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Adaptive step forward-backward matching pursuit algorithm

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Abstract. The sparseness adaptive matching pursuit algorithm (SAMP) is a classical algorithm based on compressed sensing theory. Aiming at reconstructing signals with unknown sparsity, an adaptive step forward-backward matching pursuit algorithm (AFBMP) is presented. The AFBMP select matching atoms in the forward processing by using logarithmic variable steps which under the frame of sparseness adaptive matching pursuit algorithm. At the beginning of iterations, high value of step size, causing fast convergence of the algorithm is used to realize the coarse approach of signal sparse, and in the later smaller value of step size is used to realize the precise reconstruction of the sparse signal which equal to half of the previous step. Then AFBMP amend the mistakes which caused in the former stage and delete part of the false atoms in the support set using the backward strategy. Finally it realizes the signal accurately approximate. Experiments show that the AFBMP algorithm can reconstruct the unknown signal more efficiently.

Keywords: Compressed Sensing, Reconstruction Algorithm, Forward-Backward, Sparseness Adaptive, Matching Pursuit Algorithm

1. Introduction

Compressed Sensing theory [1-2] (CS) is a new way to reconstruct signal or image. If the compression ratio is high, the reconstruction error is very small by the CS. In addition, it can compress the data with the data collecting, so the CS can save time and the efficiency is high.

The reconstruction algorithm is the core of the CS and its aim to achieve the original signal based on low dimensional data observed as much as possible. With the development of the CS theory, more and more reconstruction algorithms were proposed in recent years. At present, the common reconstruction algorithm can be divided into three categories: the combinatorial optimization reconstruction algorithm, the convex optimization algorithm and the greedy iterative algorithm. The reconstruction of combinatorial optimization algorithm is good, but not practical. The convex optimization algorithm is too complicated, so that it is not practical too. Such as the Basis pursuit algorithm (BP) [3] is one of the convex optimization algorithm. In recent years, the greedy iterative algorithm is more and more popular, because the accuracy of the reconstruction is high and it is convenient to realize. Such as the Matching Pursuit algorithm(MP)[4],the Orthogonal Matching Pursuit algorithm(OMP)[4-5],the Regularized Orthogonal Matching Pursuit algorithm (ROMP)[6],the Compressive sampling Matching Pursuit algorithm(CoSaMP) [7],the Subspace Pursuit algorithm(SP)[8]and the Sparseness Adaptive Matching Pursuit algorithm(SAMP)[9-10]etc. We all know the linear programming method can ensure the accuracy of the original signal in a certain number of iterations, but the structure of the algorithms is too complicated, so these algorithms can't be widely applied. The greedy iterative algorithm is famous for the fast speed of reconstruction, but we need to know the sparsity of the greedy algorithm in advance. Unfortunately, we can't get the sparsity in practice. In addition, if the sparsity is fixed, the algorithm may affect the precision of the reconstruction.

The Sparseness Adaptive Matching Pursuit algorithm (SAMP) is an improvement algorithm. It breaks through the traditional of the previous algorithm which need to know the sparisity. The algorithm solves the problem of the reconstruction in the case of unknown sparisity for the first time. The reconstruction speed of SAMP mainly depends on the choice of fixed step. If the step size is too large, the reconstruction speed is high and the reconstruction accuracy is low. At the same time, it may cause over estimation. In contrast, if the step size is too small, the reconstruction speed is slower and the reconstruction accuracy is high. Then it may cause under estimation. We all know the SAMP belongs to the forward greedy algorithm and cannot delete redundant atoms. Then some scholars propose some improved algorithms. MSAMP [11] use atom matching test firstly and get the sparsity of the signal. Then it achieves reconstruction in the framework of SAMP. But in the stage of "big step" or "small step", the step value is still a fixed value. So MSAMP may cause over estimation or under estimation. LSAMP [12] use logarithmic variable step and solve the over estimation or under estimation problem by controlling the dual threshold. The reconstruction effect is still good when the sampling rate is low. However, these algorithms are essentially forward greedy algorithms, the biggest drawback of forward greedy algorithm is that the error caused by the previous step iteration can't be modified, once the atom is selected to support set, it will not be deleted. For example, figure 1 supposes that feature vector x is formed by α_1, α_2 which in the observation matrix. While the other vector α_3 is more close to the feature, it will choose α_3 firstly and cannot delete the wrong vector. This result is not what we want. In fact, the forward greedy algorithm is available when the observation matrix is not relevant.

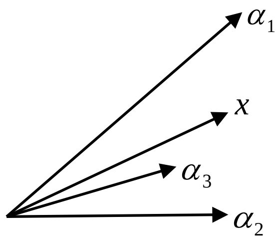

Figure 1

Now we can exploit the backward greedy algorithm to improve the shortcomings. It first selects the relevant atoms and then deletes the atoms one by one for the reconstruction error is smallest. Therefore, according to these two means, an adaptive step forward-backward matching pursuit algorithm is proposed. The new algorithm uses an improved logarithmic variable step in each stage and can delete the redundant atoms. So it can ensure the reconstruction accuracy and overcome the shortage of other algorithms.

Our paper is organized as follows: Section 2 introduces the theory of compressed sensing and reconstruction algorithm. Section 3 introduces the Sparsity Adaptive Matching Pursuit algorithm. Section 4 introduces the adaptive step forward-backward matching pursuit algorithm in detail. Section 5 compares the new algorithm with the common reconstruction algorithm by experiments. Finally, we get the conclusion.

2. The theory of compressed sensing and reconstruction algorithm

The compressed sampling theory pointed out that as long as the signal was sparse, we could reconstruct the signal by low dimensional linear observed matrix.

Suppose x is the original signal and its length is N .The sparsity is K .Set y is the observed signal and its length is M .Let $\Phi_{M\times N}(M<N)$ denotes the measurement matrix. According to the theory of Compressed Sensing, y can be expressed as:

$$y=\Phi x \tag{1}$$

What we want to know is how to reconstruct x form the observed signal y .We usually solve the following optimization problem[13-15]:

$$\min\|x\|_0, \quad s.t. \quad y=\Phi x \tag{2}$$

Obviously, if $M<<N$, equation (1) is a system of indeterminate equations. Then the equations have more than one solution. Equation (2) is a Non-deterministic Polynomial [12]. It is hard to get the solution. But when x is sparse enough and Φ meet the Restricted Isometry Property (RIP) [16-17]:

$$(1-\delta_K)\|x\|_2^2 \le \|\Phi x\|_2^2 \le (1+\delta_K)\|x\|_2^2 \tag{3}$$

Where $\delta_K \in (0,1)$ denotes restricted isometry constant degree and K is the sparsity. Now, solving equation (2) is equivalent to solve l_1 norm minimization problem:

$$\min\|x\|_1, \quad s.t. \quad y=\Phi x \tag{4}$$

The matching pursuit algorithm provides a powerful tool to achieve the solution. OMP algorithm can ensure the result of each iteration is optimal by orthogonal for the selected atoms and reduce the number of iterations. Then one scholar puts forward the regularization orthogonal matching pursuit algorithm. The algorithm selects multiple relates atoms as the candidate set each iteration. Then they use a regularization method to the candidate set and get a new support set. Finally they achieve the reconstruction of atoms. But these methods are based on the sparsity is known. In practical, signal sparsity is often unknown which brings a big challenge to signal reconstruction. So *Thong T.Do* proposed the sparsity adaptive matching pursuit algorithm for the first time and solved the problem of the sparsity was unknown. The algorithm achieves signal reconstruction by fixed step.

3. SAMP

The SAMP algorithm can achieve the reconstruction of sparse signals quickly and accurately under the sparsity is unknown. The optimal sparsity can be approached by adjusting the step a stage to a stage. Firstly, the SAMP algorithm calculates the absolute value of the inner product of the residual r and each column of the observed matrix Φ .Secondly, it selects size atoms which the correlation coefficients are optimal and combines the previous support set as candidate set. The size is the length of the support set. Then it calculate the inner product of the residual and the atoms of the candidate set and select size atoms which the correlation coefficients are optimal as a new support set F and update the residual r_{new} .If the $r_{new} > r$,enter the next stage and update the length of the support set $size = size \times step$ .Until the residual r is less than a certain threshold. If the new residual less than the residual, you should continue the iteration.

The core of the algorithm as follows:

(1) Initialization: residual $r = y$, the length of the support set $size = step$, $stage = 1$, index set $S = \varnothing$, the number of iteration is 1,the support set $F = \varnothing$.

(2) Calculate the absolute value of the inner product of the residual r and each column of the observed matrix Φ . Then select size atoms which the correlation coefficients are supreme as the index set S .

(3) Combine the index set and the support set as the candidate set $C = F \cup S$. Then calculate the absolute value of the inner product of the residual and the candidate set. At last, select size atoms which the correlation coefficients are supreme as a new support set F .

(4) Update the residual $r_{new} = y - \Phi_F \Phi_F^{'} y$.

(5) If $\|r_{new}\|_2 \geq \|r\|_2$, update the iterative phase $stage = stage + 1$ and the length of the support set $size = stage \times step$,then go to step (2). Or update the support set $F = F_{new}$ and the residual $r = r_{new}$. Let the number of iteration k=k+1.go to step (2).

We know the step size of each stage form the above algorithm is constant. So the selection of the step size is the key to the reconstruction accuracy of the algorithm. But the biggest drawback of the algorithm is that it can't delete the wrong atoms and produce some errors. In order to solve these problems, this paper proposed an adaptive step forward-backward matching pursuit algorithm based on SAMP.

4. Adaptive step forward-backward matching pursuit algorithm

The SAMP breaks the limitation of the traditional for sparsity. But the step size of each stage is fixed. Related experiments show that when the initial step is large, the algorithm needs less number of iterations. The efficiency of the algorithm is high but the precision of the sparsity will drop. On the contrary, when the initial step is small, the algorithm needs more iterations and the precision of the sparsity is high. So we may miss the best sparsity when the step is fixed. How to select a suitable step is the key to improve the algorithm of signal reconstruction precision. The experiments of the literature [12] show that with the increasing of the support set without more than real sparsity, the signal reconstruction energy difference of two adjacent iteratives is more and more small. These Experiments tell us with the increase of the number of iterations, we should increase the step size slowly and use "small step" instead of "large step" approaching the sparsity.

At the same time, with the increasing of the number of iterations the support set capacity is increasing. This will produce some unnecessary atoms in the support set. So according to the characteristics of the logarithmic function and the processing of SAMP, we propose an adaptive step forward-backward matching pursuit algorithm. The new algorithm adds the ideas of backward eliminating redundancy atoms and makes up for the greedy algorithm. Next we will describe the new algorithm from two aspects: step selection and algorithm steps.

4.1 Step Selection

Literature [12] shows that when the support set size do not reach the real sparsity, the signal energy difference of two adjacent iterative decreasing rapidly initially. Then it tends to be stable. Inspired by this thought, we adopt variable step. In the initial iterative phase, we use the "large step". With the increase of the number of iterations, we adopt the "small step" instead of "large step". Then we set two thresholds $\varepsilon_1, \varepsilon_2$. If $\|x_t - x_{t-1}\| > \varepsilon_1$, we use "large step". If $\varepsilon_2 < \|x_t - x_{t-1}\| < \varepsilon_1$, we use "small step". If $\|x_t - x_{t-1}\| < \varepsilon_2$, it shows that the energy difference tends to stable and we can make it as a condition of termination.

Based on the above ideas and the characteristics of the logarithmic function, we suppose the "large step":

$$step(s) = a \log_2 s + b \tag{5}$$

where $s \in [1, stage]$. When s=1, the initial step is $step(1) = \frac{c \cdot M}{2 \cdot \log_2 N}$.The maximum step size is equal to the length of the signal $step(stage) = N$.So we can solve (5) and obtain a function of the step:

$$step(s) = \frac{N - \frac{c \cdot M}{2 \cdot \log_2 N}}{\log_2 stage} \log_2 s + \frac{c \cdot M}{2 \cdot \log_2 N} \tag{6}$$

When we need “small step”, we can put the half step of the current stage as the “small step”:

$$step(s) = \frac{1}{2}\left[\frac{N - \frac{c \cdot M}{2 \cdot \log_2 N}}{\log_2 stage} \log_2 s + \frac{c \cdot M}{2 \cdot \log_2 N}\right] \tag{7}$$

Though setting suitable thresholds, we can get the step of each stage:

$$step(s) = \begin{cases} \frac{N - \frac{c \cdot M}{2 \cdot \log_2 N}}{\log_2 stage} \log_2 s + \frac{c \cdot M}{2 \cdot \log_2 N} & \text{当}\|x_t - x_{t-1}\| > \varepsilon_1\text{时} \\ \frac{1}{2}\left[\frac{N - \frac{c \cdot M}{2 \cdot \log_2 N}}{\log_2 stage} \log_2 s + \frac{c \cdot M}{2 \cdot \log_2 N}\right] & \text{当}\varepsilon_2 < \|x_t - x_{t-1}\| < \varepsilon_1\text{时} \end{cases}$$

All steps are positive integers. $c \ge 1 (c \in N^+)$.M represents the length of the observed signal and N represents the length of the estimated signal. $\varepsilon_1 > \varepsilon_2$.

4.2 . Algorithm Steps

(1) Initialization: residual $r = y$, the support set F, the length of the support set $size = step$, $stage = 1$, the number of iteration t=1, index set $S = \varnothing$,the candidate set $C = \varnothing$;

(2) Calculate the correlation coefficients. Then select size atoms which the correlation coefficients are optimal as the index set S.

(3) Combine the index set and the support set as the candidate set $C = F \cup S$. Then calculate the absolute value of the inner product of the residual and the candidate set C. At last, select size atoms which the correlation coefficients are optimal as a new support set F.

(4) Then we can get $x_t = \arg\min \|y - \Phi_F x_{t-1}\|_2$. Update the residual $r_t = y - \Phi_t x_{t-1}$.

(5) If $\|x_t - x_{t-1}\| > \varepsilon_1$ go to step (6), or go to step (8).

(6) If $\|r_t\| \ge \|r_{t-1}\|$ go to step (7), or go to step (9).

(7) Enter the next stage. $stage = stage + 1$.We can get a new step by formula (6) and expand the support set $size = size + step$, $t = t + 1$, return to step (2).

(8) If $\|x_t - x_{t-1}\| < \varepsilon_2$, stop the iteration, or go to step (10).

(9) Update the support and residual. Then return to step (2).

(10) Enter the “small step”. $stage = stage + 1$.We can get a new step by formula (7) and expand the support set $size = size + step$, $t = t + 1$, return to step (2).

(11) Suppose $q(F) = (1/2)\|y - \Phi_F x_t\|_2^2$.

(12) $j = \arg\min_{i \in F} q(F / i)$.

(13) $d^- = q(F / j) - q(F)$.

(14) $d^+ = \varepsilon_3$.

(15) If $d^{-} < d^{+}$,delete x_j and update the support set. Then return to (11),until $d^{-} > d^{+}$.

The step 2-10 select the relevant atoms and the step 11-15 delete the wrong atoms. The new algorithm has two advantages. One is that the adaptive step can avoid over estimation and under estimation problem. It adds to backward step so that the algorithm can delete some wrong atoms. The new algorithm not only can improve the accuracy of the reconstruction, but also can reduce the reconstruction time.

5. Experiments and analysis

In order to verify the validity of the new algorithm, we choose the MATLAB as the processing platform. We use one dimensional Gaussian sparse signal and its length is N=256. The observed matrix is the Fast Fourier Transform matrix (FFT). $\varepsilon_1 = e^{-6}$. $\varepsilon_2 = e^{-12}$. $\varepsilon_3 = e^{-14}$. In order to illustrate the effectiveness of the new algorithm further, we compare SAMP, MSAMP, LSAMP with the new algorithm(AFBMP) under different sampling rate of refactoring. All the parameters of the other algorithms in this paper are set according to the original reference.

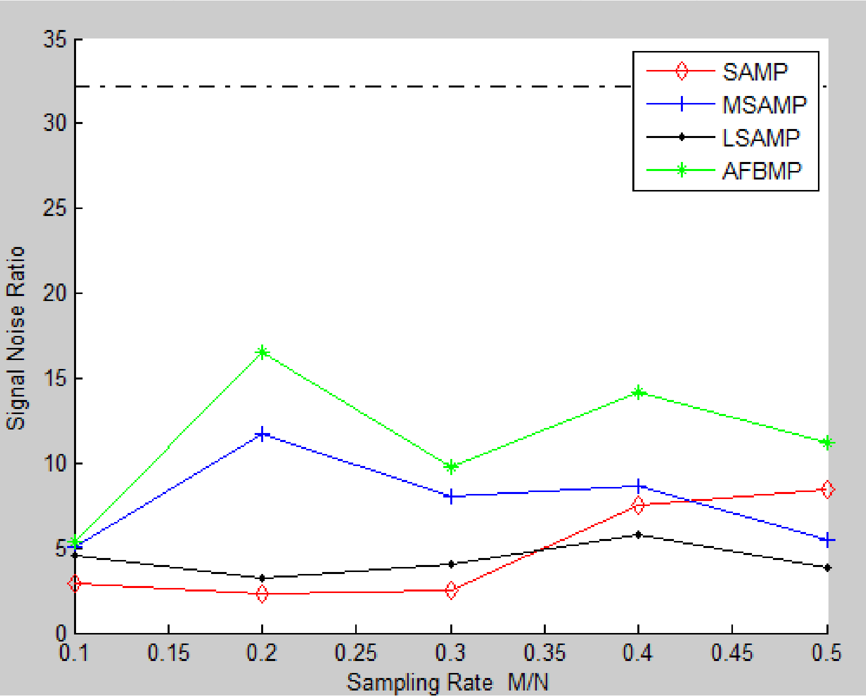

Figure 2 the signal noise ratio of the reconstructed signal under different sampling rate

Figure 2 denotes the signal noise ratio of some algorithms under different sampling rate. The horizontal ordinate represents the sampling rate and the longitudinal coordinate represents the signal noise ratio (SNR). The results of the experiment show that the algorithm enhances the performance by using the logarithmic steps and adding the backward thought. The algorithm enhances the precision of the original signal reconstruction significantly.

Figure 3 represents the refactoring relative error of four algorithms under different sampling rate. The experiments show that the relative error is high of the previous algorithms. We know MSAMP and LSAMP enhance the performance of the reconstruction and reduce the relative error. The new algorithm combines the advantages of previous algorithms and makes the relative error lower. So we can say the new algorithm guarantee the reconstruction accuracy. The experiments also prove the effectiveness of the algorithm.

In order to explain the application of the new algorithm to the practical problems, we test some images which the size is 256*256. Then we compare SAMP, MSAMP with AFBMP. Figure 4 shows that the improved algorithm is better than the classical algorithm, and the reconstruction effect of the algorithm is better than other algorithms.

Table 1 is the running time of the three sets of test charts. The sampling rate is 0.5. We can see that in the same sampling rate, the reconstruction time of different algorithms is different, and the reconstruction time of the algorithm is lower than that of the traditional algorithm.

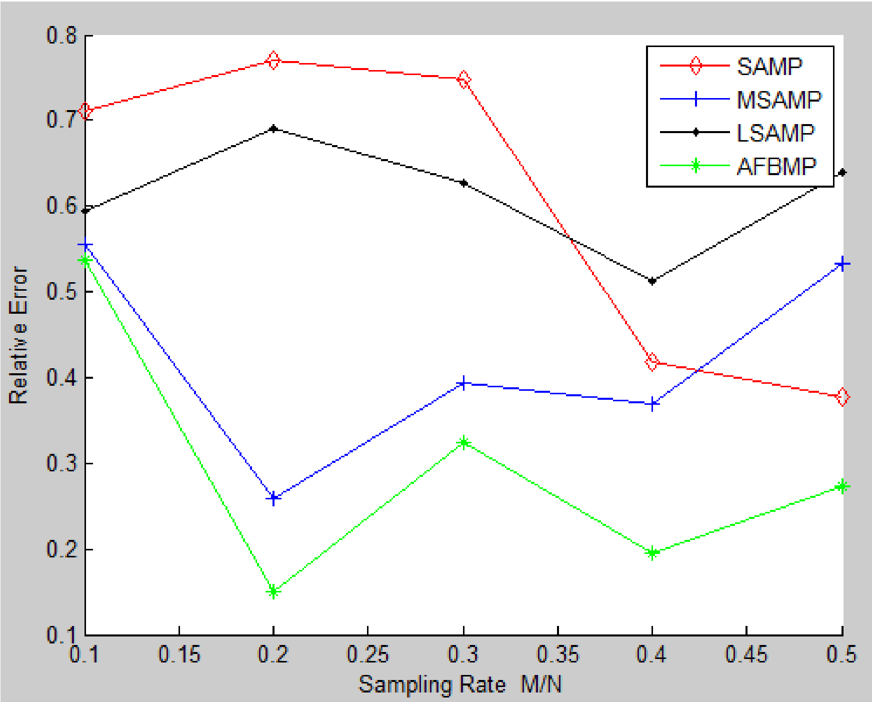

Figure 3 the relative error of the algorithms under different sampling rate

Figure 4 Two dimensional image reconstruction effect

	SAMP	MSAMP	AFBMP
test1	55.338	56.173	54.568
test2	54.389	53.522	54.095
test3	54.285	53.771	54.116

Table 1 reconstruction time of different image reconstruction algorithms

6. Conclusions

This paper studies the classic algorithm of the compressed sensing theory deeply. According to the frame of SAMP, this paper proposes an improved backward and step adaptive matching pursuit algorithm under the condition of the sparsity is unknown. SAMP may lead to over estimation or under estimation problem using the fixed step and SAMP cannot delete the redundant atoms after the iterations. According to these questions, we use the logarithmic step and backward thought under the frame of SAMP. The new algorithm set two thresholds to control the step selection and stop criterion. Then realize the sparsity of approximation gradually and finish the task of reconstruction of signal. The experiments show that the new algorithm can well realize the reconstruction under the sparsity is unknown and the quality of the reconstruction is superior to other algorithms.

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