing the state of win-win and malicious competition, if only in terms of unit price, the wholesale price of product a will be higher under the malicious environment while the wholesale price of product b is lower. The retail prices of both product a and b are reduced. Under the environment of malicious competition, the retailer R1 make the malicious competition by adjusting the price Pa1. But as shown in Fig.21, the decrease degree of the price Pa1 is the most.

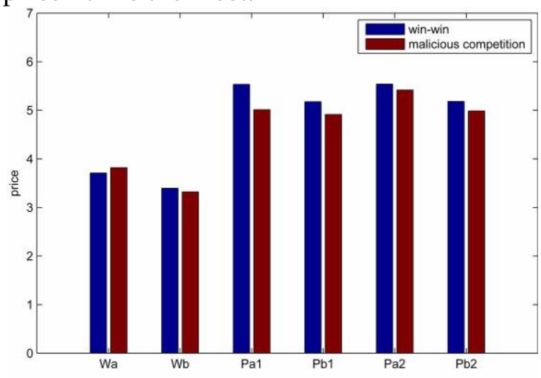


Fig.21. The wholesale and retail prices of products a and b in different competitive conditions.

5.3. Sales volume

In the condition of the malicious competition, the retailer R1 make the malicious competition by adjusting the price P_{a1}. This leads to a decline in the price P_{a1}.But the sales volume of product *a* in the Retailer1 increases, while the sales volume of both product *a* and *b* in Retailer2 are discount (Fig.22). In terms of improving sales volume,this approach also has a certain desirability.

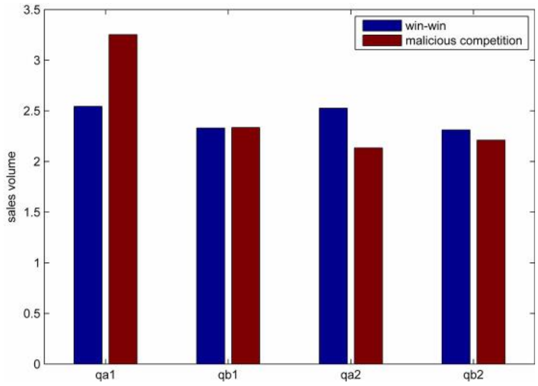


Fig.22. The sales volume of product *a* and *b*.

5.4 The vendors profits

As is seen in Fig.23-25, when retailer1 competes using the malicious method, the profits of both retailer1 and retailer2 are all decrease, although the relative benefits of the retailer 1 becomes larger.And under the malicious competition environment, the profits of vendors are more likely to be unstable.

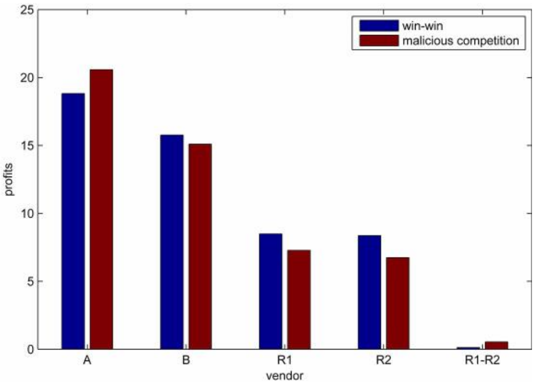


Fig.23. The vendors profits in different competitive environment.

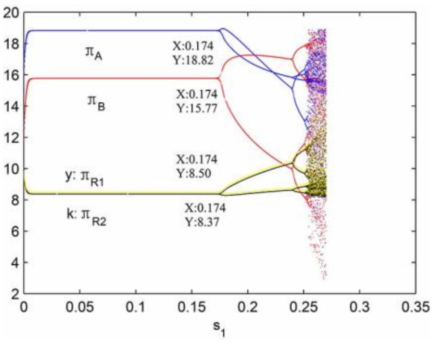


Fig.24. Bifurcation diagram of profits with $s_1 \in (0,0.27)$, s_2=0.1under the win-win environment.

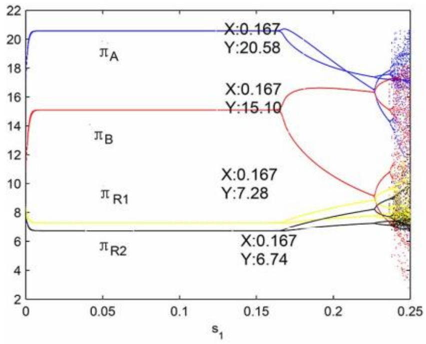


Fig.25. Bifurcation diagram of profits with $s_1 \in (0,0.25)$, s_2=0.1 under the malicious competition environment.

6. Conclusions

Based on the 2-2 suppliers-retailers situation, this paper researches on the market under two environment by using dynamic price Stackelberg game model. We find that in these two cases, the effect of price sensitivity coefficient on the stability of Nash Equilibrium is the same, while the amplitude of stable range is different. In addition, we find that besides its relative interests increase, the retailer which initiates the malicious competition bear a great loss in all other areas. These areas include the prices of goods, sales as well as the businessman profit. Moreover, compared to under the win-win environment, the market under malicious competition situation is more likely to appear unstable state. Therefore, we propose that businesses work together to build a friendly market environment in order to bring more revenue to their own. The analysis method in this paper is traditional, the author suggest that readers can use different methods to analyze, for example, the combination of complex network of the dichotomy, and the readers can also make appropriate changes in the model.

7. Acknowledge

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