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Adomian decomposition method for fuzzy differential equations with linear differential operator

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Abstract. In this paper we have taken the fuzzy differential equation with linear differential operator. We have used Adomian decomposition method (ADM) to find the approximate solution. We have given several numerical examples and by comparing the numerical results obtain from ADM with the exact solution, we have studied their accuracy.

Keywords: fuzzy differential; fuzzy differential equations; adomian decomposition method.

1. Introduction

Fuzzy differential equations are very useful to model dynamical systems whose uncertainty is characterized by a non-random process [9]. So the existence and uniqueness of the fuzzy differential equation is a area of great interest and this have been studied in [10, 13, 14, 19, 21, 23]. Various kind of fuzzy differential equation and their application have been studied by many researchers. Bede et al. interpret first order linear fuzzy differential equations by using the strongly generalized differentiability concept [11]. Chalco-Cano, Roman-Flores study the class of first order fuzzy differential equations where the dynamics is given by a continuous fuzzy mapping which is obtain via Zadeh's extension principle [12]. Solutions of first order fuzzy differential equations have been also studied in [15, 17, 18, 20, 22, 24, 25, 26]. An extension of differential transformation method using the concept of generalized H-differentiability has been studied by Allahviranloo et al. in [16].

The Adomian Decomposion Method (ADM) was first introduced by Adomian in 1980 [1]. ADM is very powerful tool to solve algebraic, differential, integral and integro-differential equations involving non-linear functional [2, 3, 4, 5]. A second-order fuzzy differential equation has been solved by using Adomian method under strongly generalized differentiability in [8]. Using ADM hybrid fuzzy differential equations have been solved in [7]. Numerical approximation of fuzzy first-order initial value problem by using ADM is presented in [6]. In this paper, we develop numerical method for fuzzy differential equations with linear differential operator by an application of the ADM. The structure of this paper is organized as follows: Section 2 contains some basic definitions of fuzzy sets and fuzzy number. Section 3 contains solution procedure of fuzzy differential equations with linear differential operator by ADM. In section 4, the proposed method is illustrated by two numerical examples and we compare ADM solution with exact solution to check the accuracy of the method. Finally the conclusion and future research is given in section 5.

2. Preliminaries

Definition 2.1. If X is a collection of objects denoted by x then a fuzzy set $\tilde{A}$ in X is a set of ordered pairs denoted and defined by:

$\tilde{A} = \{(x, \mu_{\tilde{A}}(x))/x \in X\}$, were $\mu_{\tilde{A}}(x)$ is called membership function or grade of membership (also degree of compatibility or degree of truth) of x in $\tilde{A}$ which maps X to $[0,1]$.

Definition 2.2. α-cut of a fuzzy $\tilde{A}$ set is a crisp set A_α and defined by

$A_\alpha = \{x/\mu_{\tilde{A}}(x) \geq \alpha\}$, where $\tilde{A} = \{(x, \mu_{\tilde{A}}(x))\}$.

Definition 2.3. A fuzzy set $\tilde{A}$ is said to be convex fuzzy set if A_α is a convex set for all $\alpha \in (0,1]$.

Definition 2.4. A fuzzy set $\tilde{A}$ is said to be normal fuzzy set if there exist an element $(a, 1) \in \tilde{A}$.

Definition 2.5. If a fuzzy set is convex, normalized and its membership function, defined in $\mathbb{R}$, is piecewise continuous then it is called as fuzzy number.

A triangular fuzzy number $\tilde{A}$ is denoted by $(a_1,\ a_2,\ a_3)$ and it is a fuzzy set $\{(x, \mu_{\tilde{A}}(x))\}$ where

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-a_1}{a_2-a_1}, a_1 \le x \le a_2 \\ \frac{a_3-x}{a_3-a_2}, a_2 \le x \le a_3 \\ 0, \text{ otherwise} \end{cases}.$$

$\tilde{A}$ is called positive triangular fuzzy number if $a_1 > 0$ and negative triangular fuzzy number if $a_3 < 0$.

Definition 2.6. [27]. Let E be the set of all upper semicontinuous normal convex fuzzy numbers with bounded α-cut intervals. It means if $\tilde{v} \in$ E then the α-cutset is a closed bounded interval which is denoted by

$$v_{\alpha} = [v_1, v_2].$$

For arbitrary $u_{\alpha} = [u_1, u_2]$,$v_{\alpha} = [v_1, v_2]$and $k \ge 0$, addition $(u_{\alpha} + v_{\alpha})$ and multiplication by k are defined as $(u+v)_1(\alpha) = u_1(\alpha) + v_1(\alpha)$, $(u+v)_2(\alpha) = u_2(\alpha) + v_2(\alpha)$, $(ku)_1(\alpha) = ku_1(\alpha)$, $(ku)_2(\alpha) = ku_2(\alpha)$.

Since each y ∈R can be regarded as a fuzzy number $\tilde{y}$ defined by

$$\mu_{\tilde{y}}(x) = \begin{cases} 1 \quad if \ \ x = y, \\ 0 \ \ if \ x \ne y. \end{cases}$$

The Hausdorff distance between fuzzy numbers given by D :E×E→ $R_{+} \cup \{0\}$,

$$D(\tilde{u}, \tilde{v}) = \sup_{\alpha \in [0,1]} \text{Max}\{|u_1(\alpha) - v_1(\alpha)|, |u_2(\alpha) - v_2(\alpha)|\}.$$

It is easy to see that D is a metric in E and has the following properties (see [28])

(i) $D(\tilde{u} \oplus \tilde{w}, \tilde{v} \oplus \tilde{w}) = D(\tilde{u}, \tilde{v}), \forall\ \tilde{u}, \tilde{v}, \tilde{w} \in$E,

(ii) $D(k \odot \tilde{u}, k \odot \tilde{v}) = |k| D(\tilde{u}, \tilde{v}), \forall k \in$R, $\tilde{u}, \tilde{v} \in$E,

(iii) $D(\tilde{u} \oplus \tilde{v}, \tilde{w} \oplus \tilde{e}) \le D(\tilde{u}, \tilde{w}) + D(\tilde{v}, \tilde{e}), \forall\ \tilde{u}, \tilde{v}, \tilde{w}, \tilde{e} \in$E,

(iv) (D,E) is a complete metric space.

Definition 2.7. (see [24]). Let f:R →E be a fuzzy valued function. If for arbitrary fixed t_0 ∈R and $\epsilon > 0$, a $\delta > 0$ such that

$$|t - t_0| < \delta \Longrightarrow D(f(t), f(t_0)) < \epsilon,$$

f is said to be continuous.

3. Fuzzy differential equation with linear differential operator

The fuzzy differential equation with linear differential operator is as follows:

$$L\tilde{u}(t) + R\tilde{u}(t) + N(t, \tilde{u}(t)) = \tilde{g}(t) \tag{1}$$

$\tilde{u}^j(a) = \tilde{a}_j, j = 0,1, \ldots\ldots, m-1$, where $\tilde{g}(t)$ is a fuzzy function of t, L is the highest order linear differential operator of order m, R is the remaining part of the linear differential operator and N may be linear or nonlinear function of t and $\tilde{u}(t)$. Here, in general, we take N as a nonlinear function of t and $\tilde{u}(t)$ such that

$$N_1(t, u_1, u_2) = \sum_{i=1}^{l_1} F_{1i}(t, u_1) F_{2i}(t, u_2) \tag{2}$$

$$N_2(t, u_1, u_2) = \sum_{j=1}^{l_2} G_{1j}(t, u_1) G_{2j}(t, u_2) \tag{3}$$

where $u_{\alpha}(t) = [u_1(t,\alpha), u_2(t,\alpha)], [N_1(t, u_1, u_2), N_2(t, u_1, u_2)] = N(t, [u_1(t,\alpha), u_2(t,\alpha)])$,

$F_{1i}(t, u_1)$, $G_{1j}(t, u_1)$ are functions of t and $u_1(t,\alpha)$ and $F_{2i}(t, u_1)$, $G_{2j}(t, u_1)$ are functions of t and $u_2(t,\alpha)$, i = 1,2, ..., l_1, $j = 1,2, \ldots\ldots, l_2$.

Let, $[H_1(u_1, u_2), H_2(u_1, u_2)] = R[u_1(t,\alpha), u_2(t,\alpha)]$, $a_{j\alpha} = [a_{j1}(\alpha),\ a_{j1}(\alpha)]$ and

$$g_{\alpha}(t) = [g_1(t,\alpha), g_2(t,\alpha)] \tag{4}$$

Now, from (1) we get

$$L[u_1(t,\alpha), u_2(t,\alpha)] + R[u_1(t,\alpha), u_2(t,\alpha)] + N(t, [u_1(t,\alpha), u_2(t,\alpha)]) = [g_1(t,\alpha), g_2(t,\alpha)] \tag{5}$$

Therefore,

$$Lu_1(t,\alpha) + H_1(u_1, u_2) + N_1(t, u_1, u_2) = g_1(t,\alpha) \tag{6}$$

and

$$Lu_2(t,\alpha) + H_2(u_1, u_2) + N_2(t, u_1, u_2) = g_2(t,\alpha) \tag{7}$$

Applying the inverse operator L^{-1} of L on both sides of (6) and (7) we get

$$u_1(t,\alpha) = \sum_{j=0}^{m-1} a_{j1}\frac{x^j}{(j)!} + L^{-1}g_1(t,\alpha) - L^{-1}H_1(u_1,u_2) - L^{-1}N_1(t,u_1,u_2) \quad (8)$$

$$u_2(t,\alpha) = \sum_{j=0}^{m-1} a_{j2}\frac{x^j}{(j)!} + L^{-1}g_2(t,\alpha) - L^{-1}H_2(u_1,u_2) - L^{-1}N_2(t,u_1,u_2) \quad (9)$$

The Adomian decomposition method assume an infinite series solution for the unknown functions $u_1(t,\alpha)$ and $u_2(t,\alpha)$, given by

$$u_1(t,\alpha) = \sum_{n=0}^{\infty} u_{1n}(t,\alpha) \quad (10)$$

$$u_2(t,\alpha) = \sum_{n=0}^{\infty} u_{2n}(t,\alpha) \quad (11)$$

The nonlinear functions $F_{1i}(t,u_1), F_{2i}(t,u_2), G_{1j}(t,u_1), G_{2j}(t,u_2)$ into an infinite series of polynomials given by

$$F_{1i}(t,u_1) = \sum_{n=0}^{\infty} A_{1in} \quad (12)$$

$$F_{2i}(t,u_2) = \sum_{n=0}^{\infty} A_{2in} \quad (13)$$

$$G_{1j}(t,u_1) = \sum_{n=0}^{\infty} B_{1jn} \quad (14)$$

$$G_{2j}(t,u_2) = \sum_{n=0}^{\infty} B_{2jn} \quad (15)$$

where $A_{1in}, A_{2in}, B_{1jn}, B_{2jn}$ are the so-called Adomian polynomial defined by

$$A_{1in} = \frac{1}{n!}\left[\frac{d^n}{d\lambda^n} F_{1i}\left(t, \sum_{k=0}^{n} \lambda^k u_{1k}\right)\right]_{\lambda=0}, \quad (16)$$

$$A_{2in} = \frac{1}{n!}\left[\frac{d^n}{d\lambda^n} F_{2i}\left(t, \sum_{k=0}^{n} \lambda^k u_{2k}\right)\right]_{\lambda=0}, \quad (17)$$

$$B_{1jn} = \frac{1}{n!}\left[\frac{d^n}{d\lambda^n} G_{1j}\left(t, \sum_{k=0}^{n} \lambda^k u_{1k}\right)\right]_{\lambda=0}, \quad (18)$$

$$B_{2jn} = \frac{1}{n!}\left[\frac{d^n}{d\lambda^n} G_{2j}\left(t, \sum_{k=0}^{n} \lambda^k u_{2k}\right)\right]_{\lambda=0}, \quad (19)$$

$i = 1,2,\ldots\ldots, l_1, \quad j = 1,2,\ldots\ldots, l_2, n \geq 0.$

We can see that A_{1i0} and B_{1j0} depend only on u_{10}, A_{1i1} and B_{1j1} depend only on u_{10} and u_{11}, and so on.

Similarly, A_{2i0} and B_{2j0} depend only on u_{20}, A_{2i1} and B_{2j1} depend only on u_{20} and u_{21}, and so on. Where $i = 1,2,\ldots\ldots, l_1, \; j = 1,2,\ldots\ldots, l_2$.

Using the Adomian decomposition method we set the recurrence relation as follows:

$$u_{10} = L^{-1}g_1(t,\alpha) + \sum_{j=0}^{m-1} a_{j1}\frac{x^j}{(j)!} \quad (20)$$

$$u_{20} = L^{-1}g_2(t,\alpha) + \sum_{j=0}^{m-1} a_{j2}\frac{x^j}{(j)!} \quad (21)$$

$$u_{11} = -L^{-1}H_1(u_{10},u_{20}) - L^{-1}\sum_{i=1}^{m} A_{1i0}A_{2i0}$$

$$u_{21} = -L^{-1}H_2(u_{10},u_{20}) - L^{-1}\sum_{j=1}^{l} B_{1j0}B_{2j0}$$

$$u_{12} = -L^{-1}H_1(u_{11},u_{21}) - L^{-1}\sum_{i=1}^{m} A_{1i1}A_{2i1}$$

$$u_{22} = -L^{-1}H_2(u_{11},u_{21}) - L^{-1}\sum_{j=1}^{l} B_{1j1}B_{2j1}$$

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$$u_{1k+1} = -L^{-1}H_1(u_{1k},u_{2k}) - L^{-1}\sum_{i=1}^{m} A_{1ik}A_{2ik} \quad (22)$$

$$u_{2k+1} = -L^{-1}H_2(u_{1k},u_{2k}) - L^{-1}\sum_{j=1}^{l} B_{1jk}B_{2jk}, \; k \geq 0 \quad (23)$$

Then, we define the nth term approximation to the solution $u_\alpha(t) = [u_1(t,\alpha), u_2(t,\alpha)]$ by $\psi_{n\alpha} = [\psi_{n1}(t,\alpha), \psi_{n2}(t,\alpha)]$ where, $\psi_{n1}(t,\alpha) = \sum_{i=0}^{n-1} u_{1i}(t,\alpha)$, $\psi_{n2}(t,\alpha) = \sum_{i=0}^{n-1} u_{2i}(t,\alpha)$.

Hence, $\lim_{n\to\infty}\psi_{n1}(t,\alpha) = u_1(t,\alpha)$ and $\lim_{n\to\infty}\psi_{n2}(t,\alpha) = u_2(t,\alpha)$.

4. Numerical examples

Example 4.1

Let us consider the fuzzy differential equation of the following form

$$\frac{d^2\tilde{u}}{dt^2} + \frac{d\tilde{u}}{dt} = e^t$$

with initial conditions

$$u_\alpha(0) = [1+\alpha, 3-\alpha]$$

$$u_\alpha'(0) = [\alpha, 2-\alpha]$$

The exact solution, given by classical solution method, is

$$u_1(t,\alpha) = cosht + 2\alpha - \alpha e^{-t} \quad (24)$$

$$u_2(t,\alpha) = sinht + (\alpha-1)e^{-t} + 4 - 2\alpha \quad (25)$$

Now we will use ADM to find the approximate solution.

The equation is

$$\frac{d^2u_1}{dt^2} = e^t - \frac{du_1}{dt}$$

$$\frac{d^2u_2}{dt^2} = e^t - \frac{du_2}{dt}$$

with the initial conditions

$$u_1(0,\alpha) = 1+\alpha\,,\ u_2(0,\alpha) = 3-\alpha$$

$$u_1'(0,\alpha) = \alpha\,,\quad u_2'(0,\alpha) = 2-\alpha$$

Here, $L \equiv \frac{d^2}{dt^2}$ and by operating the two sides of the above equation with the inverse operator (namely $L^{-1}(*) \equiv \iint(*)dtdt$) and using the initial conditions, we get

$$u_1(t,\alpha) = e^t + 2\alpha t + \alpha - \int_0^t u_1(x,\alpha)dx$$

$$u_2(t,\alpha) = e^t + (4-2\alpha)t + (2-\alpha) - \int_0^t u_2(x,\alpha)dx$$

Now applying the ADM we get

$$u_{10}(t,\alpha) = e^t + 2\alpha t + \alpha$$

$$u_{20}(t,\alpha) = e^t + (4-2\alpha)t + (2-\alpha)$$

$$u_{1k+1}(t,\alpha) = -\int_0^t u_{1k}(x,\alpha)dx\,,\ k \geq 0.$$

$$u_{2k+1}(t,\alpha) = -\int_0^t u_{2k}(x,\alpha)dx\,,\ k \geq 0.$$

On substituting and solving the above equation, we obtain the approximate solution after four iterations as

$$u_{10}(t,\alpha) + u_{11}(t,\alpha) + u_{12}(t,\alpha) + u_{13}(t,\alpha)$$

$$= 1 + \alpha + \alpha t + \frac{(1-\alpha)}{2}t^2 + \frac{\alpha}{2}t^3 + \frac{\alpha}{12}t^4$$

$$u_{20}(t,\alpha) + u_{21}(t,\alpha) + u_{22}(t,\alpha) + u_{23}(t,\alpha)$$

$$= (3-\alpha) + (2-\alpha)t + \frac{(\alpha-1)}{2}t^2 + \frac{(2-\alpha)}{2}t^3 + \frac{(2-\alpha)}{12}t^4$$

The comparison between the exact and approximate solutions at $t = 1$ for some $\alpha \in [0,1]$ has been shown at table 1, 2, 3 and 4. We have calculated all data for Table 1, Table 2 and Fig.1 by using MATLAB

Table 1: **Exact solution** at $t = 1$ for Example 1.

α	0	.2	.4	.6	.8	1
u_1	1.5431	1.8695	2.1959	2.5224	2.8488	3.1752
u_2	4.8073	4.4809	4.1545	3.8280	3.5016	3.1752

Table 2: **Approximate solution** at $t = 1$ for Example 1.

α	0	.2	.4	.6	.8	1
u_1	1.5000	1.8167	2.1333	2.4500	2.7667	3.0833
u_2	4.6667	4.3500	4.0333	3.7167	3.4000	3.0833

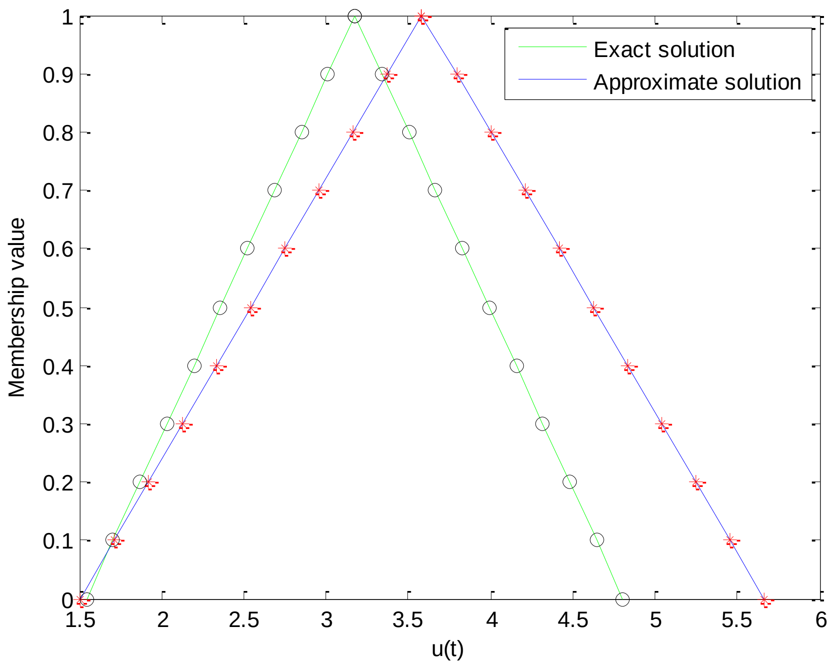

Fig.1 represents the plots of the exact solution $\tilde{u}$ and the approximate solution $\tilde{\psi}_4$ at the point $x = 1$ for Example 1.

We have also checked that for $\alpha = 0.2,\ 0.4,\ .6$ and $.8$, respectively, as the number of iteration increases, the maximal errors become gradually smaller and approach zero. For example, for $\alpha = .4$, we plot the curves of the error functions in Fig.2 and Fig.3 by using Mathematica 9.

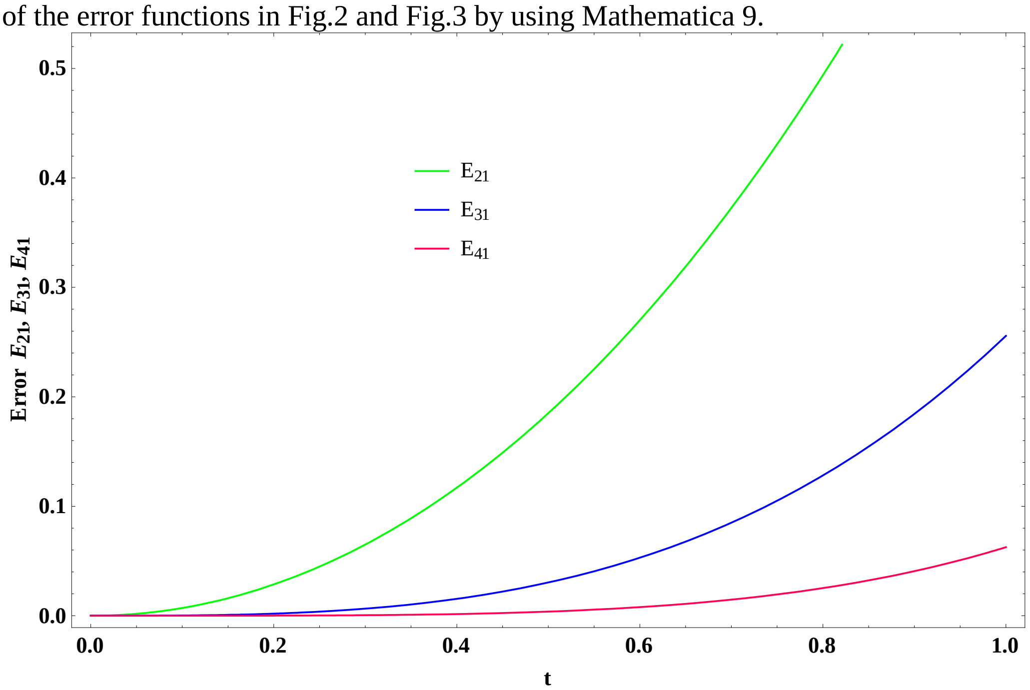

Fig.2 represents the plots of the error functions $E_{n1} = |u_1 - \psi_{n1}|$ for $n = 2, 3$ *and* 4 for $\alpha = .4$ for Example 1.

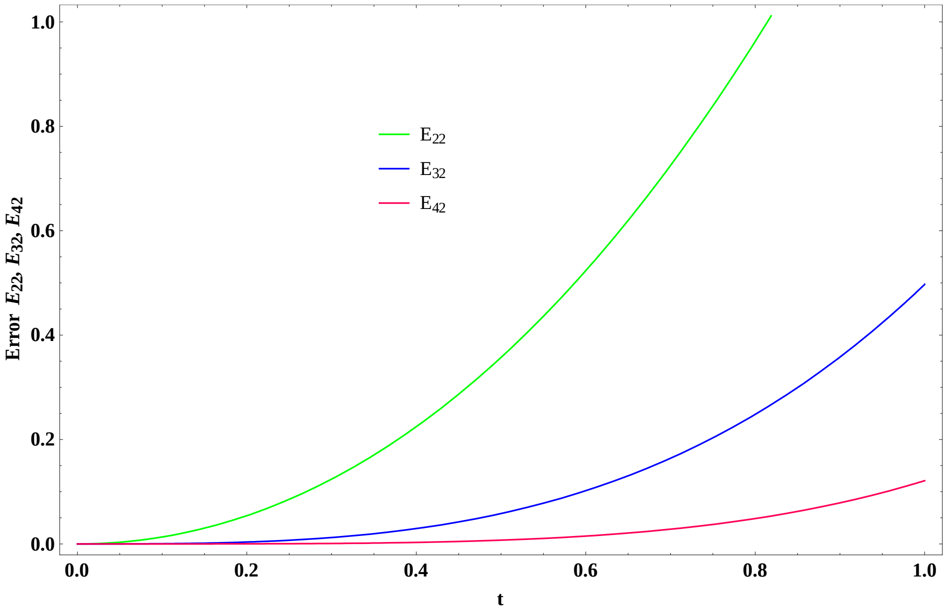

Fig.3 represents the plots of the error functions $E_{n2} = |u_2 - \psi_{n2}|$ for $n = 2, 3$ and 4 for $\alpha = .4$ for Example 1.

Example 4.2

Let us consider the fuzzy differential equation of the following form

$$\frac{d^2\tilde{u}}{dt^2} - 2\frac{d\tilde{u}}{dt} + \tilde{u} = \tilde{a} \text{ where } \tilde{a} = (-1, 0, 1) \text{ i.e. } a_\alpha = [\alpha - 1, 1 - \alpha]$$

with initial conditions

$$u_\alpha(0) = [1 + \alpha, 3 - \alpha]$$
$$u_\alpha{}'(0) = [\alpha, 2 - \alpha]$$

The exact solution, given by classical solution method, is

$$u_1(t, \alpha) = (2 - t)e^t + (\alpha - 1)te^{-t} + (\alpha - 1)$$
$$u_2(t, \alpha) = (2 - t)e^t - (\alpha - 1)te^{-t} - (\alpha - 1)$$

Now we will use ADM to find the approximate solution.

The equation is

$$\frac{d^2u_1}{dt^2} = (\alpha - 1) + 2\frac{du_2}{dt} - u_1$$
$$\frac{d^2u_2}{dt^2} = (1 - \alpha) + 2\frac{du_1}{dt} - u_2$$

with the initial conditions

$$u_1(0, \alpha) = 1 + \alpha, \quad u_2(0, \alpha) = 3 - \alpha$$
$$u_1{}'(0, \alpha) = \alpha, \quad u_2{}'(0, \alpha) = 2 - \alpha$$

Here, $L \equiv \frac{d^2}{dt^2}$ and by operating the two sides of the above equation with the inverse operator (namely, $L^{-1}(*) \equiv \iint(*)dtdt$) and using the initial conditions, we get

$$u_1(t, \alpha) = (1 + \alpha) + \int_0^t[(3\alpha - 6) + (\alpha - 1)x]dx + 2\int_0^t u_2(x, \alpha)dx - \int_0^t\int_0^x u_1(s, \alpha)\,dsdx$$
$$u_2(t, \alpha) = (3 - \alpha) + \int_0^t[-3\alpha + (1 - \alpha)x]dx + 2\int_0^t u_1(x, \alpha)dx - \int_0^t\int_0^x u_2(s, \alpha)\,dsdx$$

Now applying the ADM we get

$$u_{10}(t, \alpha) = (1 + \alpha) + \int_0^t[(3\alpha - 6) + (\alpha - 1)x]dx$$
$$u_{20}(t, \alpha) = (3 - \alpha) + \int_0^t[-3\alpha + (1 - \alpha)x]dx$$
$$u_{1k+1}(t, \alpha) = 2\int_0^t u_{2k}(x, \alpha)dx - \int_0^t\int_0^x u_{1k}(s, \alpha)\,dsdx, \; k \geq 0.$$
$$u_{2k+1}(t, \alpha) = 2\int_0^t u_{1k}(x, \alpha)dx - \int_0^t\int_0^x u_{2k}(s, \alpha)\,dsdx, \; k \geq 0.$$

On substituting and solving the above equation, we obtain the approximate solution after six iterations as

$$u_{10}(t, \alpha) + u_{11}(t, \alpha) + u_{12}(t, \alpha) + u_{13}(t, \alpha) + u_{14}(t, \alpha) + u_{15}(t, \alpha)$$
$$= (1 + \alpha)(1 + 3\frac{t^2}{2!} + 5\frac{t^4}{4!} - 61\frac{t^6}{6!} - 39\frac{t^8}{8!} - \frac{t^{10}}{10!}) + (3\alpha - 6)(t + 3\frac{t^3}{3!} + 5\frac{t^5}{5!} - 61\frac{t^7}{7!} - 39\frac{t^9}{9!} - \frac{t^{11}}{11!})$$

$$+(\alpha-1)\left(\frac{t^2}{2!}+3\frac{t^4}{4!}+5\frac{t^6}{6!}-61\frac{t^8}{8!}-39\frac{t^{10}}{10!}-\frac{t^{12}}{12!}\right)+(3-\alpha)(2t+4\frac{t^3}{3!}+6\frac{t^5}{5!}+72\frac{t^7}{7!}+10\frac{t^9}{9!})$$
$$-3\alpha\left(2\frac{t^2}{2!}+4\frac{t^4}{4!}+6\frac{t^6}{6!}+72\frac{t^8}{8!}+10\frac{t^{10}}{10!}\right)+(1-\alpha)(2\frac{t^3}{3!}+4\frac{t^5}{5!}+6\frac{t^7}{7!}+72\frac{t^9}{9!}+10\frac{t^{11}}{11!})$$

$$u_{20}(t,\alpha)+u_{21}(t,\alpha)+u_{22}(t,\alpha)+u_{23}(t,\alpha)+u_{24}(t,\alpha)+u_{25}(t,\alpha)$$
$$=(3-\alpha)(1+3\frac{t^2}{2!}+5\frac{t^4}{4!}-61\frac{t^6}{6!}-39\frac{t^8}{8!}-\frac{t^{10}}{10!})-3\alpha(t+3\frac{t^3}{3!}+5\frac{t^5}{5!}-61\frac{t^7}{7!}-39\frac{t^9}{9!}-\frac{t^{11}}{11!})$$
$$+(1-\alpha)\left(\frac{t^2}{2!}+3\frac{t^4}{4!}+5\frac{t^6}{6!}-61\frac{t^8}{8!}-39\frac{t^{10}}{10!}-\frac{t^{12}}{12!}\right)+(1+\alpha)(2t+4\frac{t^3}{3!}+6\frac{t^5}{5!}+72\frac{t^7}{7!}+10\frac{t^9}{9!})$$
$$+(3\alpha-6)\left(2\frac{t^2}{2!}+4\frac{t^4}{4!}+6\frac{t^6}{6!}+72\frac{t^8}{8!}+10\frac{t^{10}}{10!}\right)+(\alpha-1)(2\frac{t^3}{3!}+4\frac{t^5}{5!}+6\frac{t^7}{7!}+72\frac{t^9}{9!}+10\frac{t^{11}}{11!})$$

The comparison between the exact and approximate solutions at $t=1$ for some $\alpha\in[0,1]$ has been shown at table 5, 6, 7 and 8. We have calculated all for Table 3, Table 4 and Fig.4 data by using MATLAB

Table 3: **Exact solution** at $t=1$ for Example 2.

α	0	.2	.4	.6	.8	1
u_1	1.3504	1.6240	1.8976	2.1711	2.4447	2.7183
u_2	4.0862	3.8126	3.5390	3.2654	2.9919	2.7183

Table 4: **Approximate solution** at $t=1$ for Example 2.

α	0	.2	.4	.6	.8	1
u_1	1.3765	1.6189	1.8613	2.1037	2.3461	2.5885
u_2	3.8005	3.5581	3.3157	3.0733	2.8309	2.5885

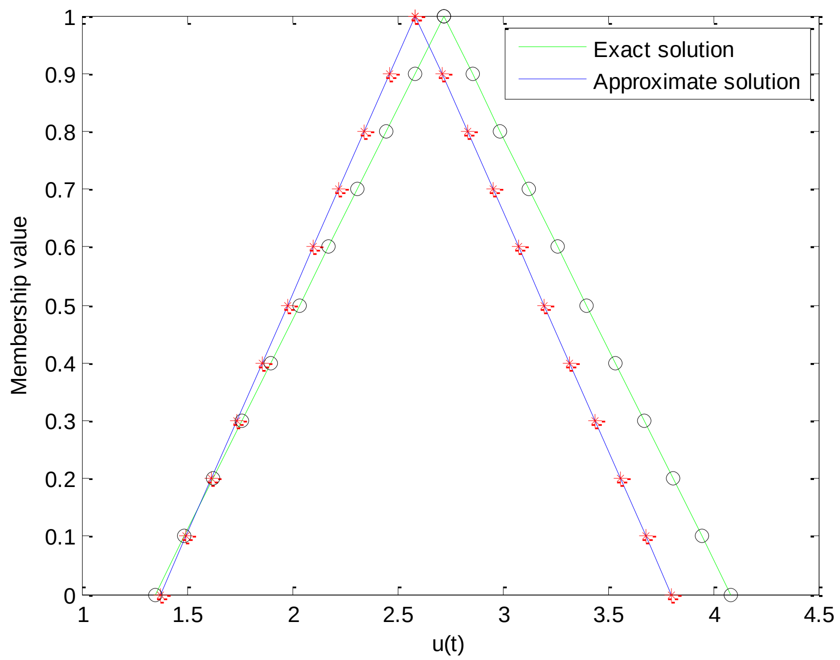

Fig.4 represents the plots of the exact solution $\tilde{u}$ and the approximate solution $\tilde{\psi}_6$ at the point $x=1$ for Example 2.

5. Conclusion

In this paper we presented fuzzy differential equation with linear differential operator which can be of any order and it also involves nonlinear functional. So our solution procedure gives the solutions of a large area of problems involving fuzzy differential equations. Note that we used ADM which gives solution even for some nonlinear problems that can’t be solved by classical methods. Future research work will be try to solve any kind of fuzzy differential equations by improving and using ADM or any other method.

6. References

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