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Finite-time chaos synchronization of the delay hyperchaotic Lü system with uncertain parameters

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Abstract. This paper deals with the finite-time chaos synchronization of the delay hyperchaotic Lü systen with uncertain parameters. Based on the finite time stability theory, a control law is proposed to realize finite-time chaos synchronization for the delay hyperchaotic Lü systen with uncertain parameters. The controller is simple, robust and only part parameters are required to be bounded. Numerical simulation results are given to demonstrate the effectiveness of the proposed finite-time chaos synchronization scheme.

Keywords: Finite-time chaos synchronization; delay hyperchaotic Lü system ; Uncertain parameters.

1. Introduction

Chaos synchronization has attracted a lot of attention from a variety of research fields since the seminal work of Pecora and Carroll [1]. From then on, chaos synchronization has been developed extensively and intensively due to its potential applications in secure communication [2, 3], complex networks [4-7], biotic science [8-14] and so on [15].

Nowadays, different techniques and methods have been put forward to achieve chaos synchronization, for instance, linear and nonlinear feedback synchronization method [16-19], impulsive synchronization method [20-22], tracking synchronization method [23-25], among many others [26-32].

As time goes on, more and more people began to realize the important role of synchronization time. To attain convergence speed, many effective methods have been introduced and finite-time control is one of them. Finite-time synchronization means the optimality in convergence time. Moreover, the finite-time control techniques have demonstrated better robustness and disturbance rejection properties [33-37].

In this paper, we present a controller to realize finite-time synchronization of he delay hyperchaotic Lü systen with uncertain parameters. The controller is robust to parameter uncertainties and simple to be constructed.

2. Prelininary definitions and lemmas

Finite-time synchronization means that the state of the slave system can track the state of the master system after a finite-time. The precise definition of finite-time synchronization is given below.

Definition 1. Consider the following two chaotic systems:

$$\dot{x}_m = f(x_m),$$
$$\dot{x}_s = h(x_m, x_s), \tag{1}$$

where x_m, x_s are two n-dimensional state vectors. The subscripts 'm' and 's' stand for the master and slave systems, respectively. $f: R^n \to R^n$ and $h: R^n \to R^n$ are vector-valued functions. If there exists a constant $T > 0$, such that

$$\lim_{t\to T}\|x_m - x_s\| = 0,$$

and $\|x_m - x_s\| \equiv 0,$ if $t \ge T$, then synchronization of the system (1) is achieved in a finite-time.

Lemma 1 [37]**.** Assume that a continuous, positive-definite function $V(t)$ satisfies the following differential inequality:

$$\dot{V}(t) \le -cV^{\eta}(t),\ \forall t \ge t_0, V(t_0) \ge 0. \tag{2}$$

Where $c > 0, 0 < \eta < 1$ are all constants. Then, for any given t_0, $V(t)$ satisfies the following inequality:

$$V^{1-\eta}(t) \le V^{1-\eta}(t_0) - c(1-\eta)(t-t_0),\ t_0 \le t \le t_1, \tag{3}$$

and

$$V(t) \equiv 0, \forall t \ge t_1 \tag{4}$$

with t_1 given by

$$t_1 = t_0 + \frac{V^{1-\eta}(t_0)}{c(1-\eta)}. \tag{5}$$

Proof. Consider the following differential equation:

$$\dot{X}(t) = -cX^{\eta}(t),\ X(t_0) = V(t_0). \tag{6}$$

Although this differential equation does not satisfy the global Lipschitz condition, the unique solution of Eq.(6) can be found as

$$X^{1-\eta}(t) = X^{1-\eta}(t_0) - c(1-\eta)(t-t_0). \tag{7}$$

Therefore, form the comparison Lemma, one obtains

$$V^{1-\eta}(t) \le V^{1-\eta}(t_0) - c(1-\eta)(t-t_0),\ t_0 \le t \le t_1, \tag{8}$$

and

$$V(t) \equiv 0, \forall t \ge t_1$$

with t_1 given in (5).

Lemma 2. For $a_1, a_2, \ldots, a_n \in R$, the following inequality holds:

$$|a_1| + |a_2| + \cdots + |a_n| \ge \sqrt{a_1^{\ 2} + a_2^{\ 2} + \ldots + a_n^{\ 2}} \tag{9}$$

3. Main results

A chaotic system is extremely sensitive to its initial condition and minor variations of parameters. In actual situation, the system is disturbed by parameter variation which cannot be exactly predicted. The consequence of these uncertainties will destroy the synchronization and even break it. Therefore, it is important and necessary to study the synchronization of systems with uncertainties. In this section, we first discuss finite-time synchronization of the hyperchaotic L ü system. Then , we turn the problem to the system with uncertain parameters.

3.1 chaos synchronization of hyperchaotic L ü system

The delay hyperchaotic L ü system (10) [38]was constructed from the L ü system. The form of the delay hyperchaotic L ü system is given by

$$\begin{aligned}
\dot{x}_1 &= a(x_2 - x_1), \\
\dot{x}_2 &= cx_2 - x_1x_3 + x_4(t-\tau), \\
\dot{x}_3 &= x_1x_2 - bx_3, \\
\dot{x}_4 &= -\alpha_1 x_1 - \alpha_2 x_2 ,
\end{aligned} \tag{10}$$

where a, b, c, α_1, α_2, τ are real positive constants. System (10) is considered as the master system and the slave system is a controlled system as follows:

$$\begin{aligned}
\dot{y}_1 &= a(y_2 - y_1) + u_1, \\
\dot{y}_2 &= cy_2 - y_1y_3 + y_4(t-\tau) + u_2 \\
\dot{y}_3 &= y_1y_2 - by_3 + u_3 \\
\dot{y}_4 &= -\alpha_1 y_1 - \alpha_2 y_2 + u_4.
\end{aligned} \tag{11}$$

Denote $e_1 = y_1 - x_1, e_2 = y_2 - x_2, e_3 = y_3 - x_3, e_4 = y_4 - x_4$. Subtracting Eq.(10) from Subtracting Eq.(11) we can get the following error system:

$$\begin{aligned}
\dot{e}_1 &= a(e_2 - e_1) + u_1, \\
\dot{e}_2 &= ce_2 + e_1e_3 - e_1y_3 - y_1e_3 + e_4(t-\tau) + u_2,
\end{aligned}$$

$$\dot{e}_3 = -e_1e_2 + e_1y_2 + y_1e_2 - be_3 + u_3,$$
$$\dot{e}_4 = -\alpha_1e_1 - \alpha_2e_2 + u_4. \tag{12}$$

Our aim is to design a controller that can achieve the finite-time synchronization of the hyperchaotic Lü system (10) and the controlled system (11). The problem can be converted to design a controller to attain finite-time stable of the error system (12). The design procedure consists of two steps as follows.

Step 1:Let $u_4 = \alpha_1e_1 + \alpha_2e_2 - e_4 - sgn(e_4)$. Substituting the control input u_4 into the equation of (12) yields

$$\dot{e}_4 = -e_4 - sgn(e_4). \tag{13}$$

Choose a candidate Lyaupunov function

$$V_1 = \frac{1}{2}e_4^{\ 2}. \tag{14}$$

The derivative of V_1 along the trajectory of (12) is

$$\dot{V}_1 = e_4\dot{e}_4 = e_4(-e_4 - sgn(e_4)) = -e_4^{\ 2} - |e_4| \le -|e_4| = -2^{\frac{1}{2}}(\frac{1}{2}e_4^{\ 2})^{\frac{1}{2}} = -2^{\frac{1}{2}}V_1^{\frac{1}{2}} \tag{15}$$

From Lemma 1, the system (15) is finite-time stable. That means there is a $T_1 > 0$ such that $e_4 \equiv 0, e_4(t-\tau) \equiv 0$ provided that $t \ge T_1 + \tau$.

Step 2:Select $u_1 = -ae_2 - sgn(e_1)$, $u_2 = -ke_2 - y_3e_1sgn(e_1e_2y_3) - sgn(e_2)$ with $k \ge c$, $u_3 = -y_2e_1sgn(e_1y_2e_3) - sgn(e_3)$. When $t > T_1 + \tau$,the last three equations of system (12) become

$$\dot{e}_1 = a(e_2 - e_1) + u_1,$$
$$\dot{e}_2 = ce_2 + e_1e_3 - e_1y_3 - y_1e_3 + u_2,$$
$$\dot{e}_3 = -e_1e_2 + e_1y_2 + y_1e_2 - be_3 + u_3, \tag{16}$$

Choose a candidate Lyaupunov function for the syetem (16) as follows:

$$V_2 = \frac{1}{2}e_1^{\ 2} + \frac{1}{2}e_2^{\ 2} + \frac{1}{2}e_3^{\ 2}. \tag{17}$$

The derivative of V_2 along the trajectory of (16) is

$$\begin{aligned}
\dot{V}_2 &= e_1\dot{e}_1 + e_2\dot{e}_2 + e_3\dot{e}_3 = ae_1e_2 + e_1u_1 - ae_1^{\ 2} + ce_2^{\ 2} - e_1e_2y_3 + e_2u_2 + e_1y_2e_3 - be_3^{\ 2} + e_3u_3 \\
&\le ae_1e_2 + e_1u_1 + ce_2^{\ 2} - e_1e_2y_3 + e_2u_2 + e_1y_2e_3 + e_3u_3 \\
&= -|e_1| - (k-c)e_2^{\ 2} - (e_1e_2y_3 + |e_1e_2y_3|) - |e_2| - (|e_1y_2e_3| - e_1y_2e_3) - |e_3| \\
&\le -|e_1| - |e_2| - |e_3| \le -2^{\frac{1}{2}}(\frac{1}{2}e_1^{\ 2} + \frac{1}{2}e_2^{\ 2} + \frac{1}{2}e_3^{\ 2})^{\frac{1}{2}}
\end{aligned} \tag{18}$$

In proof of (18) we have used lemma 2 at the inequality before the last equation. Then from Lemma 1, the error system (18) is finite-time stable. That is to say $e_1 \equiv 0, e_2 \equiv 0, e_3 \equiv 0$ after a finite-time T_2. So when $t > T_2$, $y_1 \equiv x_1$, $y_2 \equiv x_2$, $y_3 \equiv x_3$, $y_4 \equiv x_4$, i.e. the slave system (11) finite-timely synchronizes the master the master system (10) by the controller $u_1 = -ae_2 - sgn(e_1)$, $u_2 = -ke_2 - y_3e_1sgn(e_1e_2y_3) - sgn(e_2)$, $u_3 = -y_2e_1sgn(e_1y_2e_3) - sgn(e_3)$, $u_4 = \alpha_1e_1 + \alpha_2e_2 - e_4 - sgn(e_4)$.

3.2 chaos synchronization of hyperchaotic Lü system with uncertain parameters

This subsection deals with the case of uncertain hyperchaotic Lü system. It is valuable because practical systems often disturbed by different factors. It is assumed that both the master system and the slave system hold uncertainties. Consider the following hyperchaotic Lü system with uncertain parameters:

$$\dot{\hat{x}}_1 = (a + \triangleleft_1)(\hat{x}_2 - \hat{x}_1),$$
$$\dot{\hat{x}}_2 = (c + \triangleleft_2)\hat{x}_2 - \hat{x}_1\hat{x}_3 + \hat{x}_4(t-\tau),$$
$$\dot{\hat{x}}_3 = \hat{x}_1\hat{x}_2 - (b + \triangleleft_3)\hat{x}_3,$$

$$\dot{\hat{x}}_4 = -\alpha_1\hat{x}_1 - \alpha_2\hat{x}_2 \ , \quad (19)$$

where $\triangleleft_i \ (i=1,2,3)$ denote the bounded uncertain parameters, i.e $|\triangleleft_i| \le \rho_i (i=1,2,3)$. The slave system with uncertain parameters is described by

$$\dot{\hat{y}}_1 = (a+\triangleleft_4)(\hat{y}_2 - \hat{y}_1) + v_1,$$
$$\dot{\hat{y}}_2 = (c+\triangleleft_5)\hat{y}_2 - \hat{y}_1\hat{y}_3 + \hat{y}_4(t-\tau) + v_2,$$
$$\dot{\hat{y}}_3 = \hat{y}_1\hat{y}_2 - (b+\triangleleft_6)\hat{y}_3 + v_3,$$
$$\dot{\hat{y}}_4 = -\alpha_1\hat{y}_1 - \alpha_2\hat{y}_2 + v_4. \quad (20)$$

where $\triangleleft_i \ (i=4,5,6)$ denote the bounded uncertain parameters in slave system, i.e $|\triangleleft_i| \le \rho_i (i=4,5,6)$. $v_i (i=1,2,3,4)$ denote the control inputs of the slave system. Denote $z = \hat{y} - \hat{x}$. Subtracting Eq.(19) from Subtracting Eq.(20) we can get the following error system:

$$\dot{z}_1 = (a+\triangleleft_1)(z_2 - z_1) + (\triangleleft_4 - \triangleleft_1)(\hat{y}_2 - \hat{y}_1) + v_1,$$
$$\dot{z}_2 = (c+\triangleleft_2)z_2 + (\triangleleft_5 - \triangleleft_2)\hat{y}_2 + z_1z_3 - z_1\hat{y}_3 - z_3\hat{y}_1 + z_4(t-\tau) + v_2,$$
$$\dot{z}_3 = -z_1z_2 + z_1\hat{y}_2 + z_2\hat{y}_1 - (b+\triangleleft_3)z_3 - (\triangleleft_6 - \triangleleft_3)\hat{y}_3 + v_3,$$
$$\dot{z}_4 = -\alpha_1 z_1 - \alpha_2 z_2 + v_4. \quad (21)$$

The design of synchronization for the uncertain system is also divided into two steps.

Step 1: Let $v_4 = \alpha_1 z_1 + \alpha_2 z_2 - z_4 - sgn(z_4)$. Substituting the control v_4 into the equation of (21) yields

$$\dot{z}_4 = -z_4 - sgn(z_4). \quad (22)$$

As can be seen from the proof of system (13), system (22) is finite-time stable at a finite-time T_3.

Step 2: When $t > T_3 + \tau$, $z_4 \equiv 0, z_4(t-\tau) \equiv 0$. The last three equations of system (21) become

$$\dot{z}_1 = (a+\triangleleft_1)(z_2 - z_1) + (\triangleleft_4 - \triangleleft_1)(\hat{y}_2 - \hat{y}_1) + v_1,$$
$$\dot{z}_2 = (c+\triangleleft_2)z_2 + (\triangleleft_5 - \triangleleft_2)\hat{y}_2 + z_1z_3 - z_1\hat{y}_3 - z_3\hat{y}_1 + v_2,$$
$$\dot{z}_3 = -z_1z_2 + z_1\hat{y}_2 + z_2\hat{y}_1 - (b+\triangleleft_3)z_3 - (\triangleleft_6 - \triangleleft_3)\hat{y}_3 + v_3 \quad (23)$$

Now, we take $v_1 = -L_1 z_1 - sgn(z_1) - \lambda_1 z_2 sgn(z_1z_2) - \lambda_2\hat{y}_2 sgn(z_1\hat{y}_2) - \lambda_3\hat{y}_1 sgn(\hat{y}_1 z_1)$, where $L_1 \ge -a + \rho_1$, $\lambda_1 \ge \rho_1 + a$, $\lambda_2 \ge \rho_4 + \rho_1$ and $\lambda_3 \ge \rho_4 + \rho_1$

$v_2 = -L_2 z_2 - sgn(z_2) - \lambda_4\hat{y}_2 sgn(\hat{y}_2 z_2) - \lambda_5 |z_1\hat{y}_3| sgn(z_2)$, where $L_2 \ge c + \rho_2$, $\lambda_4 \ge \rho_5 + \rho_2$, $\lambda_5 \ge 1$ and $v_3 = -L_3 z_3 - \lambda_6 |z_1\hat{y}_2| sgn(z_3) - \lambda_7\hat{y}_3 sgn(\hat{y}_3 z_3) - sgn(z_3)$, where $L_3 \ge -b + \rho_3, \lambda_6 \ge 1, \lambda_7 \ge \rho_3 + \rho_6$.

Take v_1, v_2 and v_3 in to the system (23) and consider the following candidate Lyaupunov function:

$$V_3 = \frac{1}{2}z_1^{\ 2} + \frac{1}{2}z_2^{\ 2} + \frac{1}{2}z_3^{\ 2}. \quad (24)$$

The derivative of V_3 along the trajectory of (23) is

$$\begin{aligned}
\dot{V}_3 &= z_1\dot{z}_1 + z_2\dot{z}_2 + z_3\dot{z}_3 \\
&= -(L_1 + a + \triangleleft_1)z_1^{\ 2} - |z_1| - [\lambda_1 - (a+\triangleleft_1)sgn(z_1z_2)]|z_1z_2| - [\lambda_2 - (\triangleleft_4 - \triangleleft_1)sgn(z_1\hat{y}_2)]|\hat{y}_2 z_1| \\
&\quad -[\lambda_3 + (\triangleleft_4 - \triangleleft_1)sgn(\hat{y}_1 z_1)]|\hat{y}_1 z_1| - [L_2 - (c+\triangleleft_2)]z_2^{\ 2} - [\lambda_4 - (\triangleleft_5 - \triangleleft_2)sgn(\hat{y}_2 z_2)]|\hat{y}_2 z_2| \\
&\quad -[\lambda_5 + sgn(\hat{y}_3 z_1 z_2)]|\hat{y}_3 z_1 z_2| - |z_2| \\
&\quad -(L_3 + b + \triangleleft_3)z_3^{\ 2} - |z_3| - [\lambda_6 - sgn(z_1\hat{y}_2 z_3)]|z_1\hat{y}_2 z_3| - [\lambda_7 + (\triangleleft_6 - \triangleleft_3)sgn(z_3\hat{y}_3)]|\hat{y}_3 z_3| \ . \\
&\le -|z_1| - |z_2| - |z_3| \le -2^{\frac{1}{2}}(\frac{1}{2}z_1^{\ 2} + \frac{1}{2}z_2^{\ 2} + \frac{1}{2}z_3^{\ 2})^{\frac{1}{2}}
\end{aligned} \quad (25)$$

From Lemma 1, the error system (23) is finite-time stable. Thus, the uncertain slave syetem (20) can finite-timely synchronize the uncertain master system (19).

4. Simulation results

In this section, the fourth-order Runge-Kutta method is used to solve the system of differential equations with step size 0.001 in all numerical simulations. Choose initial conditions of the master system $(x_1(0), x_2(0), x_3(0), x_4(0)) = (-2.0, 0.4, 2.1, 3.0)$, and of the slave system $(y_1(0), y_2(0), y_3(0), y_4(0)) = (-2.0, 0.4, 2.1, 3.0)$. The parameters of the system are chosen as $a = 35$, $b = 3$, $c = 20$, $\alpha_1 = 2$, $\alpha_2 = 2$, $\tau = 1$, $k = 20$. Fig.1 show the error response of the delay hyperchaotic Lü system.

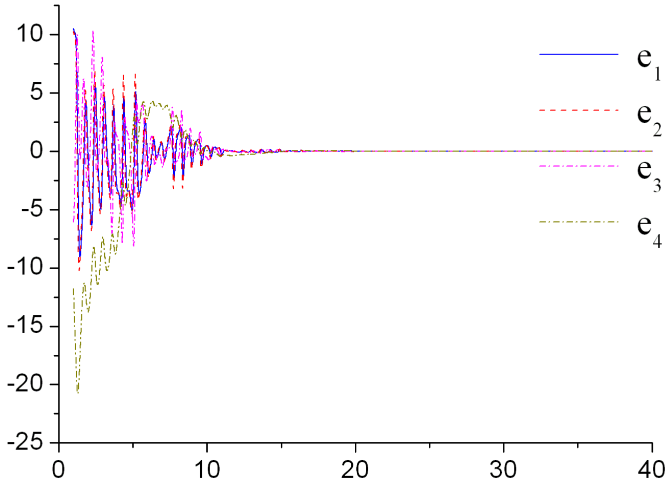

Fig.1.Synchronization errors of the delay hyperchaotic Lü system.

We now turn to the hyperchaotic Lü system with uncertain parameters, the uncertainties are adopted as $\triangleleft_1 = \sin t$, $\triangleleft_2 = \sin \hat{x}_2$, $\triangleleft_3 = \cos \hat{x}_3$, $\triangleleft_4 = \cos t$, $\triangleleft_5 = \cos \hat{y}_2$ $\triangleleft_6 = 0.2$, $L_1 = -34$, $L_2 = 21$, $L_3 = -2$, $\lambda_1 = 36$, $\lambda_2 = 4$, $\lambda_3 = 2$, $\lambda_4 = 2$, $\lambda_5 = 1$, $\lambda_6 = 1$, $\lambda_7 = 2$. Fig.2 show the error responses of the delay hyperchaotic Lü system with uncertain parameters. We can see the delay hyperchaotic Lü system with uncertain parameters have strong robustness to the uncertainties.

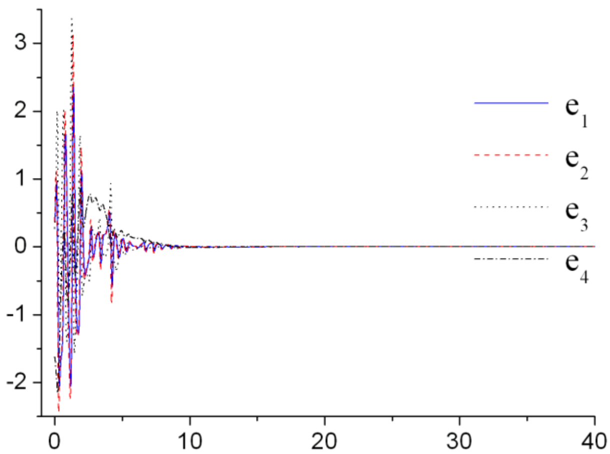

Fig.2.Synchronization errors of the delay hyperchaotic Lü system with uncertain parameters.

5. Conclusion

This paper is concerned with finite-time synchronization of the delay hyperchaotic Lü systen with uncertain parameters. The presented controller has strong robustness to uncertainties. Not all the boundaries of the uncertainties are needed for the design of the controller. From the proof process we can see that this method can be extended to other systems.

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7. References

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